Programm- & Systemverifikation

Coverage Criteria

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184.741

What happened so far

- How bugs come into being:
  - **Fault** – cause of an error (e.g., mistake in coding)
  - **Error** – *incorrect* state that may lead to failure
  - **Failure** – deviation from *desired* behaviour
- We specified *intended* behaviour using **assertions**
- We proved (simple) programs correct.
- We learned about black-box testing
  - equivalence partitioning
  - boundary testing

Black-box Testing

- Mainly applicable to higher levels of testing
  - Acceptance Testing
  - System Testing
- Focus on <u>what</u> the software does (not how it does it)
- Derive input equivalence classes by speculating on *behaviour*

Black-box Testing

```
float sqrt (float x);
```

pre: $\texttt{x} \geq 0$

post: $|\texttt{result}^2 - \texttt{x}| < \varepsilon$

Test cases from valid equivalence classes:

- $+0$, -0, `FLT_MAX`, `FLT_EPSILON`, 15.3

Test cases from invalid equivalence classes:

- `FLT_MIN`, -`FLT_EPSILON`, -7.9
- $-\infty$, $+\infty$
- `NaN`

How is `sqrt` implemented in practice?

Quake 3 implementation of inverse square root:

```c
float Q_rsqrt( float number )
{
    long i;
    float x2, y;
    const float threehalfs = 1.5F;

    x2 = number * 0.5F;
    y  = number;
    i  = * ( long * ) &y;                       // evil floating point bit level hacking
    i  = 0x5f3759df - ( i >> 1 );               // what the fuck?
    y  = * ( float * ) &i;
    y  = y * ( threehalfs - ( x2 * y * y ) );   // 1st iteration
//  y  = y * ( threehalfs - ( x2 * y * y ) );   // 2nd iteration, this can be removed

    return y;
}
```

(Original comments)

link to corresponding tweet

How is sqrt implemented in practice?

Quake 3 implementation of inverse square root:

```
float InvSqrt(float x)
      {
      float xhalf = 0.5f*x;
      int i = *(int*)&x;         // get bits for floating value
      i = 0x5f3759df - (i>>1); // gives initial guess y0
      x = *(float*)&i;           // convert bits back to float
      x = x*(1.5f-xhalf*x*x);  // Newton step, repeating increases accuracy
      return x;
      }
```

(cleaned-up version)

link to corresponding tweet

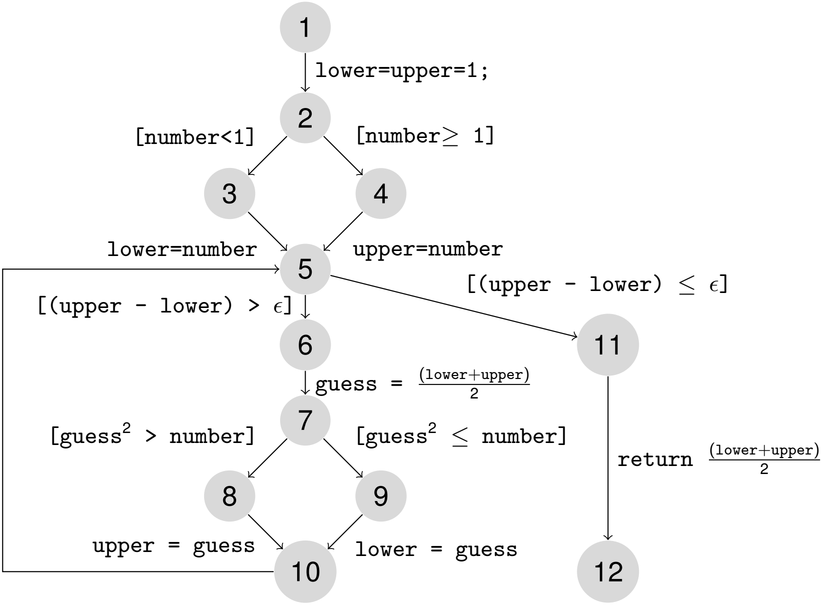
Testing our own (Non-Quake) Square-Root Implementation

```
float sqrt (float number) {
  float lower = 1, upper = 1, guess;

  if (number < 1)
    lower = number;
  else
    upper = number;

  while ((upper - lower) > EPSILON) {
    guess = (lower + upper) / 2;
    if (guess*guess > number)
      upper = guess;
    else
      lower = guess;
  }
  return (lower + upper) / 2;
}
```

Testing our Square-Root Implementation



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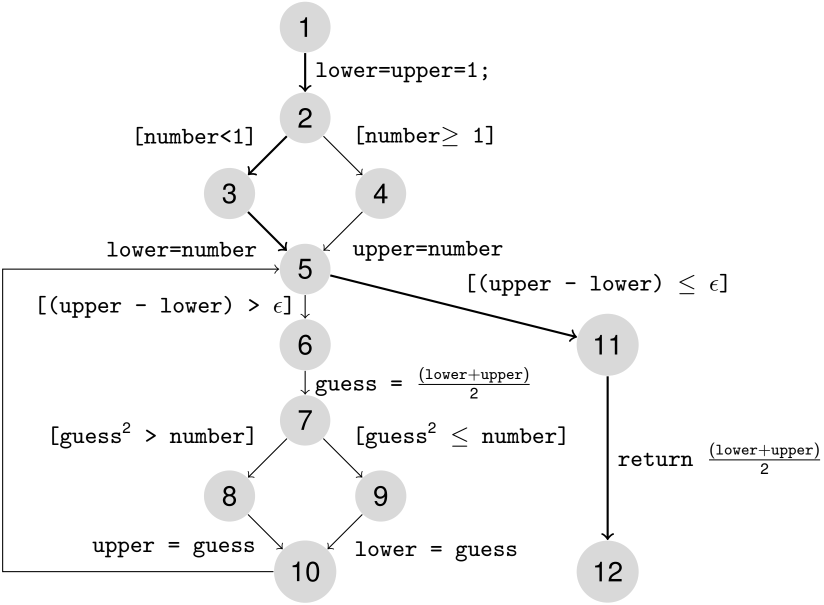

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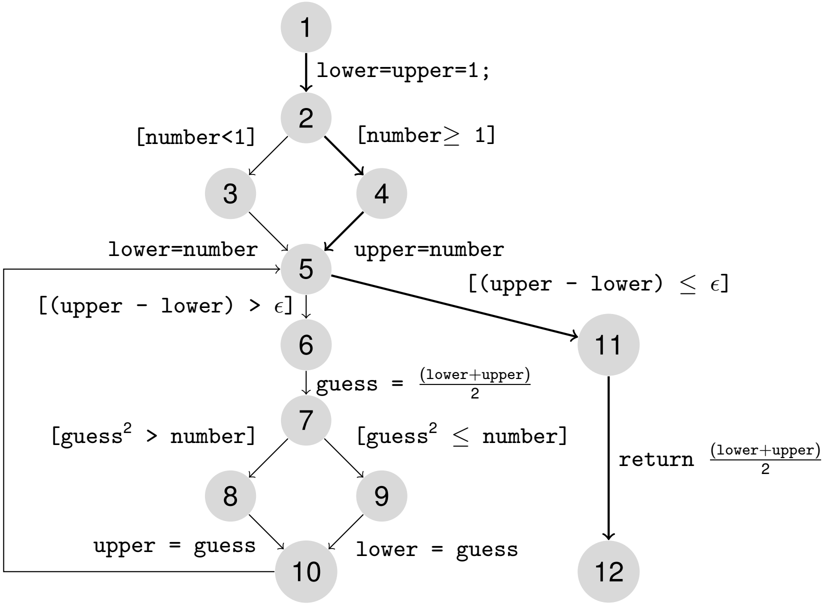

