

1 Bonusaufgabe 3

```
ratprint:false;  
false
```

1.1 Angabe

```
phi_0:-%pi/6;
```

$$-\frac{\pi}{6}$$

```
d_0_1:matrix([2],[1],[3])$  
d_1_2:matrix([4],[2],[1])$  
d_2_3:matrix([3],[1],[4])$  
d_3_4:matrix([3],[4],[1])$
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```
p_1_s1:matrix([2],[1],[1/2])$  
p_2_s2:matrix([3/2],[1/2],[2])$  
p_3_s3:matrix([3/2],[2],[1/2])$  
p_4_s4:matrix([2],[1/2],[1])$  
p_4_E:matrix([4],[1],[2])$
```

1.2 Aufgabe 1

```

R_0_1_z:matrix([cos(phi_0),-sin(phi_0),0],[sin(phi_0),cos(phi_0),0],[0,0,1])$
H_0_1_z:addrow(addcol(R_0_1_z,[0,0,0]),[0,0,0,1])$
R_0_1_x:matrix([1,0,0],[0,cos(q1),-sin(q1)],[0,sin(q1), cos(q1)])$
H_0_1_x:addrow(addcol(R_0_1_x,[0,0,0]),[0,0,0,1])$
H_0_1_v:matrix([0,0,0,d_0_1[1,1]],[0,0,0,d_0_1[2,1]],[0,0,0,d_0_1[3,1]],[0,0,0,0])$
H_0_1:H_0_1_z.H_0_1_x+H_0_1_v;

```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{\cos(q1)}{2} & -\frac{\sin(q1)}{2} & 2 \\ -\frac{1}{2} & \frac{\sqrt{3}\cos(q1)}{2} & -\frac{\sqrt{3}\sin(q1)}{2} & 1 \\ 0 & \sin(q1) & \cos(q1) & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```

R_1_2_x:matrix([1,0,0],[0,cos(q2),-sin(q2)],[0,sin(q2), cos(q2)])$
H_1_2_x:addrow(addcol(R_1_2_x,[0,0,0]),[0,0,0,1])$
H_1_2_v:matrix([0,0,0,d_1_2[1,1]],[0,0,0,d_1_2[2,1]],[0,0,0,d_1_2[3,1]],[0,0,0,0])$
H_1_2:H_1_2_x+H_1_2_v;

```

$$\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & \cos(q2) & -\sin(q2) & 2 \\ 0 & \sin(q2) & \cos(q2) & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```

R_2_3_x:matrix([1,0,0],[0,cos(q3),-sin(q3)],[0,sin(q3), cos(q3)])$
H_2_3_x:addrow(addcol(R_2_3_x,[0,0,0]),[0,0,0,1])$
H_2_3_v:matrix([0,0,0,d_2_3[1,1]],[0,0,0,d_2_3[2,1]],[0,0,0,d_2_3[3,1]],[0,0,0,0])$
H_2_3:H_2_3_x+H_2_3_v;

```

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & \cos(q3) & -\sin(q3) & 1 \\ 0 & \sin(q3) & \cos(q3) & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R_3_4:matrix([1,0,0],[0,1,0],[0,0,1])\$

H_3_4_tx:matrix([1,0,0,q4],[0,1,0,0],[0,0,1,0],[0,0,0,1])\$

H_3_4_v:matrix([0,0,0,d_3_4[1,1]],[0,0,0,d_3_4[2,1]],[0,0,0,d_3_4[3,1]],[0,0,0,0])\$

H_3_4:H_3_4_tx+H_3_4_v;

$$\begin{pmatrix} 1 & 0 & 0 & q4+3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

H_0_2:H_0_1.H_1_2\$

trigsimp(%);

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{\sin(q1)\sin(q2)-\cos(q1)\cos(q2)}{2} & -\frac{\cos(q1)\sin(q2)+\sin(q1)\cos(q2)}{2} & -\frac{\sin(q1)-2\cos(q1)-4\sqrt{3}-4}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}\sin(q1)\sin(q2)-\sqrt{3}\cos(q1)\cos(q2)}{2} & -\frac{\sqrt{3}\cos(q1)\sin(q2)+\sqrt{3}\sin(q1)\cos(q2)}{2} & -\frac{\sqrt{3}\sin(q1)-2\sqrt{3}\cos(q1)+2}{2} \\ 0 & \cos(q1)\sin(q2)+\sin(q1)\cos(q2) & \cos(q1)\cos(q2)-\sin(q1)\sin(q2) & 2\sin(q1)+\cos(q1)+3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

