B1	B2	B3	B4	B5	B6	$\sum$Bi

Nam
Student I

Knowledge Based Systems, 4.0 VU, 184.730

Exercise Test – 12.06.2019

- **The test covers the exercise sheets and background questions.**
- **Please read the questions carefully and give precise answers.** **Note that there might be differences compared to the exercise sheets.**
- **Write legibly and with a pen or a fountain-pen** *(no pencil allowed)!*
- **You need 14 of 27 points to pass the exercise part.**

Good luck!

Exercise 1 (5 pts.):

(a) Consider the theory $\Gamma := \{\forall x \exists y R(x,y), \forall x \forall y (R(x,y) \rightarrow (R(y,x) \wedge Q(y)))\}$. Use TC1, to show that the sentence $\forall x Q(x)$ is a logical consequence of Γ. Explain the intermediate steps from the consequence problem to the input of the TC1.

(b) For formulas φ and ψ, explain in detail how TC1 can be used to check

(i) validity of φ;

(ii) the entailment $\varphi \models \psi$;

A)

B) i) check ~~so~~ φ is ~~sat~~ valid $\Leftrightarrow \neg\varphi$ is uns

ii) $\varphi \models \psi \Leftrightarrow \varphi \wedge \neg\psi$ is unsat

Exercise 2 (4 pts.):

(a) Give a formal definition of an interpretation structure in first-order logic in general. Explain the elements it consists of.

(b) Give one interpretation structure under which all the following sentences become true.

$$\forall x \exists y S(x, y)$$
$$\forall x \forall y (S(x, y) \to ((P(x) \land \neg P(y)) \lor (\neg P(x) \land P(y))))$$
$$\exists x \exists y S(x, y)$$

a) $I = (U, I, \alpha)$, where U is a non-empty domain

I is an interpret. function which satisfies

- $I(c) \in U$ for constants c
- $I(f): U^n \to U$
- $I(P) \subseteq U^n$

n-ary
$n \geq 0$

n-ary

b) $I = (\{a, b\}, I, \alpha)$ where

$I(S) = \{(a, b), (b, a)\}$

$I(P) = \{(a)\}$

$\alpha = \emptyset$

Exercise 3 (5 pts.):

(a) Consider the knowledge base $\mathcal{K} := \langle \{A \sqcup B \sqsubseteq \exists R.(\forall S.\exists R.\neg A \sqcap \exists S.\forall R.\neg B)\}, \{A(a)\} \rangle$. Apply the $\mathcal{ALC}$ tableau algorithm for KB-satisfiability to $\mathcal{K}$ and give an interpretation induced by the resulting completion graph.

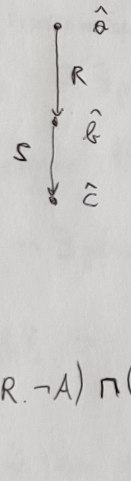
(b) Consider the $\mathcal{ALC}$ tableau algorithm for KB-satisfiability. Explain informally,

(i) why blocking is required;

(ii) the concept of cycle-detection in the context of blocking.

a)

$C_T = (\neg A \sqcap \neg B) \sqcup \exists R.((\forall S.\exists R.\neg A) \sqcap (\exists S.\forall R.\neg B))$



Interpret: $I = (\{\hat{a}, \hat{b}\}, \cdot^{I_a}$

where $R^{I_a} = \{(\hat{a}, \hat{b})\}$

$S^{I_a} = \{(\hat{b}, \hat{c})\}$

$A^{I_a} = \{\hat{a}\}$

$B^{I_a} = \{\}$

$a^{I_a} = \hat{a}$

$\hat{a}) = \{C_T, A, \exists R.((\forall S.\exists R.\neg A) \sqcap (\exists S.\forall R.\neg B))\}$

$\neg A \sqcap \neg B, \neg A, \neg B$

$= \{C_T, (\forall S.\exists R.\neg A) \sqcap (\exists S.\forall R.\neg B)),$ $\cancel{\exists R.((\forall S.\exists R.\neg A) \sqcap (\exists S.\forall R.\neg B))}$

$\forall S.\exists R.\neg A, \exists S.\forall R.\neg B \}$

$C_T, \forall R.\neg B, \neg A \sqcap \neg B, \neg A, \neg B \}$

Exercise 4 (4 pts.):

(a) Construct a $\mathcal{SROIQ}$ knowledge base $\mathcal{K} = \langle\mathcal{T}, \mathcal{A}\rangle$ (we do not need an RBox here) expressing the following knowledge.

- Every person who is a player plays chess or cards.
- Every player plays against a person.
- No card player can play against him-/herself.
- Bob and Alice play against the same player.