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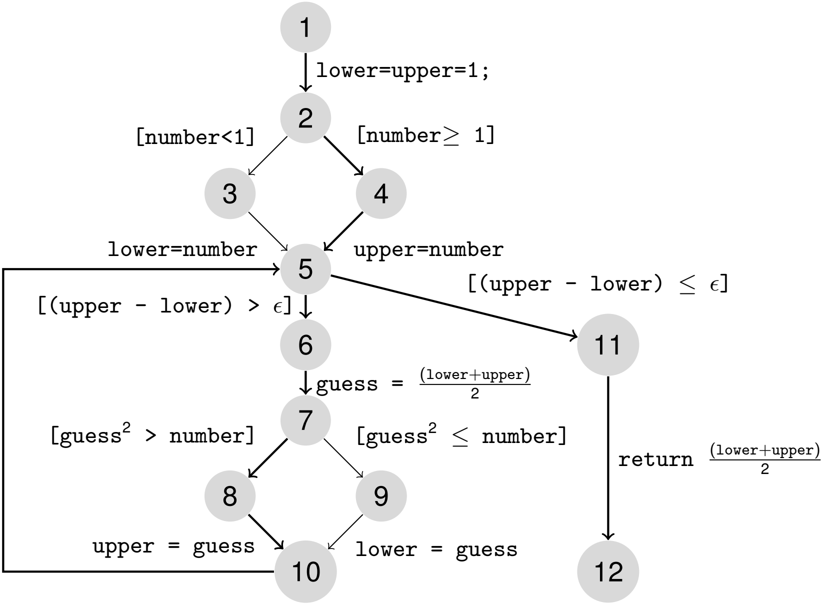

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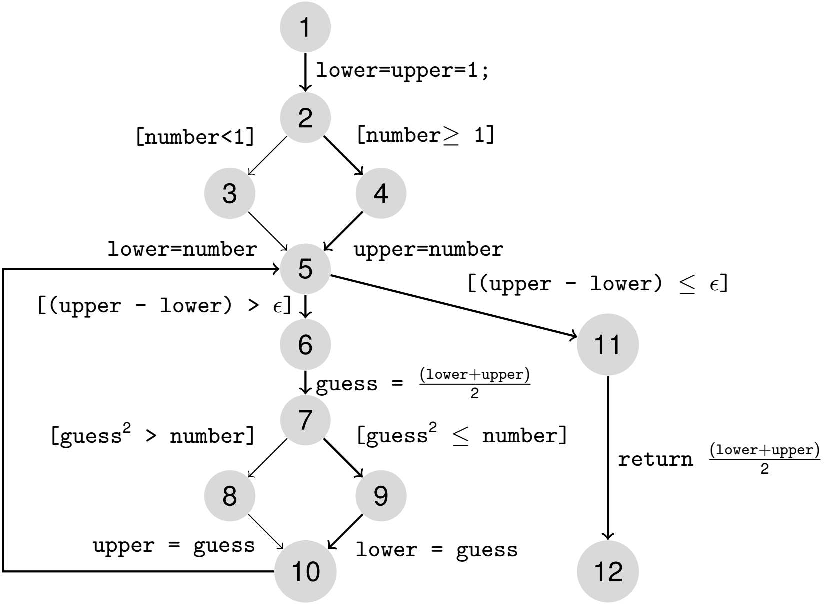

Linearly Independent Paths of Square-Root Implementation

- linearly independent $\stackrel{\text{def}}{=}$
  contains (at least) one edge not covered by other paths

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 11 \rightarrow 12$$

$$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 10 \rightarrow 5 \rightarrow 11 \rightarrow 12$$

$$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 9 \rightarrow 10 \rightarrow 5 \rightarrow 11 \rightarrow 12$$

- think of linear algebra and linearly independent equations

Linearly Independent Paths

- upper bound of test-cases necessary to test all *branches*
- in our case, 2 paths are enough:
  - $1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 9 \rightarrow 10 \rightarrow 10 \rightarrow 5 \rightarrow 11 \rightarrow 12$
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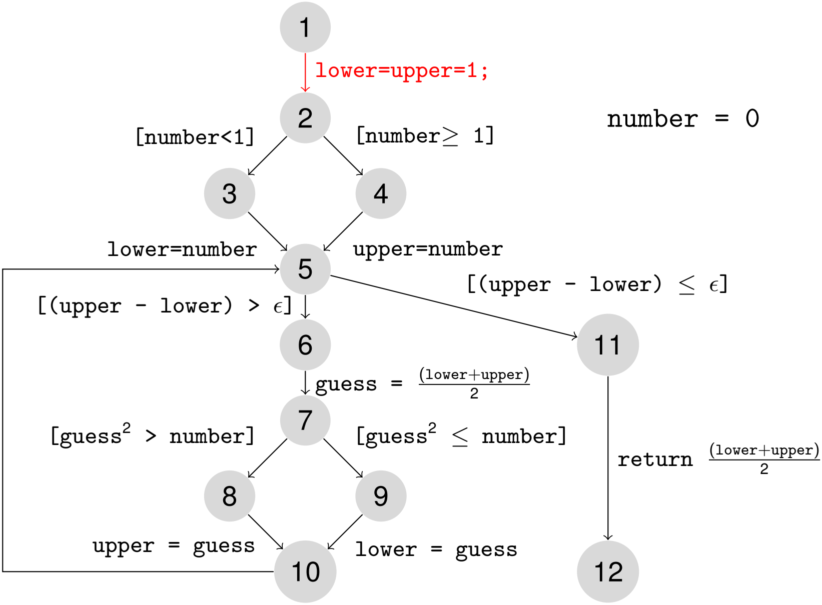
- Do our test-cases cover all branches?

Testing our Square-Root Implementation



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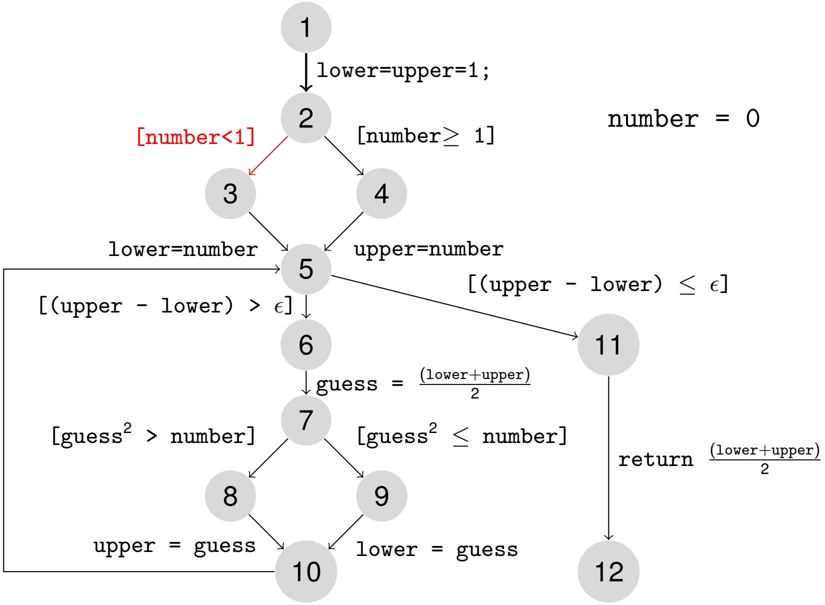
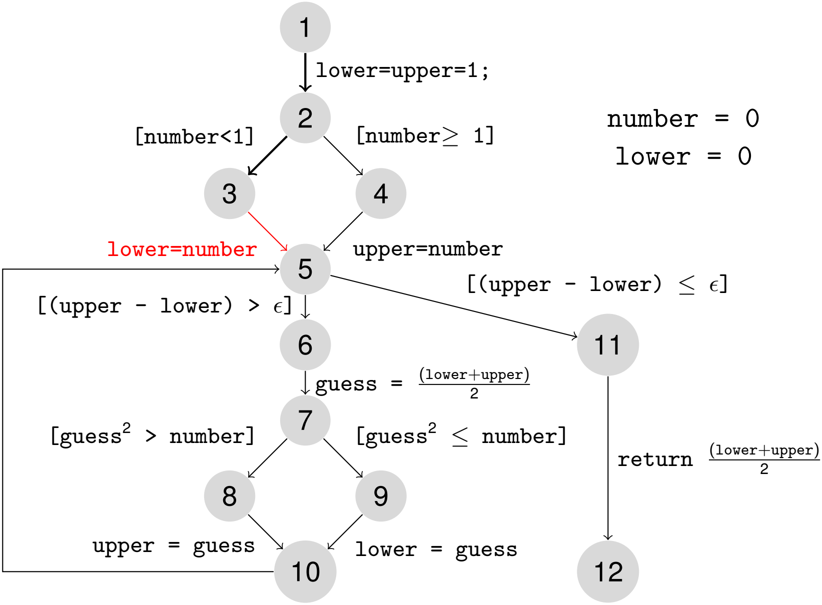