H_0_3:H_0_2.H_2_3\$

trigsimp(%);

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{(\cos(q1)\sin(q2)+\sin(q1)\cos(q2))\sin(q3)+(\sin(q1)\sin(q2)-\cos(q1)\cos(q2))\cos(q3)}{2} & \frac{(\sin(q1)\sin(q2)-\cos(q1)\cos(q2))\sin(q3)+(-\cos(q1)\sin(q2)-\sin(q1)\cos(q2))\cos(q3)}{2} \\ -\frac{1}{2} & -\frac{(\sqrt{3}\cos(q1)\sin(q2)+\sqrt{3}\sin(q1)\cos(q2))\sin(q3)+(\sqrt{3}\sin(q1)\sin(q2)-\sqrt{3}\cos(q1)\cos(q2))\cos(q3)}{2} & \frac{(\sqrt{3}\sin(q1)\sin(q2)-\sqrt{3}\cos(q1)\cos(q2))\sin(q3)+(-\sqrt{3}\cos(q1)\sin(q2)-\sin(q1)\cos(q2))\cos(q3)}{2} \\ 0 & (\cos(q1)\cos(q2)-\sin(q1)\sin(q2))\sin(q3)+(\cos(q1)\sin(q2)+\sin(q1)\cos(q2))\cos(q3) & (-\cos(q1)\sin(q2)-\sin(q1)\cos(q2))\sin(q3)+(\cos(q1)\cos(q2)-\sin(q1)\sin(q2))\cos(q3) \\ 0 & 0 & 0 \end{pmatrix}$$

H_0_4:H_0_3.H_3_4\$

trigsimp(%);

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{\langle \cos(q1)\sin(q2)+\sin(q1)\cos(q2)\rangle \sin(q3)+\langle \sin(q1)\sin(q2)-\cos(q1)\cos(q2)\rangle \cos(q3)}{2} & \frac{\langle \sin(q1)\sin(q2)-\cos(q1)\cos(q2)\rangle \sin(q3)+(-\cos(q1)\sin(q2)-\sin(q1)\cos(q2)) \sin(q3)+\langle \cos(q1)\cos(q2)-\sin(q1)\sin(q2)\rangle \cos(q3)}{2} \\ -\frac{1}{2} & -\frac{\langle \sqrt{3}\cos(q1)\sin(q2)+\sqrt{3}\sin(q1)\cos(q2)\rangle \sin(q3)+\langle \sqrt{3}\sin(q1)\sin(q2)-\sqrt{3}\cos(q1)\cos(q2)\rangle \cos(q3)}{2} & \frac{\langle \sqrt{3}\sin(q1)\sin(q2)-\sqrt{3}\cos(q1)\cos(q2)\rangle \sin(q3)+(-\sqrt{3}\cos(q1)\sin(q2)-\sqrt{3}\sin(q1)\cos(q2)) \sin(q3)+\langle \cos(q1)\cos(q2)-\sin(q1)\sin(q2)\rangle \cos(q3)}{2} \\ 0 & \frac{\langle \cos(q1)\cos(q2)-\sin(q1)\sin(q2)\rangle \sin(q3)+\langle \cos(q1)\sin(q2)+\sin(q1)\cos(q2)\rangle \cos(q3)}{0} & \frac{(-\cos(q1)\sin(q2)-\sin(q1)\cos(q2)) \sin(q3)+\langle \cos(q1)\cos(q2)-\sin(q1)\sin(q2)\rangle \cos(q3)}{0} \\ 0 & 0 & 0 \end{pmatrix}$$

1.3 Aufgabe 2

Wir müssen eine Reihe zum Vektor hinzufügen, damit die Multiplikation erfolgen kann. Danach kann diese Reihe wieder entfernt werden.

