# Computernumerik (Visual Computing) Test 2

22.January 2025

Time: 100 minutes

## 2.1

Given is the matrix A = $\begin{pmatrix} 2 & 0 & 0 \\ 4 & 3 & 0 \\ 6 & 6 & 6 \end{pmatrix}$

**a)** Calculate the LU-factorization of A with the normalized lower triangular matrix L and the upper triangular matrix U.

**b)** Assume the matrix $B \in \mathbb{R}^{n \times n}$. What is the computational cost of the LU-factorization of B ? Let U be an upper triangluar matrix. What is the cost of solving the system Ux = b using backwards substituion ?

**c)** Assume you know the matrix $B \in \mathbb{R}^{n \times n}$ and its LU-factorization B = LU. How can you solve the system Bx = b with a cost of $\mathcal{O}(n^2)$ ?

## 2.2

Given is the matrix A = $\begin{pmatrix} 0 & 2 \\ 4 & 1 \end{pmatrix}$

**a)** Prove that A has no LU-factorization.

**b)** Does A have a Cholesky-factorization? Why, or why not?

**c)** Calculate the Cholesky-factorization of B = $\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$

## 2.3

**a)** Let $m > n$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ be given. How is the least squares solution for the system $Ax = b$ defined?

**b)** Given is Rectangle with sides $a$ and $b$. 3 independent measurements are made that yield the following approximate results:

- $a \approx 15cm$
- $b \approx 21cm$
- $a + b \approx 39cm$

Determine the refined values of a and b with least squares.

## 2.4

Given is the function $f(x) = x^2 - 2$.

**a)** Suppose you want to solve $f(x*) = 0$ using Newton's method with the start value $x_o = 2$. Perform the first 3 steps of the calculation.

**b)** Under which conditions does Newton's Method (1D) converge quadratically? What does *quadratic convergence* mean in this context?

**c)** Let $g(x) = x^2$ and you want to solve $g(x*) = 0$ using Newton's Method (1D) starting at $x_0 = 1$. Do you expect quadratic convergence? Why, or why not?

## 2.5

Decide if True or False.

**a)**

1. The cost for calculating the Cholesky-factorization is twice the cost of the LU-factorization.
2. Let $A \in \mathbb{R}^{n \times n}$ with the $QR$-factorization A = QR. The factor Q has to be an orthogonal matrix.
3. Gaussian elimination solves a system $Ax = b$ by transforming it into upper triangular form and then using backwards substitution to solve it.

**b)**

1. The proof of quadratic convergence of 1D Newton's method is based on the theory of fixed point iterations.
2. Crout's algorithm is a method to calculate the $LU$-factorization of a matrix.
3. Every invertible matrix $A \in \mathbb{R}^{n \times n}$ has a $QR$-factorization.

**c)**

1. Given is $m > n$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$. The normal equation to $Ax = b$ reads as $A^\top Ax = b$.
2. The Armijo-rule is a way to choose the step-length in a descent method.
3. Given is $m < n$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$. $Ax = b$ is called an underdetermined system.

**d)** Fill in the blanks:

1. Calculating the $QR$-decomposition of a matrix $A \in \mathbb{R}^{n \times n}$ has complexity _.
2. $A \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$. Solving $Ax = b$ with gaussian elimination has complexity _.
3. A matrix $Q \in \mathbb{R}^{n \times n}$ is called orthogonal if _.