number = 0

Testing our Square-Root Implementation



number = 0
lower = 0

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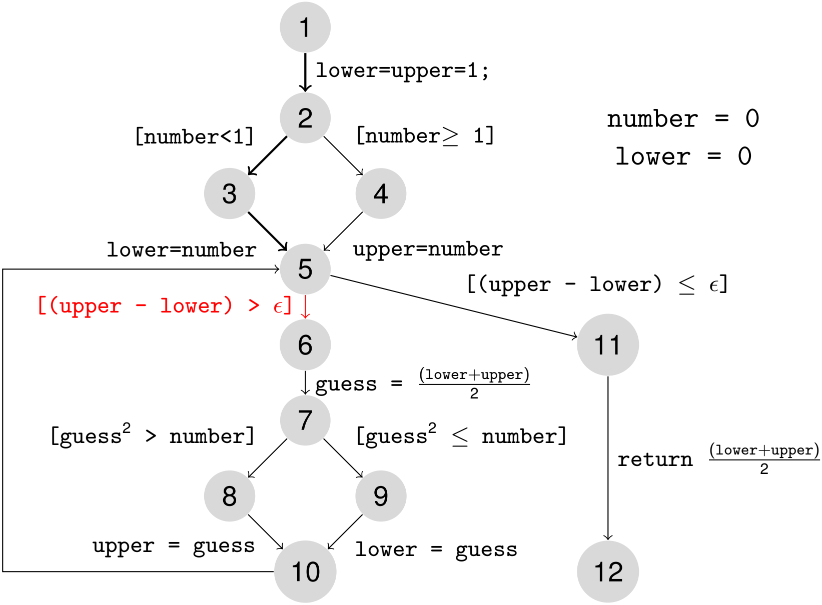

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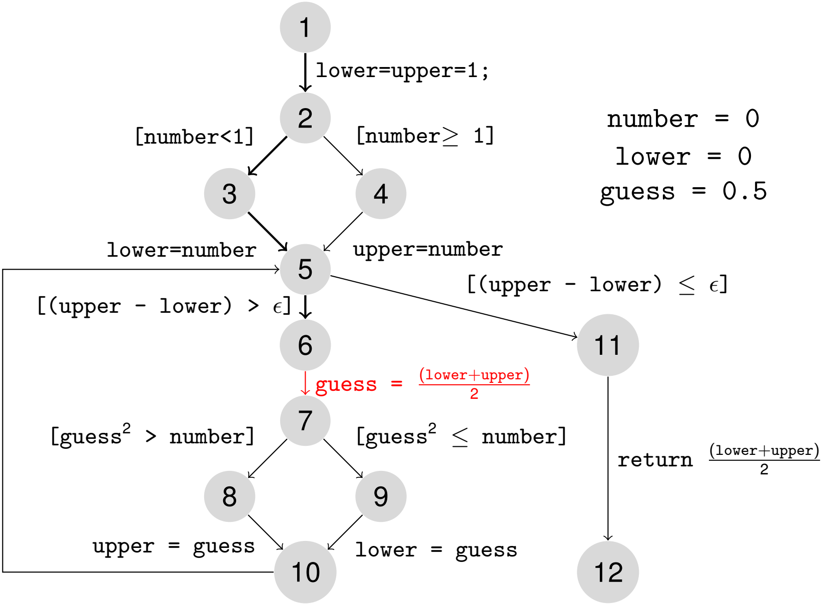

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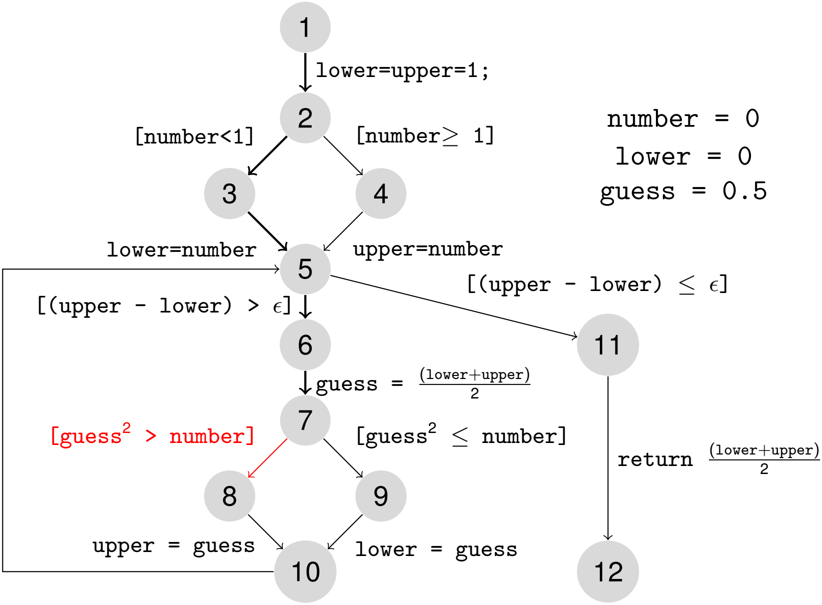

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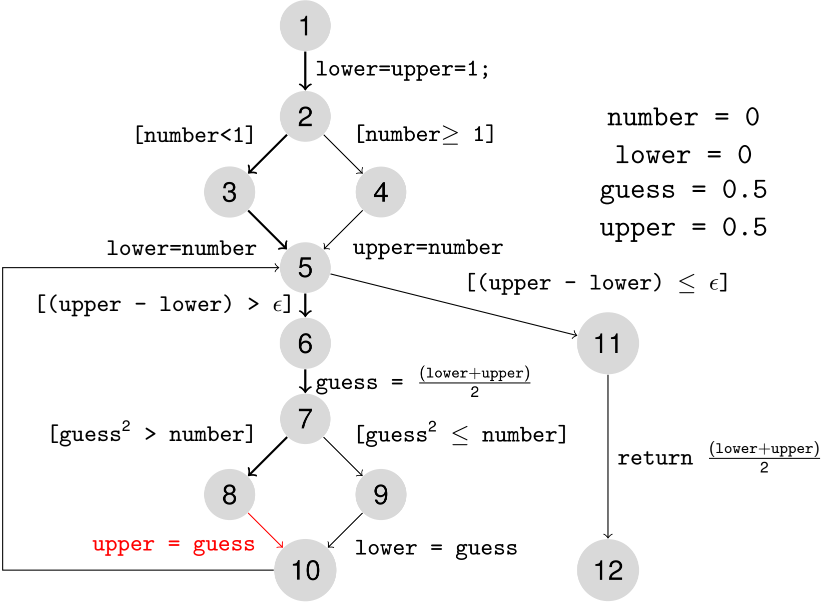