j_p_s: jacobian([p_0_s1,p_0_s2,p_0_s3,p_0_s4,p_0_E],[q1,q2,q3,q4])\$

j_v_0_s1:trigsimp(addcol(j_p_s[1,1],j_p_s[1,2],j_p_s[1,3],j_p_s[1,4]));

$$\begin{pmatrix} -\frac{2\sin(q1)+\cos(q1)}{4} & 0 & 0 & 0 \\ -\frac{2\sqrt{3}\sin(q1)+\sqrt{3}\cos(q1)}{4} & 0 & 0 & 0 \\ -\frac{\sin(q1)-2\cos(q1)}{2} & 0 & 0 & 0 \end{pmatrix}$$

j_v_0_s2:trigsimp(addcol(j_p_s[2,1],j_p_s[2,2],j_p_s[2,3],j_p_s[2,4]));

$$\begin{pmatrix} \frac{(4\sin(q1)-\cos(q1))\sin(q2)+(-\sin(q1)-4\cos(q1))\cos(q2)-4\sin(q1)-2\cos(q1)}{4} & \frac{(4\sin(q1)-\cos(q1))\sin(q2)+(-\sin(q1)-4\cos(q1))\cos(q2)-4\sin(q1)-2\cos(q1)}{4} \\ \frac{(4\sqrt{3}\sin(q1)-\sqrt{3}\cos(q1))\sin(q2)+(-\sqrt{3}\sin(q1)-4\sqrt{3}\cos(q1))\cos(q2)-4\sqrt{3}\sin(q1)-2\sqrt{3}\cos(q1)}{4} & \frac{(4\sqrt{3}\sin(q1)-\sqrt{3}\cos(q1))\sin(q2)+(-\sqrt{3}\sin(q1)-4\sqrt{3}\cos(q1))\cos(q2)-4\sqrt{3}\sin(q1)-2\sqrt{3}\cos(q1)}{4} \\ -\frac{(\sin(q1)+4\cos(q1))\sin(q2)+(4\sin(q1)-\cos(q1))\cos(q2)+2\sin(q1)-4\cos(q1)}{2} & -\frac{(\sin(q1)+4\cos(q1))\sin(q2)+(4\sin(q1)-\cos(q1))\cos(q2)+2\sin(q1)-4\cos(q1)}{2} \end{pmatrix}$$

j_v_0_s3:trigsimp(addcol(j_p_s[3,1],j_p_s[3,2],j_p_s[3,3],j_p_s[3,4]));

$$\begin{pmatrix} \frac{(4\sin(q1)-\cos(q1))\sin(q2)+(-\sin(q1)-4\cos(q1))\cos(q2)-4\sin(q1)-2\cos(q1)}{4} & \frac{(4\sin(q1)-\cos(q1))\sin(q2)+(-\sin(q1)-4\cos(q1))\cos(q2)-4\sin(q1)-2\cos(q1)}{4} \\ \frac{(4\sqrt{3}\sin(q1)-\sqrt{3}\cos(q1))\sin(q2)+(-\sqrt{3}\sin(q1)-4\sqrt{3}\cos(q1))\cos(q2)-4\sqrt{3}\sin(q1)-2\sqrt{3}\cos(q1)}{4} & \frac{(4\sqrt{3}\sin(q1)-\sqrt{3}\cos(q1))\sin(q2)+(-\sqrt{3}\sin(q1)-4\sqrt{3}\cos(q1))\cos(q2)-4\sqrt{3}\sin(q1)-2\sqrt{3}\cos(q1)}{4} \\ \frac{(\sin(q1)+4\cos(q1))\sin(q2)+(4\sin(q1)-\cos(q1))\cos(q2)+2\sin(q1)-4\cos(q1)}{2} & \frac{(\sin(q1)+4\cos(q1))\sin(q2)+(4\sin(q1)-\cos(q1))\cos(q2)+2\sin(q1)-4\cos(q1)}{2} \end{pmatrix}$$

j_v_0_s4:trigsimp(addcol(j_p_s[4,1],j_p_s[4,2],j_p_s[4,3],j_p_s[4,4]));

