Einführung in Wissensbasierte Systeme WS 2022/23, 3.0 VU, 184.737

Exercise Sheet 3 – Answer-Set Programming and Probabilistic Reasoning

For the presentation part of this exercise, mark and upload in pdf format your solved exercises in **TUWEL** until Tuesday, December 06, 23:55 CET. Be sure that you tick only those exercises that you can solve and explain in detail with the necessary theoretical background. In particular, note that ticking exercises which you do not understand can result in a low number of points in the exercise part!

If exercises have subtasks, each subtask counts as one exercise and you will be able to cross them separately.

Please ask questions in the **TUWEL** Forum or visit our tutors during the tutor hours (see **TUWEL**).

Exercise 3.1: Consider the following answer-set programs:

- $\mathcal{P}_1 := \{P(b). \quad Q(a). \quad Q(b). \quad R(X) \leftarrow Q(X).\}$.
- $\mathcal{P}_2 := \{P(a) \vee \neg P(a). \quad \neg Q(a).\}$.
- $\mathcal{P}_3 := \{Q(a). \quad R(a) \leftarrow Q(a). \quad \neg P(a) \leftarrow R(a).\}$.
- $\mathcal{P}_4 := \{Q(a). \quad \neg P(X) \vee S(X) \leftarrow Q(X), \mathit{not}\, P(X).\}$.
- $\mathcal{P}_5 := \{Q(a). \quad P(X) \vee R(X) \leftarrow Q(X).\}$.
- $\mathcal{P}_6 := \{Q(a). \quad R(X) \leftarrow Q(X), \mathit{not}\, P(X). \quad P(X) \leftarrow Q(X), \mathit{not}\, R(X).\}$.

Classify them based on their syntax. That is, decide whether they are ground, Horn, normal, non-disjunctive, basic, or merely a collection of facts. Moreover, use the translation presented in the lecture to transform the given answer-set programs into their corresponding default theories and determine the cases where there is a one-to-one correspondence between the answer sets of the original program and the extensions of its respective translation.

Solution.

	ground	Horn	normal	non-disjunctive	basic	facts
$\mathcal{P}_1$	0	1	1	1	1	0
$\mathcal{P}_2$	1	0	0	0	1	1
$\mathcal{P}_3$	1	0	0	1	1	0
$\mathcal{P}_4$	0	0	0	0	0	0
$\mathcal{P}_5$	0	0	0	0	1	0
$\mathcal{P}_6$	0	0	1	1	0	0

Given the translation given in the lecture one obtains:

- $\delta(\mathcal{P}_1) = \left(\emptyset, \left\{\frac{\top\,:\,\emptyset}{P(b)}, \frac{\top\,:\,\emptyset}{Q(a)}, \frac{\top\,:\,\emptyset}{Q(b)}, \frac{Q(x)\,:\,\emptyset}{R(x)}\right\}\right)$ with its only extension being $Cn(\{P(b), Q(a), Q(b), R(a), R(b)\})$ which corresponds to the only answer set of $\mathcal{P}_1$, i.e. $\{P(b), Q(a), Q(b), R(a), R(b)\}$.