number = 0
lower = 0
guess = 0.5
upper = 0.5

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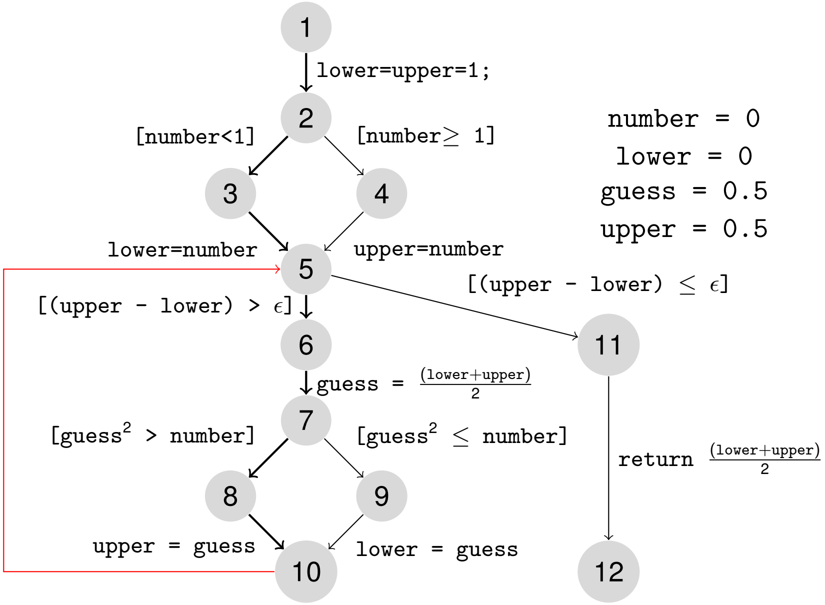

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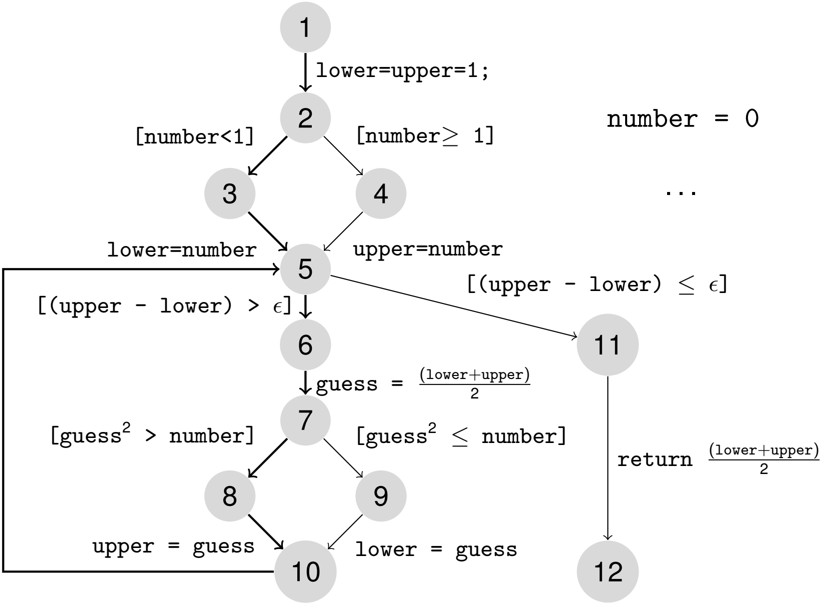

number = 0

...

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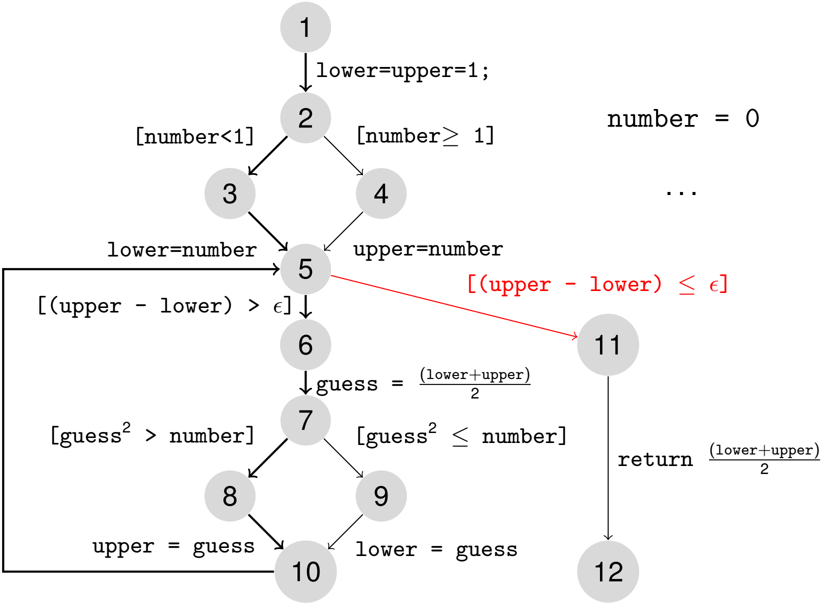

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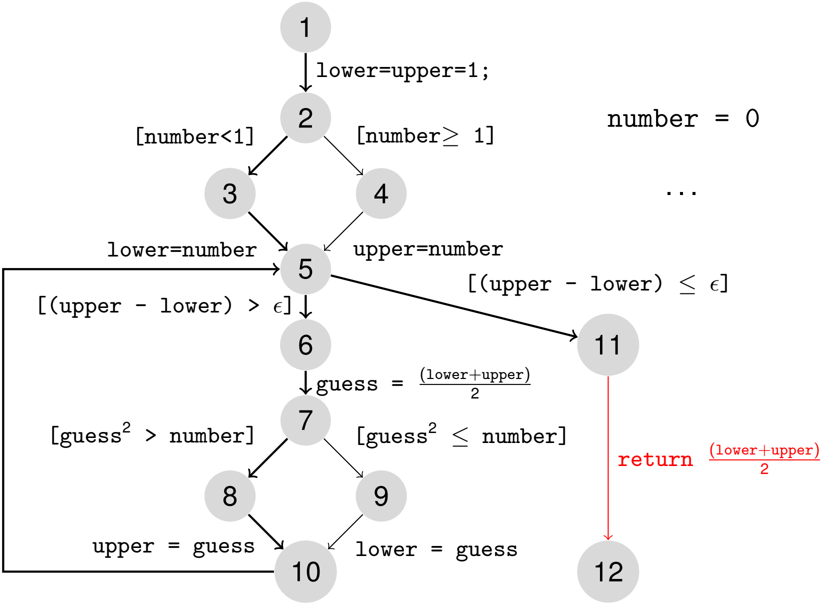

number = 0

...

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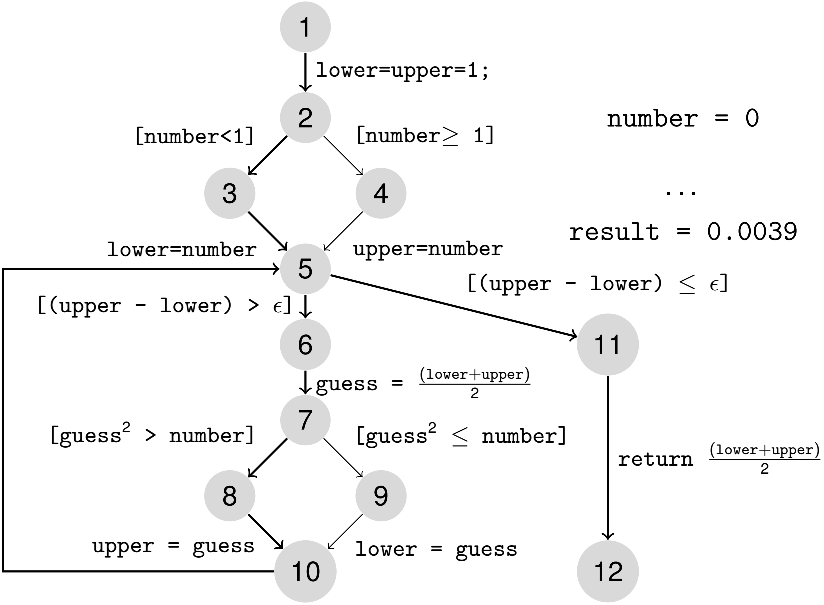

number = 0

...

result = 0.0039

Which "independent" path fragments were executed?

