Project and Enterprise Financing

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Lectures 3, 4, and 5: Corporate Bond Financing

Content

1. Zero Bonds and Coupon Bonds
2. Floater
3. Convertible Bonds, Warrant Bonds
4. Spot Rates, Forward Rates, Forward Rate Agreement
5. Interest Rate Swaps
6. Duration

1. Zero Bonds and Coupon Bonds

1.1 Zerobond

- Basis for valuing fixed income securities: **discount function P(T)**
- **P(T)**: **Present value** of a future cash flow of **one currency unit** at time T

Example:

Investment of €0.9759 over a period of 6 month

And we get exactly one Euro after 6 month

➔ Value of the discount function for half a year is: **P(0.5) = 0.9759**

➔ We can already value simple fixed income securities, like a zerobond.

Example:

A zerobond has a maturity of 6 months and a repayment amount of €500.

$$\textbf{Present value} = 500 \cdot 0.9759 = 487.95$$

➔ **Present value** equals the **arbitrage free price**.

On a **perfect market** (small transaction costs, no short selling restrictions), the **actual market price** of such a zerobond has to be **equal** to the (calculated) **present value**.

Otherwise, **arbitrage opportunities exist**, resulting in buy and sell orders that force the market price towards the present value (arbitrage free price) of the zerobond.

Example:

Let us assume the market price of the above zerobond is €480. What will happen with the price?

Arbitrage portfolio:

	Today	**in 6 months**
Buy zerobond	-480	+500
Loan	+ 487.95 (= 500 · 0.9759)	-500
Sum	**+7.95**	0

➔ **Arbitrage profit = 487.95 - 480 = 7.95**

Assumption: We can borrow at current market conditions, i.e., at a P(0.5) of 0.9759, thus, borrowing €0.9759 today and repaying €1.000 in 6 months.

Implied borrowing costs in % p.a.:

$$0.9759 = \frac{1.000}{(1 + \mathrm{YTM}_{0.5})^{0.5}}$$

YTM Yield to maturity

$$\mathrm{YTM}_{0.5} = \left(\frac{1.000}{0.9759}\right)^{\frac{1}{0.5}} - 1 = 0.05 \;\rightarrow\; 5\%\ \text{p.a.}$$

The implied borrowing costs for the 6 months period are 5% p.a.

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More general:

$$YTM_n = \left(\frac{FV}{P}\right)^{\frac{1}{n}} - 1$$

YTM_n Yield to maturity for n periods to maturity

n Periods (i.e. years) to maturity

P Price at t = 0

FV Face value

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Valuation rule 1: Valuation of zerobonds using the discount function

The present value *PV(Z)* of a zerobond with a repayment amount of *Z* currency units and a time to maturity of *T* is defined as:

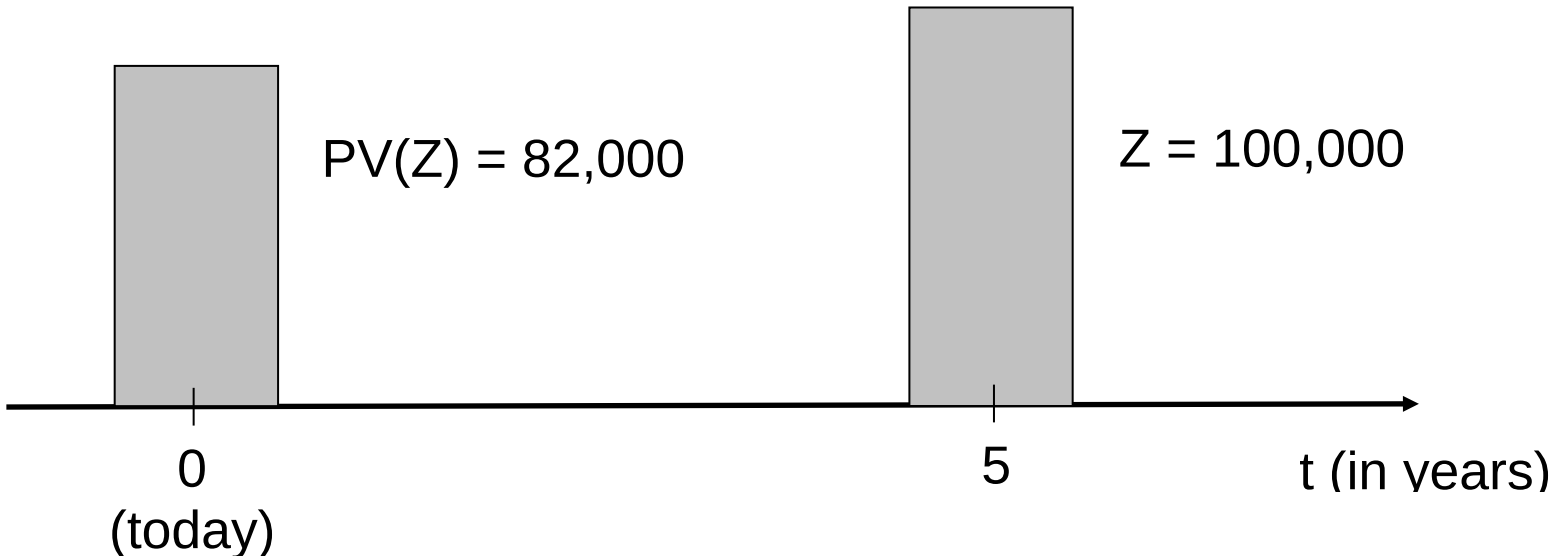
$$PV(Z) = Z \cdot P(T)$$

In a perfect market, the actual market price should correspond to the present value.

Example

Calculate the present value of a zerobond with a repayment amount of Z = €100,000.- and a time to maturity of T = 5 years. The maturity conform discount factor P(5) = 0.82.

$$PV(Z) = Z \cdot P(T) = 100{,}000 \cdot 0.82 = 82{,}000€$$



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Statements about discount functions:

(1) For a **particular** time to maturity T only **one discount factor** $P(T)$ exists. (Otherwise, arbitrage possibilities are available).

(2) $P(0) = 1$. (**Today** the **value** of **one currency unit** is exactly worth **one currency unit**).

(3) $P(\infty) \rightarrow 0$. (With **increasing time to maturity** the **present value converges to zero**).

(4) $P(T) \leq 1$. (One currency unit is **worth less in the future** than today).

(5) $P(T) > 0$. (The **present value** of one currency unit is **always positive**).

(6) $P(T) \leq P(S)$, *if* $T > S$. (The **larger the time to maturity** of a cash flow is, the **lower is its value today**).