- $\delta(\mathcal{P}_2) = \left(\emptyset, \left\{\frac{\top : \emptyset}{P(a) \vee \neg P(a)}, \frac{\top : \emptyset}{\neg Q(a)}\right\}\right)$ with its only extension being $Cn(\{P(a) \vee \neg P(a), \neg Q(a)\})$. By contrast the two answer sets of $\mathcal{P}_2$ are $\{P(a), Q(a)\}$ and $\{\neg P(a), Q(a)\}$.
- $\delta(\mathcal{P}_3) = \left(\emptyset, \left\{\frac{\top : \emptyset}{Q(a)}, \frac{Q(a) : \emptyset}{R(a)}, \frac{R(a) : \emptyset}{\neg P(a)}\right\}\right)$ with its only extension being $Cn(\{Q(a), R(a), \neg P(a)\})$ which corresponds to the only answer set of $\mathcal{P}_3$, i.e. $\{Q(a), R(a), \neg P(a)\}$.
- $\delta(\mathcal{P}_4) = \left(\emptyset, \left\{\frac{\top : \emptyset}{Q(a)}, \frac{Q(x) : \neg P(x)}{\neg P(x) \vee S(X)}\right\}\right)$ with its only extension being $Cn(\{Q(a), \neg P(a) \vee S(a)\})$. By contrast the two answer sets of $\mathcal{P}_4$ are $\{Q(a), \neg P(a)\}$ and $\{Q(a), S(a)\}$.
- $\delta(\mathcal{P}_5) = \left(\emptyset, \left\{\frac{\top : \emptyset}{Q(a)}, \frac{Q(a) : \emptyset}{P(a) \vee R(a)}\right\}\right)$ with its only extension being $Cn(\{Q(a), P(a) \vee R(a)\})$. By contrast the two answer sets of $\mathcal{P}_5$ are $\{Q(a), P(a)\}$ and $\{Q(a), R(a)\}$.
- $\delta(\mathcal{P}_6) = \left(\emptyset, \left\{\frac{\top : \emptyset}{Q(a)}, \frac{Q(x) : \neg P(x)}{R(x)}, \frac{Q(x) : \neg R(x)}{P(x)}\right\}\right)$ with its extensions being $Cn(\{Q(a), R(a)\})$ and $Cn(\{Q(a), P(a)\})$ which corresponds to the answer sets of $\mathcal{P}_6$, i.e. $\{Q(a), R(a)\}$ and $\{Q(a), P(a)\}$.

$\square$

Exercise 3.2: Consider the following disjunctive logic program $\mathcal{P}$:

$$\mathcal{P} = \left\{ \begin{array}{c} b \leftarrow a, \text{not}\, c. \\ a \vee d \leftarrow \text{not}\, b. \\ d \leftarrow c. \\ c \leftarrow \text{not}\, b. \end{array} \right\}.$$

(i) Determine all answer sets of $\mathcal{P}$. For each proposed answer set S, argue formally that it is indeed an answer set (using the Gelfond-Lifschitz reduct $\mathcal{P}^S$).

(ii) Is it possible to add new rules Q to $\mathcal{P}$ such that $S' = \{a, d\}$ is an answer set of $\mathcal{P} \cup Q$? If yes, give the new rules Q, if not, argue why.

(iii) The same as in (ii), but for $S' = \{a, b\}$.

Solution.

(i)
- We consider $S_1 = \{b\}$, whose reduct is $\mathcal{P}^{S_1} = \{b \leftarrow a.\ d \leftarrow c.\}$, whose minimal model is $\{\} \neq S_1$, hence S_1 is no answer-set.
- For all supersets of S_1 the respective reduct will be $\{\}$ and the minimal model of theses reducts is also $\{\}$, hence no superset of S_1 is an answer-set.
- We consider $S_2 = \{a\}$, whose reduct is $\mathcal{P}^{S_2} = \{b \leftarrow a.\ a \vee d.\ d \leftarrow c.\ c.\}$. Since any model of $\mathcal{P}^{S_2}$ must contain c (is a fact in the reduct) S_2 is not an answer-set.
- We consider $S_3 = \{a, c\}$, whose reduct is $\mathcal{P}^{S_3} = \{a \vee d.\ d \leftarrow c.\ c.\}$. Since any model of $\mathcal{P}^{S_3}$ must contain d, S_3 is not an answer-set.