- Test case 0 traversed
  - $2 \rightarrow 3 \rightarrow 5$,
  - $7 \rightarrow 8 \rightarrow 10$, and
  - $5 \rightarrow 11 \rightarrow 12$
- It did *not* traverse
  - $2 \rightarrow 4 \rightarrow 5$ and
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- Could we have predicted that one test case is not enough?

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- It did *not* traverse
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  - $7 \rightarrow 9 \rightarrow 10$
- Could we have predicted that one test case is not enough?
  - Not without knowing the implementation!

Have we done "enough" testing?

- Reasonable to assume that "all of the code" should be tested!
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  - Let’s have a look at 15.3, ok?

GCOV Usage

This is tedious, can't we automate this?

- `gcc -g -fprofile-arcs -ftest-coverage -o sqrt sqrt.c`
  (use `clang` instead of `gcc` on newer Macs)
- `gcov sqrt`
- `cat sqrt.c.gcov`
- `./sqrt ; gcov sqrt`
- `cat sqrt.c.gcov`

Coverage information for `sqrt(0.0)`

```
        1:    6:float squrt (float number) {
        1:    7:  float lower = 1, upper = 1, guess;
        -:    8:
        1:    9:  if (number < 1)
        1:   10:    lower = number; // sqrt < 1, but > number
        -:   11:  else
    #####:   12:    upper = number; // sqrt > 1, but < number
        -:   13:
        9:   14:  while ((upper - lower) > EPSILON) {
        7:   15:    guess = (lower + upper) / 2;
        7:   16:    if (guess*guess > number)
        7:   17:      upper = guess;
        -:   18:    else
    #####:   19:      lower = guess;
        7:   20:  }
        1:   21:  return (lower + upper) / 2;
        -:   22:}
```

Coverage information for `sqrt(15.3)`

```
        1:    6:float squrt (float number) {
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        -:    8:
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  - How many different inputs are there to `sqrt(float)`?
    - `sizeof(float)` = 4 bytes, so roughly 2^{32}
  - How many different inputs are there to our `AVL` implementation?
- Maybe visit all possible *states*?

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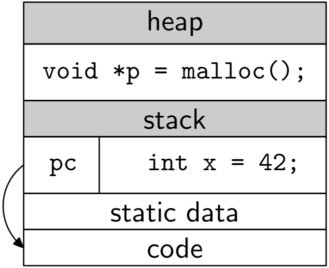
What is a *state*?

Values of

- global variables



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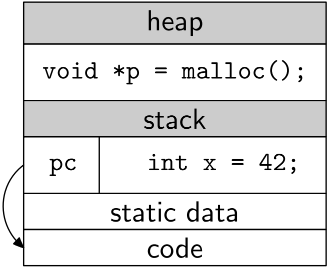
- stack variables



Have we done "enough" testing?

What is a *state*?

Values of

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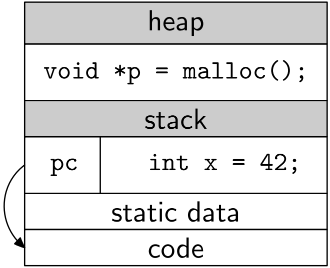
- heap…



Reachable Sets of States

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(what about false?)

Collecting Semantics

We can now define a *collecting semantics* for our CFG

- Let $\langle V, E\rangle$ be a control-flow graph with entry node $\rightarrow\!\bigcirc \in V$
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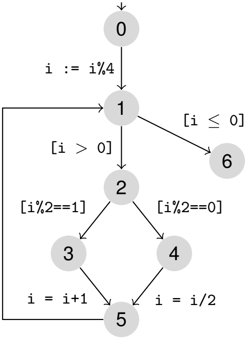
- Initialize all other nodes with $\emptyset$
- Iterate computation until *fixpoint* is found

Simple Example: Collecting Semantics



#	0	1	2	3	4	5	6
0	$\mathbb{N}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

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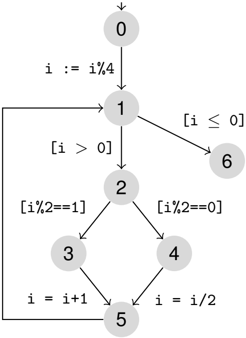

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1	$\mathbb{N}$	[0..3]	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

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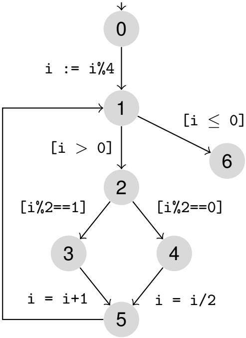

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1	$\mathbb{N}$	[0..3]	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
2	$\mathbb{N}$	[0..3]	[1..3]	$\emptyset$	$\emptyset$	$\emptyset$	$\{0\}$

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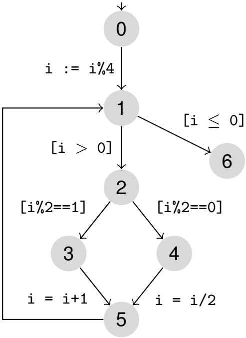

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3	$\mathbb{N}$	[0..3]	[1..3]	$\{1,3\}$	$\{2\}$	$\emptyset$	$\{0\}$

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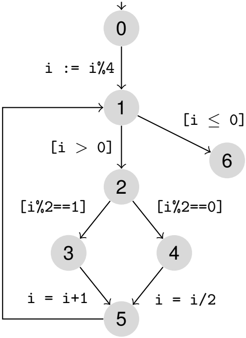

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4	$\mathbb{N}$	[0..3]	[1..3]	$\{1, 3\}$	$\{2\}$	$\{1, 2, 4\}$	$\{0\}$

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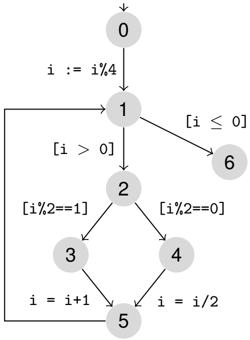

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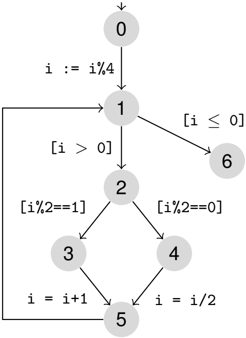
Simple Example: Collecting Semantics



#	0	1	2	3	4	5	6
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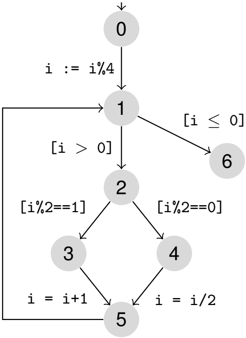

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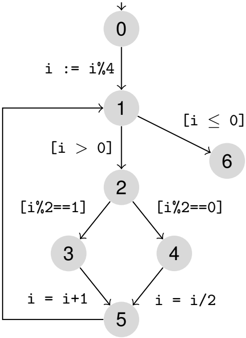

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Collecting Semantics

How many possible states are there in general?

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- ∞, in theory

Collecting Semantics

How many possible states are there in general?

- ∞, in theory
- Collecting semantics generally defined as *least fixpoint* of

$$\lambda S_j \,.\, \begin{cases} \{\sigma \mid \sigma \text{ is possible assignment}\} & \text{if } i = \textit{entrynode} \\ \bigcup_{i \in \{n \mid \langle n,j \rangle \in E\}} \texttt{stmt}_{\langle i,j \rangle}(S_i) & \text{if } j \in V \setminus \{\rightarrow\!\circ\} \end{cases}$$

Finite State Programs

How about finite state programs?

- Assume that there are only n different elements that we can insert into our AVL tree.

Element	1	2	...	n
Inserted	✓	✗	...	✓

- Finitely many *states*

How about finite state programs?

- Assume that there are only n different elements that we can insert into our AVL tree.

Element	1	2	...	n
Inserted	✓	✗	...	✓

- Finitely many *states*
  - But still 2^n possible sets (and even more trees)!

Have we done "enough" testing?

But aren't many trees "*similar*"?

Elements	1	2	3	4	5
State 1	✓	✓	✗	✗	✗
State 2	✗	✓	✓	✗	✗

- Maybe, we don't need to "cover" all of them?
- What is the problem with this argument?

Have we done "enough" testing?

But aren't many trees "*similar*"?

Elements	1	2	3	4	5
State 1	✓	✓	✗	✗	✗
State 2	✗	✓	✓	✗	✗

- Maybe, we don't need to "cover" all of them?
- What is the problem with this argument?
  - it is not *formally proven* (maybe even wrong)
  - it is specific to one program

Coverage Criteria

- Common agreement on what “sufficiently tested” means
  - coverage criteria are about *confidence*, trust
  - required for *certification* (according to industry standards)
- Important: achieving coverage is not a goal in itself
  - “The journey is the reward:” <u>Testing</u> until coverage is reached
  - Test-cases should be generated from *requirements*

Coverage Criteria

Coverage criteria define equivalence classes with respect to program behaviour

- Control flow-based coverage
  - Path coverage
  - Statement/basic block coverage
  - Branch coverage
  - Decision coverage
  - Condition coverage
  - Condition/Decision coverage
  - Modified condition/decision coverage (MC/DC)
  - Multiple decision coverage
- Data flow-based coverage
  - Definition/use pairs
- Mutation testing
- ...

Coverage Criteria: Path coverage

- Goal: Execute every *path* of the program
  - Independently of the variable values along that path
  - Every path is an equivalence class
- What's the number of paths through the following program?