$$\begin{pmatrix} \frac{(4\sin(q1)-\cos(q1))\sin(q2)+(-\sin(q1)-4\cos(q1))\cos(q2)-4\sin(q1)-2\cos(q1)}{4} & \frac{(4\sin(q1)-\cos(q1))\sin(q2)+(-\sin(q1)-4\cos(q1))\cos(q2)-4\sin(q1)-2\cos(q1)}{4} \\ \frac{(4\sqrt{3}\sin(q1)-\sqrt{3}\cos(q1))\sin(q2)+(-\sqrt{3}\sin(q1)-4\sqrt{3}\cos(q1))\cos(q2)-4\sqrt{3}\sin(q1)-2\sqrt{3}\cos(q1)}{4} & \frac{(4\sqrt{3}\sin(q1)-\sqrt{3}\cos(q1))\sin(q2)+(-\sqrt{3}\sin(q1)-4\sqrt{3}\cos(q1))\cos(q2)-4\sqrt{3}\sin(q1)-2\sqrt{3}\cos(q1)}{4} \\ \frac{(\sin(q1)+4\cos(q1))\sin(q2)+(4\sin(q1)-\cos(q1))\cos(q2)+2\sin(q1)-4\cos(q1)}{2} & \frac{(\sin(q1)+4\cos(q1))\sin(q2)+(4\sin(q1)-\cos(q1))\cos(q2)+2\sin(q1)-4\cos(q1)}{2} \end{pmatrix}$$

```
j_v_0_E:trigsimp(addcol(j_p_s[5,1],j_p_s[5,2],j_p_s[5,3],j_p_s[5,4]));
⌈
⌋
⌈(5 sin(q1)+3 cos(q1)) sin(q2)+(3 sin(q1)-5 cos(q1)) cos(q2)⌋ sin(q3)+⌈(5 sin(q1)-3 cos(q1)) sin(q2)+(5 cos(q1)-3 sin(q1)) cos(q2)⌋ cos(q3)+(-sin(q1)-cos(q2)) sin(q3)
⌋
R_0_1:R_0_1_z.R_0_1_x$
R_1_2:R_1_2_x$
R_2_3:R_2_3_x$

R_0_2:R_0_1.R_1_2$
R_0_3:R_0_2.R_2_3$
R_0_4:R_0_3.R_3_4$
R_0_E:R_0_4$

R:[R_0_1,R_0_2,R_0_3,R_0_4,R_0_E]$
q:[q1,q2,q3,q4]$
q_d:[qd1,qd2,qd3,qd4]$
```

Figure 1:

$$\mathbf{S}(\boldsymbol{\omega}_k^l) = \dot{\mathbf{R}}_k^l(\mathbf{q})\mathbf{R}_k^l(\mathbf{q})^T = \sum_{j=1}^n \left(\frac{\partial}{\partial q_j} \mathbf{R}_k^l(\mathbf{q}) \right) \mathbf{R}_k^l(\mathbf{q})^T \dot{q}_j$$

```

sum:0$
calc:R_0_1$
for y:1 while y <= 4 do (sum:sum+(diff(calc,q[y],1).transpose(calc)·q_d[y]))$
trigsimp(sum)$
matrix([sum[3][2]], [sum[1][3]], [sum[2][1]])$
J_sum:jacobian([%],q_d)$
J_w_0_1:trigsimp(addcol(J_sum[1][1],J_sum[1][2],J_sum[1][3],J_sum[1][4]));

```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```

sum:0$
calc:R_0_2$
for y:1 while y <= 4 do (sum:sum+(diff(calc,q[y],1).transpose(calc)·q_d[y]))$
trigsimp(sum)$
matrix([sum[3][2]], [sum[1][3]], [sum[2][1]])$
J_sum:jacobian([%],q_d)$
J_w_0_2:trigsimp(addcol(J_sum[1][1],J_sum[1][2],J_sum[1][3],J_sum[1][4]));

```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$


```

sum:0$
calc:R_0_3$
for y:1 while y <= 4 do (sum:sum+(diff(calc,q[y],1).transpose(calc)·q_d[y]))$
trigsimp(sum)$
matrix([sum[3][2]], [sum[1][3]], [sum[2][1]])$
J_sum:jacobian([%],q_d)$
J_w_0_3:trigsimp(addcol(J_sum[1][1],J_sum[1][2],J_sum[1][3],J_sum[1][4]));

```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```

sum:0$
calc:R_0_4$
for y:1 while y <= 4 do (sum:sum+(diff(calc,q[y],1).transpose(calc)·q_d[y]))$
trigsimp(sum)$
matrix([sum[3][2]], [sum[1][3]], [sum[2][1]])$
J_sum:jacobian([%],q_d)$
J_w_0_4:trigsimp(addcol(J_sum[1][1],J_sum[1][2],J_sum[1][3],J_sum[1][4]));