- We consider $S_4 = \{a, d\}$, whose reduct is $\mathcal{P}^{S_4} = \{b \leftarrow a.\ a \vee d.\ d \leftarrow c.\ c.\}$. Since any model of $\mathcal{P}^{S_4}$ must contain c (is a fact in the reduct), S_4 is not an answer-set.
- Consider $S_5 = \{c, d\}$. The reduct is $\mathcal{P}^{S_5} = \{a \vee d.\ d \leftarrow c.\ c.\}$, of which S_5 is the only minimal model. Hence S_5 is an answer set.
- Since S_5 is an answer set, no subset of S_5 can be an answer set (answer-sets without choice rules or constraints are subset-minimal). So, $\emptyset$, $\{c\}$, $\{d\}$ are no answer-sets.
- Next, we will consider the supersets of S_5 (which do not contain b - see above why). Consider $S_6 = \{c, d, a\}$. The reduct is $\mathcal{P}^{S_6} = \{a \vee d.\ d \leftarrow c.\ c.\}$, whose minimal model is $\{c, d\} \neq S_2$, so S_6 is not an answer-set.

(ii) Not possible. The rule $b \leftarrow a, \mathrm{not}\, c$ is not satisfied by S', so S' is not even a classical model of $\mathcal{P}$.

(iii) It suffices to add support for a, which is possible with a single fact, e.g., $Q = \{a.\}$.

$\square$

Exercise 3.3: (i) We consider an ASP Core-2 program that calculates the number of classes taught on each floor of a university building. Following pieces of information are given:

- The number of floors k
- The number of classes n
- Set of pairs (c, i) such that class c is taught on floor i.

What rule needs to be added in the line 6 given below to achieve our goal?

```
1: floor(1..k).     % for number of floors k
2: class(1..n).     % for number of classes n
3:
4: taught(c, i).    % for each class c that is taught on floor i
5:
6: how_many(F, N) :- ?    % iff the number of classes taught on floor F is N.
7:
8: #show how_many/2.
```

(ii) Under the assumption that at least one class is taught on the top floor, there is no need to include the value of k in the input of the program considered in (i). Which rule will you place in line 5 below, if the value k is not given?

(*Remark*: Note that in the new program below, line 1 is empty. Furthermore, consider that the rule which is correct answer to (i) stays in line 6. You can however solve this exercise, without solving (i).)

```
1:
2: class(1..n).     % for number of classes n
3:
4: taught(c, i).    % for each class c that is taught on floor i
5:
6: how_many(F, N) :- ?    % iff the number of classes taught on floor F is N.
```

```
7:
8: #show how_many/2.
```

(iii) Consider the following rule which defines a large country as a country inhabited by more people than France.

$$\texttt{large(C)} :- \ \texttt{size(C, S1), size(fr, S2), S1} > \texttt{S2}.$$

How will you modify this rule if "large country" should be understood as a country with a population that places it among the top half of the countries on the given list?

Solution.

(i) Line 6 should be replaced by the following rule:

$$\texttt{howmany(F, N)} :- \ \texttt{N} = \#\texttt{count}\{\texttt{C} : \texttt{taught(C, F), class(C)}\}, \texttt{floor(F)}.$$

(ii) Following rule should be indtroduced in line 5:

$$\texttt{floor(1..F)} :- \ \texttt{F} = \#\texttt{max}\{\texttt{J} : \texttt{taught(C, J)}\}.$$

(iii)

$$\begin{aligned}\texttt{large(C)} :- \ & \texttt{size(C, S)}, \#\texttt{count}\{\texttt{C1} : \texttt{size(C1, S1)}\} = \texttt{N},\\ & \#\texttt{count}\{\texttt{C1} : \texttt{size(C1, S1), S1} < \texttt{S}\} >= \texttt{N/2}.\end{aligned}$$

□

Exercise 3.4: For a program $\mathcal{P}$, let again $AS(\mathcal{P})$ denote the set of all answer sets of $\mathcal{P}$. Let $\mathcal{P}$ and $\mathcal{Q}$ be programs. We say that $\mathcal{P}$ and $\mathcal{Q}$ are

(i) *equivalent* if $AS(\mathcal{P}) = AS(\mathcal{Q})$ and

(ii) *strongly equivalent* if $AS(\mathcal{P} \cup \mathcal{R}) = AS(\mathcal{Q} \cup \mathcal{R})$, for every program $\mathcal{R}$.