```
while (1) {
  if (getchar() == EOF)
     break;
}
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Coverage Criteria: Path coverage

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while (1) {
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```

- In general, path coverage can't be achieved

Coverage Criteria: Statement Coverage

- Goal: Execute every program statement at least once
  - All traces visiting that statement build equivalence class

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- Goal: Execute every program statement at least once
  - All traces visiting that statement build equivalence class
  - Let ℓ be program location of said statement

$$\{\pi | \pi = \sigma_0, \ldots \sigma_n \wedge \exists i . 0 \leq i \leq n \wedge \sigma_i(pc) = \ell\}$$

Coverage Criteria: Statement Coverage

- Bad criterion:
  - consider test case `x = 5` for following code fragment:

```
if (x > 1) {
  x++;
}
int y = x/y;
```

Coverage Criteria: Statement Coverage

- Bad criterion:
  - consider test case `x = 5` for following code fragment:

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```

- All statements executed, but `else` branch never taken
- May not exercise all outcomes of a conditional statement

Coverage Criteria: Branch Coverage

- Goal: Execute all branches in a program
  - Equivalence class: paths execute a certain branch
- Usually implies statement coverage (but see comments later)

Coverage Criteria: Decision Coverage

- Goal: Exercise every decision outcome at least once
  - decision is a "Boolean expression composed of conditions and zero or more Boolean operators"
  - EC: traces in which decision evaluates to same value

Coverage Criteria: Decision Coverage

- Goal: Exercise every decision outcome at least once
  - decision is a "Boolean expression composed of conditions and zero or more Boolean operators"
  - EC: traces in which decision evaluates to same value
  - For each program location ℓ with decision B
    - Need one path from

$$\{\pi | \pi = \sigma_0, \ldots \sigma_n \wedge \exists i. 0 \leq i \leq n \wedge \sigma_i(pc) = \ell \wedge \sigma_i \models B\}$$

    - and one from

$$\{\pi | \pi = \sigma_0, \ldots \sigma_n \wedge \exists i. 0 \leq i \leq n \wedge \sigma_i(pc) = \ell \wedge \sigma_i \models \neg B\}$$

Coverage Criteria: Decision Coverage

- Subtly different from “branch coverage”
  - *Vacuously* true for the following program:

```
x = y;
x++;
```

  - *all* decisions covered even *without testing*
  - Therefore, does *not* imply statement coverage

Coverage Criteria: Decision Coverage vs. Branch Coverage

Danger, Will Robinson:

branch coverage $\neq$ decision coverage

At least not in general!

- *Numerous* subtle differences
- *Inconsistent* definitions (in industry standards)
- In particular, neither metric subsumes the other

Coverage Criteria: Definition of "Branch"

- **branch (1) (software).** (A) A computer program construct in which one of two or more alternative sets of programs statements is selected for execution. (B) A point in a computer program at which one of two or more alternative sets of program statements is selected for execution. Syn: *branchpoint*. [. . .]
- **branch testing.** Testing designed to execute each outcome of each decision point in a computer program. Contrast with: path testing; statement testing.

IEEE Std 100-1992 Standard Dictionary of Electrical and Electronic Terms

Coverage Criteria: Definition of “Branch”

imprecise definitions of “branch”

- Some definitions may or may not include
  - unconditional branches (`goto`)
  - function calls
  - fall-throughs (in `switch`/`case` constructs
- Example

```
x++;
goto L;
...
L: y++;
```

  - Contains *no decisions*
  - But: contains a *non-conditional* branch

Coverage Criteria: Definition of “Decision”

expressions with side effects

- Consider the following example:

```
if ((y > 1) && (( z > 1) || foo())
  x = y;
else
  x = z;
```

  - “Decision” evaluates to true if `y > 1` and `z > 1`
  - “Decision” evaluates to false if `y <= 1`
  - `foo` is never executed (short-circuited evaluation!)
  - covered by branch coverage, *if* function call a is branch

Coverage Criteria: Definition of “Decision”

- “Decision” is defined as “Boolean expression”
  - *not necessarily only* at branching points!

```
x = x + 1;
b = (x>0);
y = x;
```

  - Strictly speaking, have to cover every outcome of `x>0`
  - E.g., enforced in DO-178B standard
  - This code doesn’t contain any branch!

Coverage Criteria: Branch Coverage vs Decision Coverage

Branch coverage implies decision coverage

- if “decision” means Boolean expressions at branching points only

Decision coverage is stronger than branch coverage

- if “branch” doesn’t include unconditional jumps
- if “decision” refers to *all* Boolean expressions

Often branch and decision outcome are used synonymously

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Often branch and decision outcome are used synonymously

Meaning varies, depending on industry standard that applies

- Also: interpreted differently by different coverage tools

Coverage Criteria: Condition Coverage

- Goal: Exercise every sub-expression/atom/condition outcome
  - atom is a Boolean expression not containing Boolean operators (e.g., `&&`, `||`)
  - Equivalence class: paths in which condition evaluates to same value
- Does not imply decision coverage!
  - Consider the following program fragment:

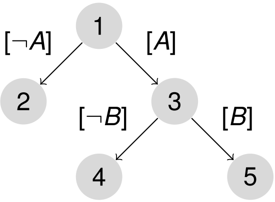
```
if ((x > 0) && (y > 0))
  x++;
```

  - Inputs: x = 5, y = -3 and x = -1 and y = 2
  - All condition outcomes considered, but decision always false

Coverage Criteria: Condition Coverage

- Goal: Exercise every sub-expression/atom/condition outcome
  - atom is a Boolean expression not containing Boolean operators (e.g., `&&`, `||`)
  - Equivalence class: paths in which condition evaluates to same value
- Can be considered as path partitioning if evaluation follows some order
  - think of generated intermediate representation

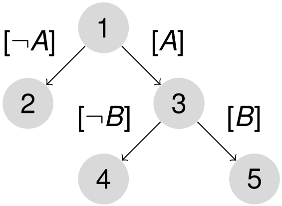
```
if (A && B)
```



Coverage Criteria: Condition/Decision Coverage

- Combination of decision and condition coverage
  - Cover all condition outcomes
  - Cover all decision outcomes
- not all branches in intermediate code might be executed!
- Consider the following cases:

A	B	A && B
0	0	0
1	1	1



- Coverage criterion is satisfied; $1 \rightarrow 3 \rightarrow 4$ never executed!

