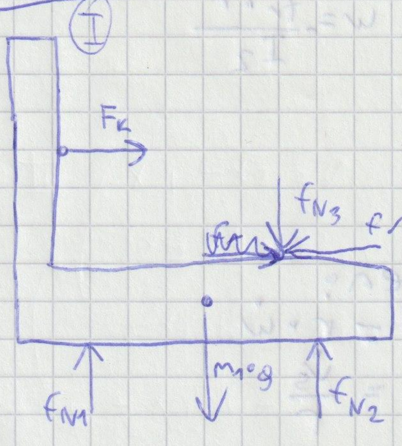
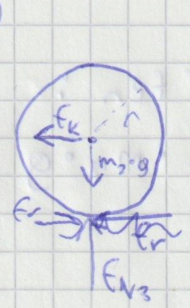
Bonus 2.1





$\dot{s} = v_s$
$\dot{x} = v_x$
$\dot{\phi} = w$

$x_{I}: \; f_k + f_r = m_1 \cdot \dot{v}_x$

$y_{I}: \; f_{N1} + f_{N2} - f_{N3} - m_1 \cdot g = 0$

$x_{II}: \; -f_k - f_r = m_2 \cdot (\ddot{x}_{S2} + \dot{v}_x)$

$y_{II}: \; f_{N3} - m_2 \cdot g = 0$

$R_{II}: \; \theta_2 \cdot \ddot{\phi} = f_r \cdot r$

$f_k = c \cdot (s - s_0) + d \cdot \dot{s} = c \cdot s + d \cdot v_s$

$m_1 \cdot \dot{v}_x = c \cdot s + d \cdot v_s + f_r \Rightarrow \dot{v}_x = \frac{c \cdot s + d \cdot v_s + f_r}{m_1}$

$f_{N1} + f_{N2} - g(m_1 + m_2) = 0$

$m_2 \cdot (\ddot{x}_{S2} + \dot{v}_x) = -c \cdot s - d \cdot v_s - f_r$

$\theta_2 \cdot \ddot{\phi} = + f_r \cdot r \Rightarrow \dot{w} = \ddot{\phi} = + \frac{f_r \cdot r}{\theta_2} \xrightarrow{\theta_2 = I_2} \dot{w} = + \frac{f_r \cdot r}{I_2}$

$\cancel{\ddot{x}_{S2} = \dot{v}_s}$

Rollen

$\cancel{\ddot{x}_{S2} = r \cdot \ddot{\phi}}$

$m_2 \cdot \ddot{x}_{S2} = -c \cdot s - d \cdot v_s - f_r - m_2 \cdot \dot{v}_x$

$\dot{v}_s = \ddot{x}_{S2} = -\left(\frac{c \cdot s + d \cdot v_s + f_r}{m_2} + \frac{c \cdot s + d \cdot v_s + f_r}{m_1} \right)$

$\dot{v}_s = -(c \cdot s + d \cdot v_s + f_r) \left(\frac{1}{m_1} + \frac{1}{m_2} \right)$

Frage 2)

$$\dot{\omega} = -\frac{f_r \cdot r}{I_2}$$

Es muss gelten:

$$|f_r| \leq \mu_r \cdot m_2 \cdot g$$

Rollen:

$$\dot{v}_S = r \cdot \dot{\omega}$$

$$\dot{\omega} = \frac{\dot{v}_S}{r}$$

$$f_r = -\frac{\dot{\omega} \cdot I_2}{r}$$

$$f_r = -\frac{\frac{\dot{v}_S}{r} \cdot \frac{I_2}{1}}{\frac{r}{1}} = -\frac{\dot{v}_S \cdot I_2}{r^2}$$

$$f_r = -\frac{I_2}{r^2} \cdot (c \cdot s + d \cdot v_S + f_r)\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$f_r = -\frac{I_2}{r^2}(c \cdot s + d \cdot v_S)\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - \frac{I_2}{r^2} \cdot \left(\frac{1}{m_1} + \frac{1}{m_2}\right) \cdot f_r$$

$$f_r\left(1 + \frac{I_2}{r^2} \cdot \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\right) = \frac{I_2 \cdot (c s + d v_S)\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}{r^2}$$

$$f_r = \frac{I_2(c \cdot s + d \cdot v_S)\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}{r^2 + I_2 \cdot \left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$$

$$\mu_r \geq \left| \frac{I_2(c \cdot s + d \cdot v_S)\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}{\left(r^2 + I_2 \cdot \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\right) \cdot m_2 \cdot g} \right|$$

$$\Downarrow$$

$$\frac{I_2(c \cdot \phi \cdot r + d \cdot \omega \cdot r) \cdot \left(\frac{1}{m_1} + \frac{1}{m_2}\right)}{\left(r^2 + I_2 \cdot \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\right) \cdot m_2 \cdot g}$$

Frage 3

$$\dot{v}_x = \frac{c \cdot s + d \cdot v_S + f_r}{m_1}$$

$$\dot{v}_x = \frac{c \cdot s + d \cdot v_S}{m_1} + \frac{-I_2(c \cdot s + d \cdot v_S)\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}{m_1 \cdot \left(r^2 + I_2\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\right)}$$

$$= \frac{c s + d \cdot v_S}{m_1}\left(1 - \frac{I_2\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}{r^2 + I_2\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}\right)$$

$$= \frac{c \cdot \phi \cdot r + d \omega \cdot r}{m_1} \cdot \frac{r^2}{r^2 + I_2 \cdot \left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$$

$$\dot{\omega} = \frac{F_r \cdot r}{I_2} = \frac{-r\,(c\cdot\phi r + d\,\omega r)\cdot\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}{r^2 + I_2\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}$$

Frage 4

ungedämpft: $\phi(0) = \max \Rightarrow \phi(t) = \phi_0 \cdot \cos(\omega_0 t)$

$$\ddot{\phi} + \phi \cdot \frac{c\cdot r^2\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}{r^2 + I_2\left(\frac{1}{m_1}+\frac{1}{m_2}\right)} = 0$$

$$\ddot{\phi} + \phi \cdot \omega_0^2 = 0$$

$$\phi(t) = \phi_0 \cdot \cos\left(\sqrt{\omega_0^2} \cdot t\right) =$$

$$= \phi_0 \cdot \cos\left(\sqrt{\frac{c\cdot r^2\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}{r^2 + I_2\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}} \cdot t\right)$$

Frage 5

$$T = \frac{2\pi}{\omega_S} = \frac{2\pi}{\omega_0}$$