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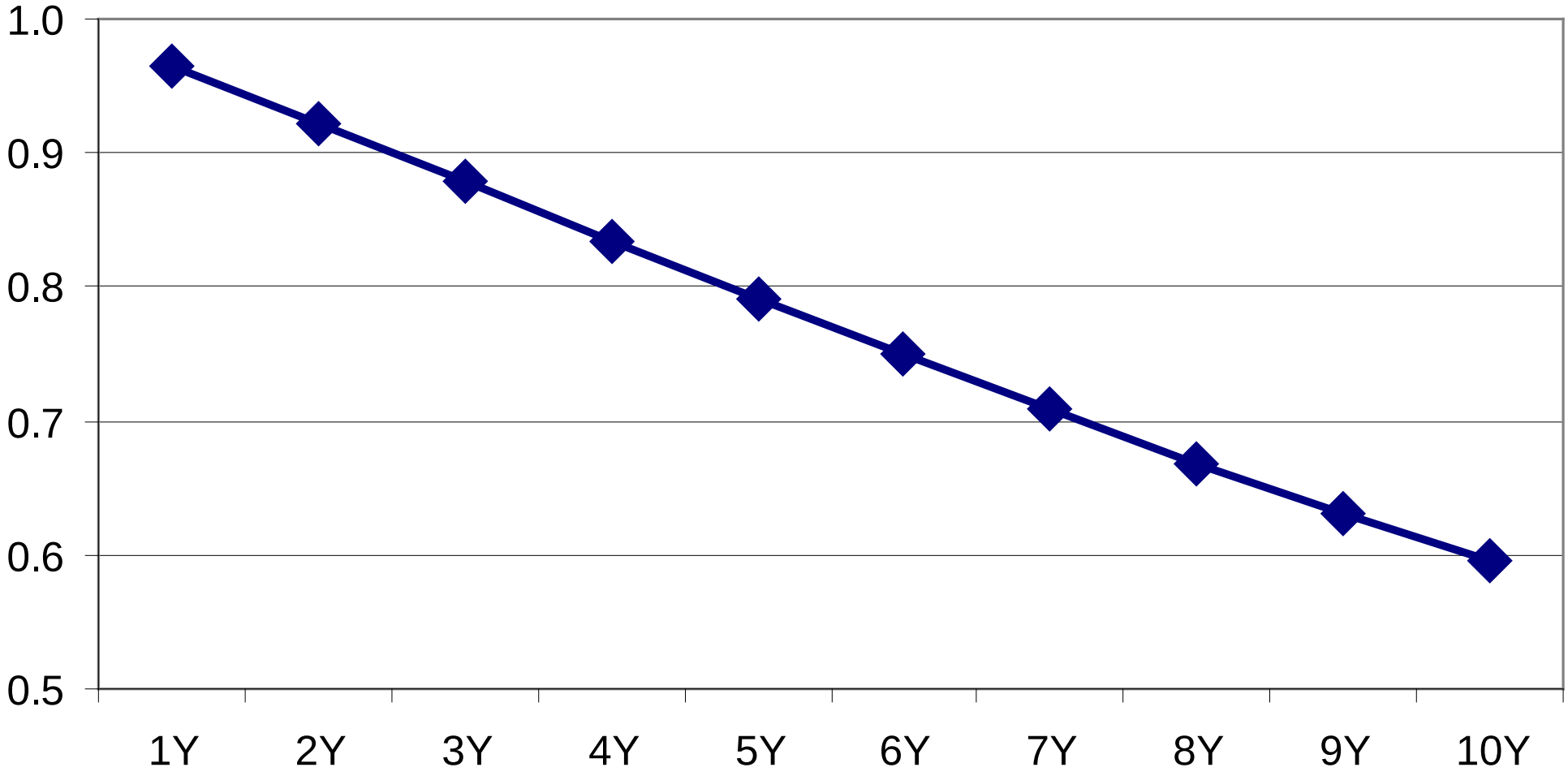
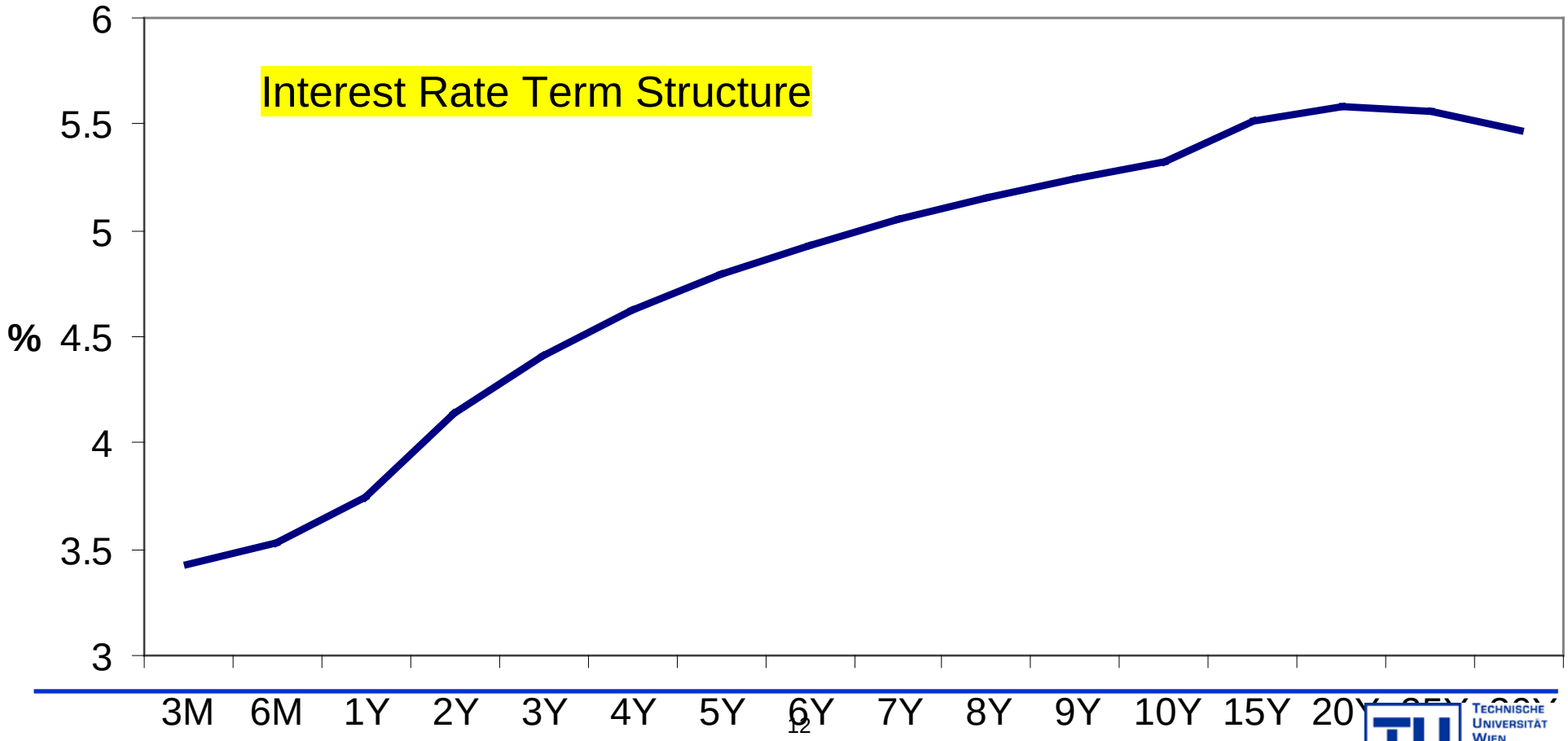
❍ **Example: Euro Discount Function** for 1.7.2002 (time to maturity: 1Y - 10Y)