Prove or refute whether

(a) (i) implies (ii),

(b) (ii) implies (i).

Solution. Obviously, (ii) implies (i) (just take $R = \emptyset$). However, (i) does not imply (ii): take $P = \{a \vee b \leftarrow .\}$, $Q = \{a \leftarrow not\ b.\ b \leftarrow not\ a.\}$, and $R = \{a \leftarrow b. b \leftarrow a.\}$. Then $Q \cup R$ has no answer set, while $P \cup R$ has the answer set $\{a, b\}$. However, P and Q are equivalent since both have the answer sets $\{a\}$ and $\{b\}$. □

Exercise 3.5: Consider a group of four people: Olivia, Mia, Liam and Peter. They hold eight different jobs, such that each of them holds exactly two jobs. The jobs include: accountant, dancer, gardener, baker, designer, librarian, pilot and trader. None of the jobs is gender implied. Furthermore, following constraints are given:

- The jobs of the gardener is held by a male, whereas the accountant is a female.
- The husband of the accountant is the baker.
- Olivia is not a trader.
- Peter has no experience with working with clients.
- Olivia and the accountant and the designer play chess together.
- For the jobs librarian, designer and gardener experience with working with clients is needed.

Determine who holds which jobs.

(a) Define all required predicates, rules, and constraints to represent the problem as an ASP Core-2 program $\mathcal{P}$ such that

- each person p of the group is denoted by a fact $person(p)$,
- each job j is denoted by a fact $job(j)$,
- each job j which required experience with working with clients is denoted by a fact $requires\_experience(j)$,
- $female(p)$ denotes that person p is female,
- $male(p)$ denotes that person p is male,
- $has\_job(p, j)$ denotes that person p has the job j, and
- each answer set of $\mathcal{P}$ represents a valid solution of the described problem.

(b) How many different solutions are there? Justify how you obtained your answer.

Solution.

(a) Facts:

```
person(olivia; mia; liam; peter).
job(accountant; dancer; gardener; baker; designer; librarian; pilot; trader).
female(olivia; mia).
male(liam; peter).
requires_experience(librarian; designer; gardener).
```

Just one person has a specific job:

```
1 {has_job(P, J) : person(P)} 1 :- job(J).
```

Each person has exactly 2 jobs:

```
2 {has_job(P, J) : job(J)} 2 :- person(P).
```

The jobs of the gardener is held by a male, whereas the accountant is a female:

$$:-\ \texttt{person}(\texttt{P}), \texttt{has\_job}(\texttt{P}, \texttt{gardener}), \texttt{not male}(\texttt{P}).$$
$$:-\ \texttt{person}(\texttt{P}), \texttt{has\_job}(\texttt{P}, \texttt{accountant}), \texttt{not female}(\texttt{P}).$$

The husband of the accountant is the baker:

$$:-\ \texttt{person}(\texttt{P}), \texttt{has\_job}(\texttt{P}, \texttt{accountant}), \texttt{has\_job}(\texttt{P}, \texttt{baker}).$$
$$:-\ \texttt{person}(\texttt{P}), \texttt{has\_job}(\texttt{P}, \texttt{baker}), \texttt{not male}(\texttt{P}).$$

Roberta is not a trader:

$$:-\ \texttt{has\_job}(\texttt{roberta}, \texttt{trader}).$$

Peter has no experience with working with clients:

$$:-\ \texttt{has\_job}(\texttt{peter}, \texttt{J}), \texttt{requires\_experience}(\texttt{J}).$$

Olivia and the accountant, and the designer play chess together:

$$:-\ \texttt{has\_job}(\texttt{olivia}, \texttt{accountant}).$$
$$:-\ \texttt{has\_job}(\texttt{olivia}, \texttt{designer}).$$
$$:-\ \texttt{person}(\texttt{P}), \texttt{has\_job}(\texttt{P}, \texttt{accountant}), \texttt{has\_job}(\texttt{P}, \texttt{designer}).$$

(b) There are in total 9 solutions that satisfy all conditions, which can be easily obtained by clingo.