```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```

sum:0$
calc:R_0_E$
for y:1 while y <= 4 do (sum:sum+(diff(calc,q[y],1).transpose(calc)·q_d[y]))$
trigsimp(sum)$
matrix([sum[3][2]], [sum[1][3]], [sum[2][1]])$
J_sum:jacobian([%],q_d)$
J_w_0_E:trigsimp(addcol(J_sum[1][1],J_sum[1][2],J_sum[1][3],J_sum[1][4]));

```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

1.4 Aufgabe 3

```

m: [m1,m2,m3,m4]$
J_w: [J_w_0_1,J_w_0_2,J_w_0_3,J_w_0_4]$
J_v: [j_v_0_s1, j_v_0_s2, j_v_0_s3, j_v_0_s4]$

```

1.4.1 Massematrix

1.4.1.1 Translatorischer Anteil

```

sum:0$
for i:1 while i <= 4 do (sum:sum+((transpose(J_v[i])·m[i]).J_v[i]))$
MM_v:trigsimp(sum);

```

$$\begin{pmatrix} -(488 m^4 + 36 m^3) \sin(q_2) + (-4 m^4 - 8 m^3) \cos(q_2) + 18 m^2 \sin(q_2) + 128 m^4 - 60 m^3 \sin(q_3) + (100 m^4 + 32 m^3) \cos(q_3) + 165 m^4 + 85 m^3 + 17 m^2 & (64 m^4 + 30 m^3) \sin(q_3) + (50 m^4 + 16 m^3) \cos(q_3) + 97 m^4 + 17 m^3 & (64 m^4 + 30 m^3) \sin(q_3) + (50 m^4 + 16 m^3) \cos(q_3) + 97 m^4 + 17 m^3 & 0 \\ -(444 m^4 + 18 m^3) \sin(q_2) + (-2 m^4 - 4 m^3) \cos(q_2) - 128 m^4 - 60 m^3 \sin(q_3) + ((-2 m^4 - 4 m^3) \sin(q_2) + (-44 m^4 - 18 m^3) \cos(q_2) - 100 m^4 - 30 m^3) \cos(q_3) + (28 m^4 + 18 m^3) \sin(q_3) & (64 m^4 + 30 m^3) \sin(q_3) + (50 m^4 + 16 m^3) \cos(q_3) + 97 m^4 + 17 m^3 & (64 m^4 + 30 m^3) \sin(q_3) + (50 m^4 + 16 m^3) \cos(q_3) + 97 m^4 + 17 m^3 & 0 \\ -(444 m^4 + 18 m^3) \sin(q_2) + (-2 m^4 - 4 m^3) \cos(q_2) - 64 m^4 - 30 m^3 \sin(q_3) + ((-2 m^4 - 4 m^3) \sin(q_2) + (-44 m^4 - 18 m^3) \cos(q_2) - 50 m^4 - 16 m^3) \cos(q_3) - 97 m^4 - 17 m^3 & (64 m^4 + 30 m^3) \sin(q_3) + (50 m^4 + 16 m^3) \cos(q_3) + 97 m^4 + 17 m^3 & (64 m^4 + 30 m^3) \sin(q_3) + (50 m^4 + 16 m^3) \cos(q_3) + 97 m^4 + 17 m^3 & 0 \\ 0 & 0 & 0 & m^4 \end{pmatrix}$$

1.4.1.2 Rotatorischer Anteil

```

I1:matrix([I1xx,0,0],[0,I1yy,0],[0,0,I1zz])$
I2:matrix([I2xx,0,0],[0,I2yy,0],[0,0,I2zz])$
I3:matrix([I3xx,0,0],[0,I3yy,0],[0,0,I3zz])$
I4:matrix([I4xx,0,0],[0,I4yy,0],[0,0,I4zz])$