Coverage Criteria: *Modified* Condition/Decision Coverage (MC/DC)

- Each condition outcome must affect the decision outcome independently
  - “fix” the value of all conditions in a decision except for one
  - flipping that one condition must change the decision outcome
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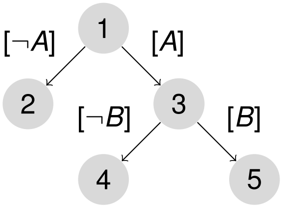

- MC/DC not satisfied: neither $A = 0$ nor $B = 0$ influence outcome 0 of A && B independently!
  - need to add $A = 0, B = 1$ and $A = 1, B = 0$

Coverage Criteria: MC/DC as defined in DO-178B

1. Every entry and exit point in the program has to be visited
2. Every conditional statement (i.e., branchpoint) has to take all possible outcomes (i.e., branches)
3. Every non-constant Boolean expression has to evaluate at least once to 1 and at least once to 0
4. Every non-constant condition in a Boolean expression has to evaluate at least once to 1 and at least once to 0
5. Every non-constant condition in a Boolean expression has to affect that expression's outcome independently

- Decision coverage requires (1, 2, 3)
- Decision/Condition coverage requires (1, 2, 3, 4)
- MC/DC requires 1 through 5
  - Note: equating branch and decision coverage violates MC/DC definition in DO-178B

DO-178B (Software Considerations in Airborne Systems and Equipment Certification)

- Safety standard
- Used for certification of safety critical software
- defines levels of criticality depending on potential damage of fault:
  - catastrophic
  - hazardrous/sever-major
  - major
  - minor
- Defines corresponding criticality levels A, B, C, D

Coverage Criteria in DO-178B

- For certification, following coverage criteria apply:

A	MC/DC
B	Decision and Statement coverage
C	Statement coverage
D	None

- (also specifies other criteria, e.g., documentation, traceability of requirements to test-cases, etc.)

Coverage Criteria: Multiple Condition Coverage

- All combinations of conditions in each decision have to be tested
- Consider the expression `(A || B) && C`
  - Condition/Decision coverage:

A	B	C
1	1	1
0	0	0

  - MC/DC (bold values influence decision outcome):

A	B	C
0	**0**	1
1	0	**1**
0	**1**	**1**
1	1	**0**

  - Multiple condition overage: *all* 2^3 combinations!

Coverage Criteria: Can they always be satisfied?

- Some coverage criteria might not be 100% satisfiable
  - Simple example: Statement coverage and unreachable code
- Coverage goal could be lower than 100%
- If coverage goal *unreachable*, justification may be necessary

Statement Coverage: Can it be satisfied?

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$$\forall i \in V \,.\, \alpha(S_i) = \top$$

Decision Coverage: Can it be satisfied?

- We define α as

$$\alpha(S_i) \stackrel{\text{def}}{=} \begin{cases} \{\sigma(B_i) \mid \sigma \in S_i\} & \text{if } i \text{ has decision } B_i \\ \top & \text{otherwise} \end{cases}$$

(We assume each location has only one decision)

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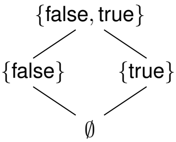

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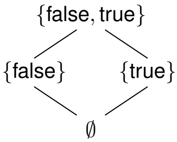

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Coverage Metrics: Can they be satisfied?

Note:

- Abstract domains for metrics we defined are *finite*
- even if set of reachable states is infinite

How to determine coverage

- Same principle: use abstraction function α
- But use states reached via *tests*
  - Each $\sigma \in S_{\rightarrow\bigcirc}$ is now starting state of a test
    (assuming we have non-reactive programs)

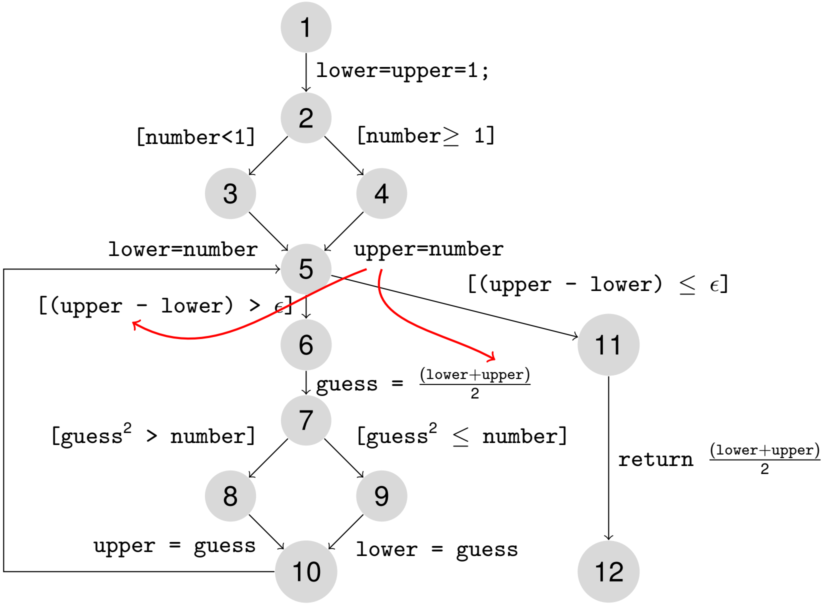
Coverage Criteria

- Control flow-based coverage
  - Path coverage
  - Statement/basic block coverage
  - Branch coverage
  - Decision coverage
  - Condition coverage
  - Condition/Decision coverage
  - Modified condition/decision coverage (MC/DC)
  - Multiple decision coverage
- **Data flow-based coverage**
  - Definition/use pairs
- Mutation testing
- ...

Coverage Criteria: Data-Flow

- Data flow: how do values propagate through program?
  - **definition:** assignment of a value to a variable
  - **use:** statement where the value is read
  - **def-use chain:** cycle-free path, first statement defines value, last statement uses value; value not re-defined in between

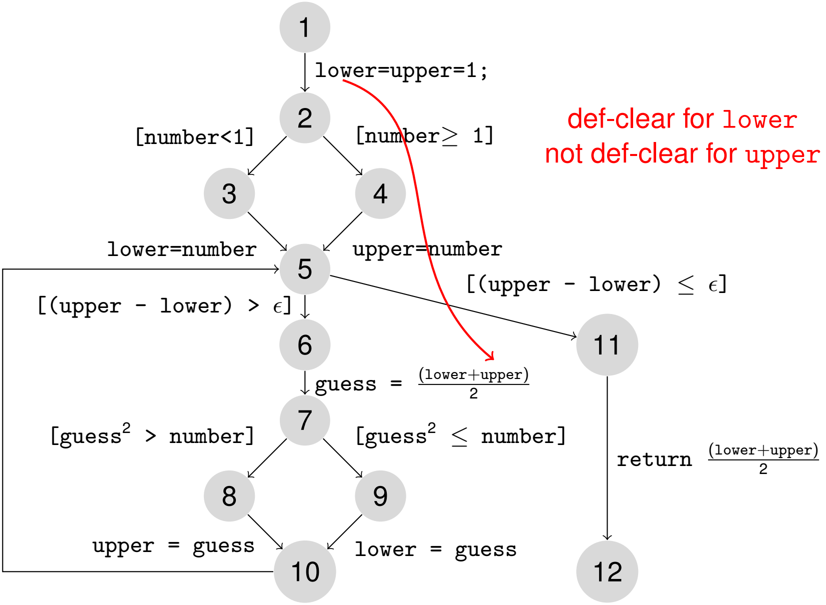
Coverage Criteria: Data-Flow



Coverage Criteria: Data-Flow

- Definitions can "flow into"
  - Boolean expressions ("predicates") in conditional statements
  - variables used to define ("compute") other values (right-hand-side of assignment)
- Some notation:
  - defs(`x`): locations where `x` is defined
  - p-use(`x`): locations where `x` is used in predicate
  - c-use(`x`): locations where `x` is used to compute other value
- A path is def-clear for `x` if
  - `x` is not re-defined between first and last node