□

Exercise 3.6: A campus bookstore sells three types of computers: laptops, desktops and tablets. In the last semester it sold 450 laptops, 160 desktops and 190 tablets. Reliability rates for the three types of machines are quite different: 22 laptops, 24 desktops and 32 tablets did not work properly and required service. If you choose a random device, what is the probability that it is a properly working device? Moreover, what is the probability that, given that a device is properly working, it is a laptop?

Use the random variable W to express that a device is working properly and the random variables X_L, X_D, X_T to express that a device is a laptop, desktop or tablet respectively.

Solution. Firstly, we compute

- $P(X_L) := \frac{450}{800} = 0.5625;$
- $P(X_D) := \frac{160}{800} = 0.2;$
- $P(X_T) := \frac{190}{800} = 0.2375;$

with X_L, X_D and X_T indicating that a device is a laptop, a desktop or a tablet respectively. Secondly, we compute

- $P(W|X_L) := \frac{450-22}{450} = 0.9511;$
- $P(W|X_D) := \frac{160-24}{160} = 0.85;$
- $P(W|X_T) := \frac{190-32}{190} = 0.8316;$

where W indicates that a device works properly.
The probability that a device works properly:

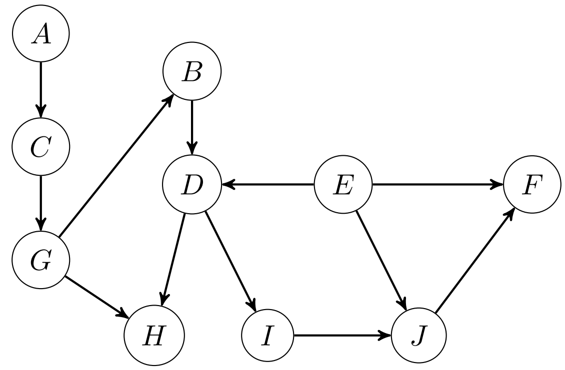
$$\begin{aligned} P(W) &= P(W|X_L)P(X_L) + P(W|X_D)P(X_D) + P(W|X_T)P(X_T) \\ &= 0.9511 \cdot 0.5625 + 0.85 \cdot 0.2 + 0.8316 \cdot 0.2375 = 0.9025 \approx 90,25\,\% \end{aligned}$$

The conditional probability that a device is a laptop, given that is properly working:

$$P(X_L|W) = \frac{P(W|X_L)P(X_L)}{P(W)} = \frac{0.9511 \cdot 0.5625}{0.9025} = 0.5928 \approx 59,28\,\%$$

□

Exercise 3.7: Consider the following graph of a Bayesian network:



Answer the following questions and give a justification for your answer:

(i) Is A conditionally independent of D given evidence B and H?

(ii) Is A conditionally independent of D given evidence B?

(iii) Which subset-minimal evidence would be sufficient such that B is conditionally independent of F given this evidence?

(iv) Which subset-minimal evidence would be sufficient such that A is conditionally independent of I given this evidence?

Solution.

(i) No. Since path $A \rightarrow C \rightarrow G \rightarrow H \rightarrow D$ in the undirected path of the network is not blocked, A is not conditionally independent on D given evidence B and H.

(ii) Yes. The path that was not blocked in the previous case, is now blocked by H, because it is not in the evicence anymore.

(iii) D blocks 2 paths when it is not in the evidence, so only one path remains that needs to be blocked. We can achieve this by adding I to the evidence.

(iv) C blocks all paths between A and I, given that it is in the evidence. Another acceptable answer would be to add B to the evidence (the paths that are not blocked by B will be blocked by H provided that it is not in the evidence).

□