Name: ____________________

# Distributed Algorithms 182.702

Quiz 1 (Ch.2 + Pre, SS 2021), Form: A

Prog. code, registr.. no.: ____________________

Date: ____________________

*Achieved points*: ____________________

## Question 1. If $f(n) = n + O(1)$ and $g(n) = \Omega(\log n)$ for $n \to \infty$, then

Hint: Multiple choices possible.

1. $f(n)/g(n) =$
   - (a) $\Omega(\frac{n}{\log n})$
   - (b) $O(\frac{n}{\log n})$
   - (c) $\Omega(n^{1-\varepsilon})$, for any $\varepsilon > 0$

2. $f(n)g(n) =$
   - (a) $\Omega(n \log n)$
   - (b) $O(n \log n)$
   - (c) $\Omega(n^{1+\varepsilon})$, for any $\varepsilon > 0$

## Question 2. Mark (= tick) the correct statements (for $n \to \infty$):

1. (a) For $k$ large but fixed, $k^n = \Omega(n^k)$
   - (b) $\Theta(n) + \Theta(n) = \Omega(n)$
   - (c) $\log n = o(n^\varepsilon)$, for any $\varepsilon > 0$

## Question 3. Mark (= tick) the correct statements (for $n \to \infty$):

1. (a) Termination is an example of a safety property.
   - (b) In the synchronous model, processors are assumed to execute in lock-step rounds.
   - (c) The message complexity of broadcasting via a given spanning tree of $n$ nodes is $n(n-1)/2$.
   - (d) In an initial configuration, no message can be in transit.

## Question 4. Fill in the missing words:

1. Of which components does the local accessible state of processor $p_i$ consist of?

   ____________________

2. How many ways are there to choose a weakly monotonic sequence of length $k$ from the integers $\{1, \ldots, n\}$? [Repetitions of the same element in the sequence are of course allowed.] ____________

3. Let $G = (V, E)$ be a simple undirected graph, and $deg(v)$ the node degree of $v \in V$. Then, $\sum_{v \in V} deg(v) =$ __________

4. How many complete matchings, i.e., $n$ (unordered) pairs of distinct elements, can be formed from $2n$ distinct elements? ____________________

## Question 5. Prove or disprove in a mathematically sound way:

Write readable!



[*More on backside!*]

1. Using the pigeon hole principle, prove that among $n \geq 2$ positive integers $a_1, \ldots, a_n$ there are $a_i$, $a_j$ such that $a_i \equiv a_j \mod (n-1)$.

2. Using induction on $n \geq 1$, show that $\sum_{k=0}^{m}(-1)^k\binom{n}{k} = (-1)^m\binom{n-1}{m}$, for every $m \geq 0$. [Hint: Recall that $\binom{0}{0} = 1$ and $\binom{0}{1} = 0$.]

3. What is the result of $(\log n + 2 + O(1/n)) \cdot (n + O(\sqrt{n}))$? [Simplify!]

4. Using an indirect proof, show that the removal of any edge $e = (p, q)$ with $p, q \in V(T)$ of a tree $T$ disconnects the resulting graph $T'$.