Coverage Criteria: Data-Flow



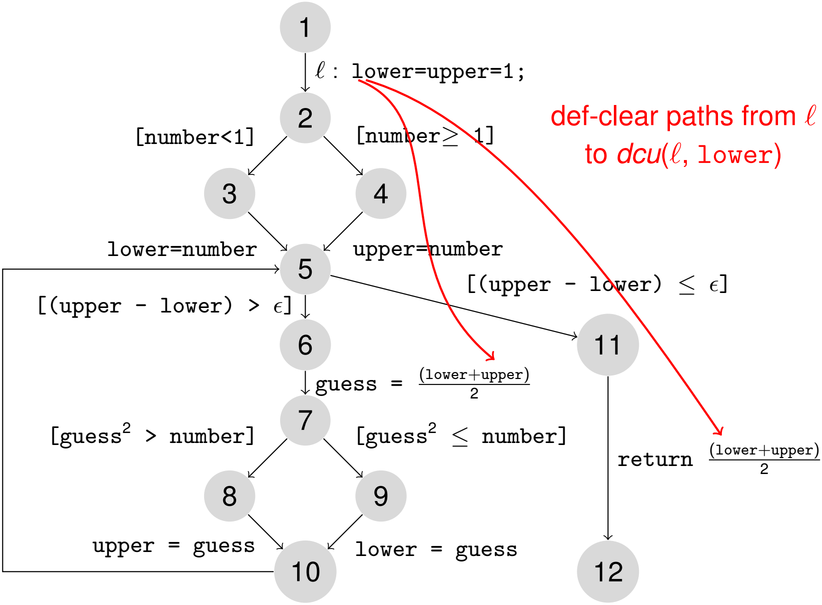
Coverage Criteria: Data-Flow

- dpu(ℓ, $\mathtt{x}$) locations $\ell' \in$ p-use$(\mathtt{x})$ such that there is a def-clear path from ℓ to ℓ'

  This are the locations which use $\mathtt{x}$ in a predicate and can potentially be influenced by the definition of $\mathtt{x}$ at ℓ
- dcu(ℓ, $\mathtt{x}$) locations $\ell' \in$ c-use$(\mathtt{x})$ such that there is a def-clear path from ℓ to ℓ'

  This are the locations which use $\mathtt{x}$ in a computations and can potentially be influenced by the definition of $\mathtt{x}$ at ℓ

Coverage Criteria: Data-Flow, Example for dcu



Frankl & Weyuker's Data-Flow Coverage Criteria

For each definition of variable $\mathtt{x}$ and for every $\ell \in \mathsf{defs}(\mathtt{x})$, the test suite traverses:

- all-defs: one path to *some* $\ell' \in (\mathsf{dpu}(\ell, \mathtt{x}) \cup \mathsf{dcu}(\ell, \mathtt{x}))$

  $\Rightarrow$ all definitions get used
- all-c-uses: one path to *each* $\ell' \in \mathsf{dcu}(\ell, \mathtt{x})$

  $\Rightarrow$ all computations affected by each definition are executed
- all-p-uses: one path to *each* $\ell' \in \mathsf{dpu}(\ell, \mathtt{x})$

  $\Rightarrow$ all decisions affected by each definition are executed

Frankl & Weyuker's Data-Flow Coverage Criteria

- all-c-uses/some-p-uses: one path to *each* $\ell' \in \mathtt{dcu}(\ell, \mathtt{x})$, but if $\mathtt{dcu}(\ell, \mathtt{x}) = \emptyset$, then at least one path to $\ell' \in \mathtt{dpu}(\ell, \mathtt{x})$

  $\Rightarrow$ all definitions used, and if they affect computations, then all affected computations are executed

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Frankl & Weyuker's Data-Flow Coverage Criteria

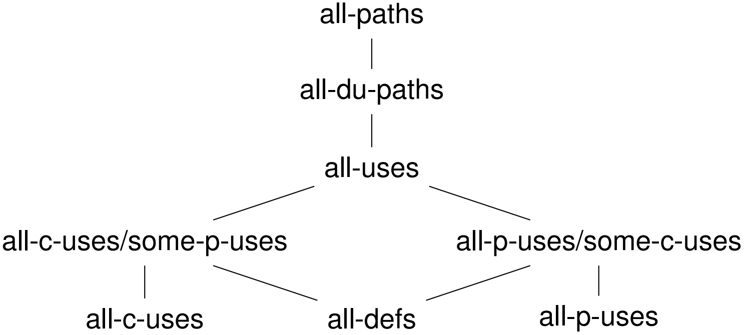
- all-uses: one path to *each* node $\ell' \in (\mathrm{dpu}(\ell, \mathtt{x}) \cup \mathrm{dcu}(\ell, \mathtt{x}))$

  $\Rightarrow$ every computation and decision affected by definition executed

- all-du-paths: *all paths* to each node $\ell' \in (\mathrm{dpu}(\ell, \mathtt{x}) \cup \mathrm{dcu}(\ell, \mathtt{x}))$

  $\Rightarrow$ like above, but *all* def-use paths

Subsumption Lattice



Coverage Criteria: Data-Flow

- Data-flow criteria track dependencies between variables
- Set of all pairs can be approximated by static analysis
  - typically covered in course on compiler design

Coverage Criteria

- Control flow-based coverage
  - Path coverage
  - Statement/basic block coverage
  - Branch coverage
  - Decision coverage
  - Condition coverage
  - Condition/Decision coverage
  - Modified condition/decision coverage (MC/DC)
  - Multiple decision coverage
- Data flow-based coverage
  - Definition/use pairs
- **Mutation testing**
- ...

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  - After designing a test-suite, mutants are applied one-by-one
  - Each mutant should be caught (*killed*) by one of the test cases!
- Typical mutations: simple syntactic modifications
  - Delete a statement
  - Change `&&` to `||`, `-` to `+`, `<` to `<=`, ...
  - Replace variables with others in scope

Coverage Criteria: Mutation Testing

- **Weak mutation testing**
  Test case must trigger the injected fault and result in an error
- **Strong mutation testing** Test case must trigger the injected fault and result in a failure

- Obstacles:
  - *Equivalent mutants:* Some faults can't be triggered (e.g., changing == to <= in `for (i=10; i==0; i--)`)
  - Also, most "real world" bugs aren't that simple (does mutation testing evaluate the ability of a test-suite to catch "real" bugs?)

Coverage Criteria: Mutation Testing – Fuzzing

- *Fuzzing:* a variation of Mutation Testing
  - “mutate” (or randomly vary) input data
  - monitor program for resulting crashes, failed assertions, memory leaks
  - c.f. fault injection

Summary: Coverage Metrics

- Coverage criteria for when program is “sufficiently” tested
- Widely used, also in certification of safety critical systems
- Are effectively a confidence measure
  - do not guarantee that program is bug-free
  - also, some of the definitions are *ambiguous*
- Never forget:
  - Test-case generation driven by <u>specification</u>, *not* by coverage!