Use the class names Person, Player, Chess, Cards, the role names plays, playsAgainst, and the individual names Bob, Alice.
As a reminder: Individual names are denoted using set brackets, for example $\{Alice\}$. To inverse a role we use the raised $-$, for example $plays^{-}.Cards$ to express that cards are played by someone. The self-reference is denoted by $playsAgainst.Self$.

(b) What is an $\mathcal{ALC}$ concept? Provide an intuitive explanation and an inductive definition.

a) $(\text{Person} \sqcap \text{Player}) \sqsubseteq (\exists \text{plays}.\text{Chess} \sqcup \exists \text{plays}.\text{Cards})$

$\text{Player} \sqsubseteq \exists \text{playsAgainst}.\text{Person}$

$\exists \text{plays}.\text{Cards} \sqsubseteq \neg \exists \text{playsAgainst}.\text{Self}$

$\exists \text{playsAgainst}.\{\text{Bob}\} \equiv \exists \text{playsAgainst}.\{\text{Alice}\}$

b) ~~The~~ An ALC concept is an attribute to describe an elemen[t] of a domain. Evaluated under an interpretation, it gi[ves] us a subset of domain elements.

ALC
a concept C can be constructed with the following con[structors]:

- $\bot$
- $\top$
- $C_1 \sqcap C_2$
- $\neg C_1$
- $C_1 \sqcup C_2$
- $\exists R.C$
- $\forall R.C$

where C[...] are con[cepts] and R [...]

4 3 1
R > Q, S
3 Q > P 2
P > S
2 1

Exercise 5 (4 pts):

(a) Is the following logic program $\mathcal{P}$ stratified (a, b are constants and P, Q, T are unary predicates and R is a binary predicate)?

$$\begin{aligned}\mathcal{P} := \{&R(a,b) \leftarrow P(a), \text{not } Q(b), \text{not } S(a).\\ &R(b,a) \leftarrow P(b), \text{not } Q(a), \text{not } S(b).\\ &Q(a) \leftarrow T(a), \text{not } P(b).\\ &Q(b) \leftarrow T(b), \text{not } P(a).\\ &P(a) \leftarrow T(a), \text{not } S(a).\\ &P(b) \leftarrow T(b), \text{not } S(b).\}\end{aligned}$$

(b) (i) For a logic program $\mathcal{P}$ with a signature Σ, define the Herbrand universe and the Herbrand basis $\mathcal{H}(\mathcal{P})$.

(ii) What kinds of logic programs were introduced in the lecture? For each of them name its syntactic elements and name its semantics.

a) yes, we give a stratification:

~~$\|R\| = 4, \|Q\|$~~

$\|R(a,b)\| = 4$, $\|Q(b)\| = \|Q(a)\| = 3$, $\|S(a)\| = \|S(b)\| = 1$

$\|R(b,a)\| = 4$, $\|P(a)\| = \|P(b)\| = 2$, $\|T(a)\| = \|T(b)\| =$

b) 1) herbrand universe: all possible ground terms, which can be co
by the FS of Σ

herb. basis: all possible ground atoms, which ca
constructed by FS and PS of Σ

2) • classical log. pr.
model:
• ground atoms (no neg., no def. neg)
• least herb. model

• normal log. pr.
• no classical neg.
model:
• ground atoms
• stable model

• extended
• cl. neg
• model
• ans

Exercise 6 (5 pts.):

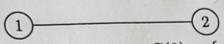
(a) Consider the following extension of the three colourability problem, where a further condition restricts the domain of allowed colors per node.

INSTANCE: Let $G = \langle V, E \rangle$ be a graph, where V is the set of nodes and E is the set of edges and $C(v)$ be a domain of colours for each vertex v from V.

QUESTION: Is there a an assignments of colours to vertices such that,

- no adjacent vertices are assigned the same color;
- every vertex must be assigned *exactly* one of the colours in its associated list.

For example



$C(1) = \{r\}$ $C(2) = \{r, g, b\}$

has the solutions $\{1 \mapsto r, 2 \mapsto g\}$ and $\{1 \mapsto r, 2 \mapsto b\}$.

Given the encoding

- $n(X)\ldots$ X is a vertex;
- $e(X,Y)\ldots$ there is an edge between X and Y;
- $dr(X), dg(X), db(X)\ldots$ red, green, blue occur in the domain of allowed colors of X;
- $r(X), g(X), b(X)\ldots$ X is coloured red, green, blue.

Write an answer set program $\mathcal{P}_C$ such that its answer sets correspond to solutions of the problem specified above.

(b) Explain the difference between queries and inferences with respect to an answer set program $\mathcal{P}$, by defining those two notions formally.

```
r(X) v g(X) v b(X) :- n(X)
:- e(X,Y), r(X), r(Y)
:- e(X,Y), g(X), g(Y)
:- e(X,Y), b(X), b(Y)
:- r(X), not dr(X)
:- g(X), not dg(X)
:- b(X), not db(X)
```