I_0_s1:trigsimp(R_0_1.I1.transpose(R_0_1))$
I_0_s2:trigsimp(R_0_2.I2.transpose(R_0_2))$
I_0_s3:trigsimp(R_0_3.I3.transpose(R_0_3))$
I_0_s4:trigsimp(R_0_4.I4.transpose(R_0_4))$
I_0_s:[I_0_s1,I_0_s2,I_0_s3,I_0_s4]$

sum:0$
for i:1 while i <= 4 do (sum:sum+((transpose(J_w[i]).I_0_s[i]).J_w[i]))$
MM_w:trigsimp(sum);

$$\begin{pmatrix} I_{4xx}+I_{3xx}+I_{2xx}+I_{1xx} & I_{4xx}+I_{3xx}+I_{2xx} & I_{4xx}+I_{3xx} & 0 \\ I_{4xx}+I_{3xx}+I_{2xx} & I_{4xx}+I_{3xx}+I_{2xx} & I_{4xx}+I_{3xx} & 0 \\ I_{4xx}+I_{3xx} & I_{4xx}+I_{3xx} & I_{4xx}+I_{3xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$


```

1.4.1.3 Gesamt

```

MM:trigsimp(MM_v+MM_w);

$$\begin{pmatrix} -(488 m^4 + 36 m^3) \sin(q_2) + (-4 m^4 - 8 m^3) \cos(q_2) - (444 m^4 + 18 m^3) \sin(q_3) + (-2 m^4 - 4 m^3) \cos(q_3) - (2004 m^4 + 44 m^3) \sin(q_2) \cos(q_3) + 4496 m^4 + 128 m^3 & -(128 m^4 + 60 m^3) \sin(q_3) + (100 m^4 + 32 m^3) \cos(q_3) + 165 m^4 + 85 m^3 + 17 m^2 + 4 I_{4xx} + 4 I_{3xx} + 4 I_{2xx} & (64 m^4 + 30 m^3) \sin(q_3) + (50 m^4 + 16 m^3) \cos(q_3) + 97 m^4 + 17 m^3 + 4 I_{4xx} + 4 I_{3xx} & 0 \\ -(444 m^4 + 18 m^3) \sin(q_2) + (-2 m^4 - 4 m^3) \cos(q_2) - (444 m^4 + 18 m^3) \sin(q_3) + (-2 m^4 - 4 m^3) \cos(q_3) - (2004 m^4 + 44 m^3) \sin(q_2) \cos(q_3) + 4496 m^4 + 128 m^3 & -(128 m^4 + 60 m^3) \sin(q_3) + (100 m^4 + 32 m^3) \cos(q_3) + 165 m^4 + 85 m^3 + 17 m^2 + 4 I_{4xx} + 4 I_{3xx} + 4 I_{2xx} & (64 m^4 + 30 m^3) \sin(q_3) + (50 m^4 + 16 m^3) \cos(q_3) + 97 m^4 + 17 m^3 + 4 I_{4xx} + 4 I_{3xx} & 0 \\ -(444 m^4 + 18 m^3) \sin(q_2) + (-2 m^4 - 4 m^3) \cos(q_2) - (444 m^4 + 18 m^3) \sin(q_3) + (-2 m^4 - 4 m^3) \cos(q_3) - (2004 m^4 + 44 m^3) \sin(q_2) \cos(q_3) + 4496 m^4 + 128 m^3 & -(128 m^4 + 60 m^3) \sin(q_3) + (100 m^4 + 32 m^3) \cos(q_3) + 165 m^4 + 85 m^3 + 17 m^2 + 4 I_{4xx} + 4 I_{3xx} + 4 I_{2xx} & (64 m^4 + 30 m^3) \sin(q_3) + (50 m^4 + 16 m^3) \cos(q_3) + 97 m^4 + 17 m^3 + 4 I_{4xx} + 4 I_{3xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$


```

1.4.2 Coriolismatrix