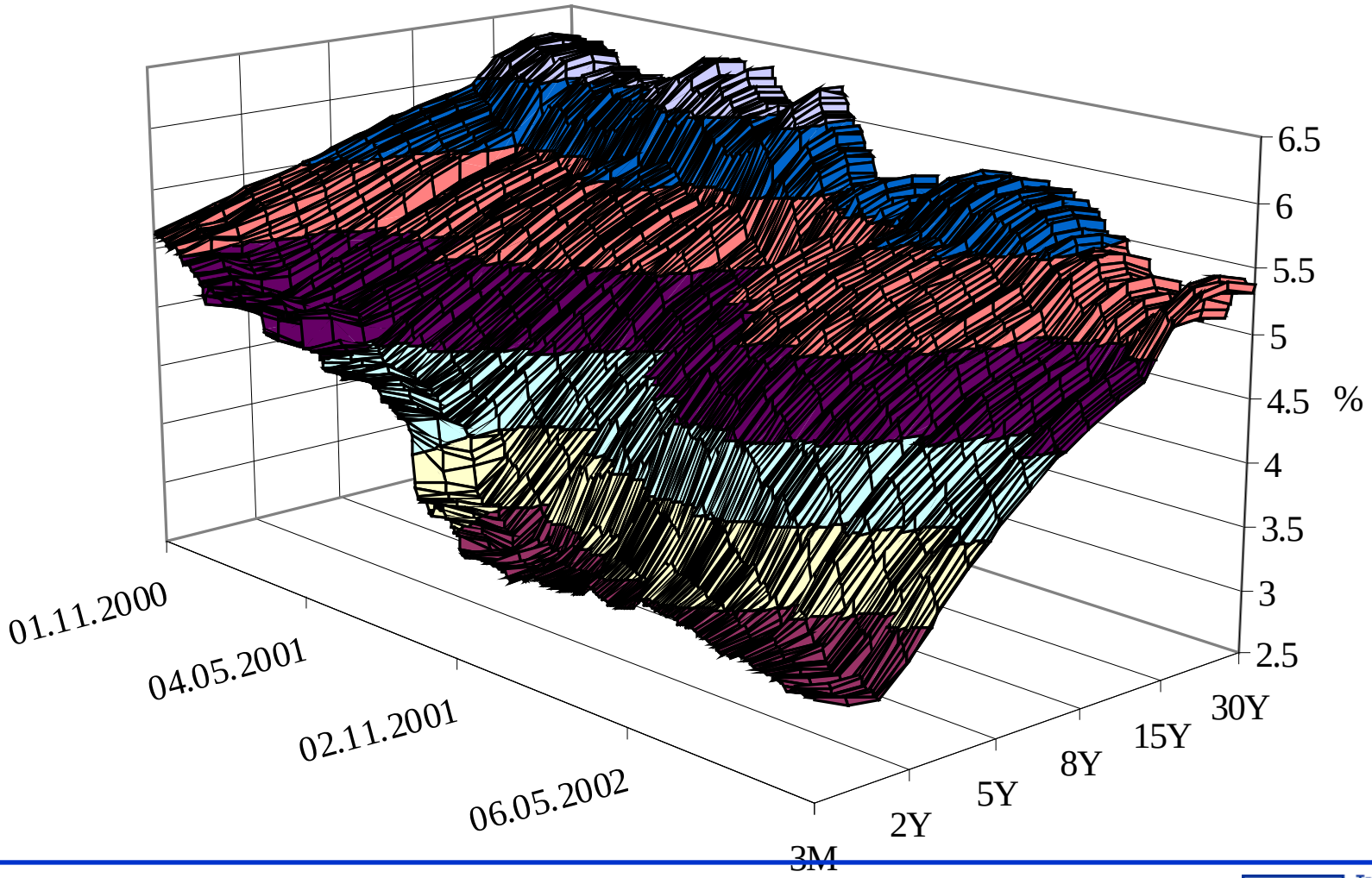
Alternative to the discount function: **Spot rates** (zero-coupon-yields)

Spot rate r(T): annual interest rate underlying a zerobond with maturity of T years.

