Efficient Algorithms Exam Topics from July 1st (Reconstruction) 🥂

1. Bicolored Subset Sum

   Input: A multiset P of numbers that are either "red" or "blue", and a target integer t.

   Question: Does there exist a subset $S \subseteq P$ such that $\sum_{p \in P} p = t$ and there are an equal amount of numbers for each color?

   (a) Describe a polynomial-time Dynamic programming algorithm solving the above problem.

   (b) Give a running time bound in $\mathcal{O}$-notation.

2. (a) For the 5-Coloring problem, there exists a trivial branching algorithm with runtime $\mathcal{O}(5^n \cdot n^{O(1)})$. Give an improved single-exponential algorithm with runtime $\mathcal{O}(c^n \cdot n^{O(1)})$, where $c < 5$.

   (b) Assume there exists a linear-time reduction from 3-SAT to some other problem. Given the following running times, which can we exclude under the Exponential Time Hypothesis? (you don't have to justify your answer)

   - ...
   - ...
   - ...
   - ...

3. For this problem, there was a textual description of the two algorithms for Min Vertex Cover from the lecture on approximation algorithms. The task was to decide which one was a factor-2-approximation algorithm and to prove it.

4. The task description included a description of the CC voting rule (in particular, the problem asking for a k-committee with maximal CC score). The task was to complete a partial ILP (only needed to fill in the parts marked with ❓) encoding to determine a committee maximizing the CC score:

   We have variables x_i with $i \in [n]$, where x_i is true iff voter v_i is represented. Variable y_i ($i \in [m]$) is true iff alternative c_i is in the committee.

   Maximize ❓
   Subject to ❓ $\leq$ ❓ $\forall v_i \in V$
   ❓ $=$ ❓
   $x_i \in \{0,1\}, y_i \in \{0,1\}$

   Additionally, the multiple choice questions asked about the runtime of ILP.

5. Prove the following statement:

   A condorcet winner x comes first in any Kemeny-consensus. 🚬

6. Give an polynomial time algorithm solving the problem $1 \mid\mid L_{\max}$ optimally. If the algorithm is different to the one in the lecture, also give a proof of correctness.

   The additional MC-question were about the complexity of $1 \mid r_j \mid L_{\max}$.

7. The final task was two sets of Multiple choice questions about Matroids (circuits, number of independent sets given k bases of cardinality n), and Max2SAT (FPTAS, pseudopolynomial, etc.)

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