```

CM:zeromatrix(4,4)$
for j:1 while j<=4 do(
  for k:1 while k<=4 do(
    for i:1 while i<=4 do(
      cc_ijk:1/2*(diff(MM[k][j],q[i])+diff(MM[k][i],q[j])-diff(MM[i][j],q[k])),
      CM[k][j]:CM[k][j]+cc_ijk*q_d[i]
    )))$
trigsimp(CM);

```

$$\begin{pmatrix}
 -((m^4+2m^3)\sin(q_2)+(22m^4+9m^3)\cos(q_2)+25m^4+8m^3)\sin(q_3)+((22m^4+9m^3)\sin(q_2)+(m^4+2m^3)\cos(q_2)+25m^4+8m^3)\cos(q_3) & ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d3} & ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d1} & ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d2} \\
 -((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d3} & ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d3} & ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d1} & ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d2} \\
 ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d2} & ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d1} & ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d1} & ((25m^4+8m^3)\sin(q_3)+(-32m^4-15m^3)\cos(q_3))q_{d2} \\
 0 & 0 & 0 & 0
 \end{pmatrix}$$

1.4.3 Vektor der verallgemeinerten Kräfte

```

vk: trigsimp(transpose(j_v_0_E).matrix([fxe],[fye],[fze]));

```

$$\begin{pmatrix}
 (6fze+5\sqrt{3}fye+5fxe)\sin(q_1)+(-10fze+3^{3/2}fye+3fxe)\cos(q_1) & (-10fze+3^{3/2}fye+3fxe)\sin(q_1)+(-6fze-5\sqrt{3}fye-5fxe)\cos(q_1) & (-10fze+3^{3/2}fye+3fxe)\sin(q_1)+(-6fze-5\sqrt{3}fye-5fxe)\cos(q_1) & (-10fze+3^{3/2}fye+3fxe)\sin(q_1)+(-6fze-5\sqrt{3}fye-5fxe)\cos(q_1) \\
 (6fze+5\sqrt{3}fye+5fxe)\sin(q_1)+(-10fze+3^{3/2}fye+3fxe)\cos(q_1) & (-10fze+3^{3/2}fye+3fxe)\sin(q_1)+(-6fze-5\sqrt{3}fye-5fxe)\cos(q_1) & (-10fze+3^{3/2}fye+3fxe)\sin(q_1)+(-6fze-5\sqrt{3}fye-5fxe)\cos(q_1) & (-10fze+3^{3/2}fye+3fxe)\sin(q_1)+(-6fze-5\sqrt{3}fye-5fxe)\cos(q_1) \\
 (6fze+5\sqrt{3}fye+5fxe)\sin(q_1)+(-10fze+3^{3/2}fye+3fxe)\cos(q_1) & (-10fze+3^{3/2}fye+3fxe)\sin(q_1)+(-6fze-5\sqrt{3}fye-5fxe)\cos(q_1) & (-10fze+3^{3/2}fye+3fxe)\sin(q_1)+(-6fze-5\sqrt{3}fye-5fxe)\cos(q_1) & (-10fze+3^{3/2}fye+3fxe)\sin(q_1)+(-6fze-5\sqrt{3}fye-5fxe)\cos(q_1) \\
 -\frac{fye-\sqrt{3}fxe}{2} & & &
 \end{pmatrix}$$

1.4.4 Vektor der Potentialkräfte

```

p:[p_0_s1,p_0_s2,p_0_s3,p_0_s4,p_0_E]$

```

V:0\$

```
for i:1 while i <= 4 do (V:V+((-m[i].g).matrix([0],[0],[-1]).p[i]))$
```

```
vp: trigsimp(transpose(jacobian([V], q)));
```

$$\begin{bmatrix} 2(4gm^4 + gm^3)\sin(q_1) + (-9gm^4 - 4gm^3)\cos(q_1)\sin(q_2) + ((-9gm^4 - 4gm^3)\sin(q_1) + (-4gm^4 - gm^3)\cos(q_1))\cos(q_2) \\ 2(4gm^4 + gm^3)\sin(q_1) + (-9gm^4 - 4gm^3)\cos(q_1)\sin(q_2) + ((-9gm^4 - 4gm^3)\sin(q_1) + (-4gm^4 - gm^3)\cos(q_1))\cos(q_2) \\ 2(4gm^4 + gm^3)\sin(q_1) + (-9gm^4 - 4gm^3)\cos(q_1)\sin(q_2) + ((-9gm^4 - 4gm^3)\sin(q_1) + (-4gm^4 - gm^3)\cos(q_1))\cos(q_2) \\ 0 \end{bmatrix}$$