- **Example: Euro Spot Rates (Zero Coupon Yields) for 1.7.2002**



- **Example: Euro Spot Rates** for the period: 1.11.2000 - 1.11.2002



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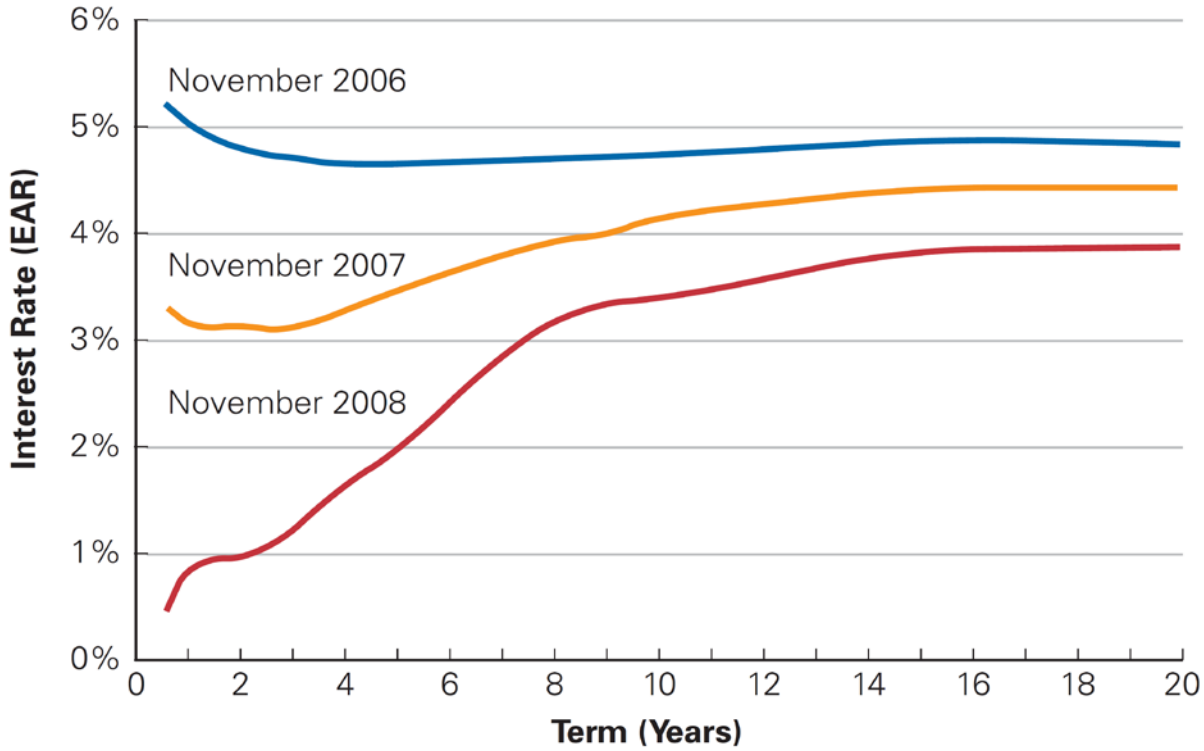
❍ Example: Term Structure of credit risk-free U.S. Interest Rates[1]

Term (years)	Date Nov-06	Nov-07	Nov-08
0.5	5.23%	3.32%	0.47%
1	4.99%	3.16%	0.91%
2	4.80%	3.16%	0.98%
3	4.72%	3.12%	1.26%
4	4.63%	3.34%	1.69%
5	4.64%	3.48%	2.01%
6	4.65%	3.63%	2.49%
7	4.66%	3.79%	2.90%
8	4.69%	3.96%	3.21%
9	4.70%	4.00%	3.38%
10	4.73%	4.18%	3.41%
15	4.89%	4.44%	3.86%
20	4.87%	4.45%	3.87%



The figure shows the interest rate available from investing in risk-free U.S. Treasury securities with different investment terms. In each case, the interest rates differ depending on the horizon. (Data from U.S. Treasury STRIPS.)

[1] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 151.

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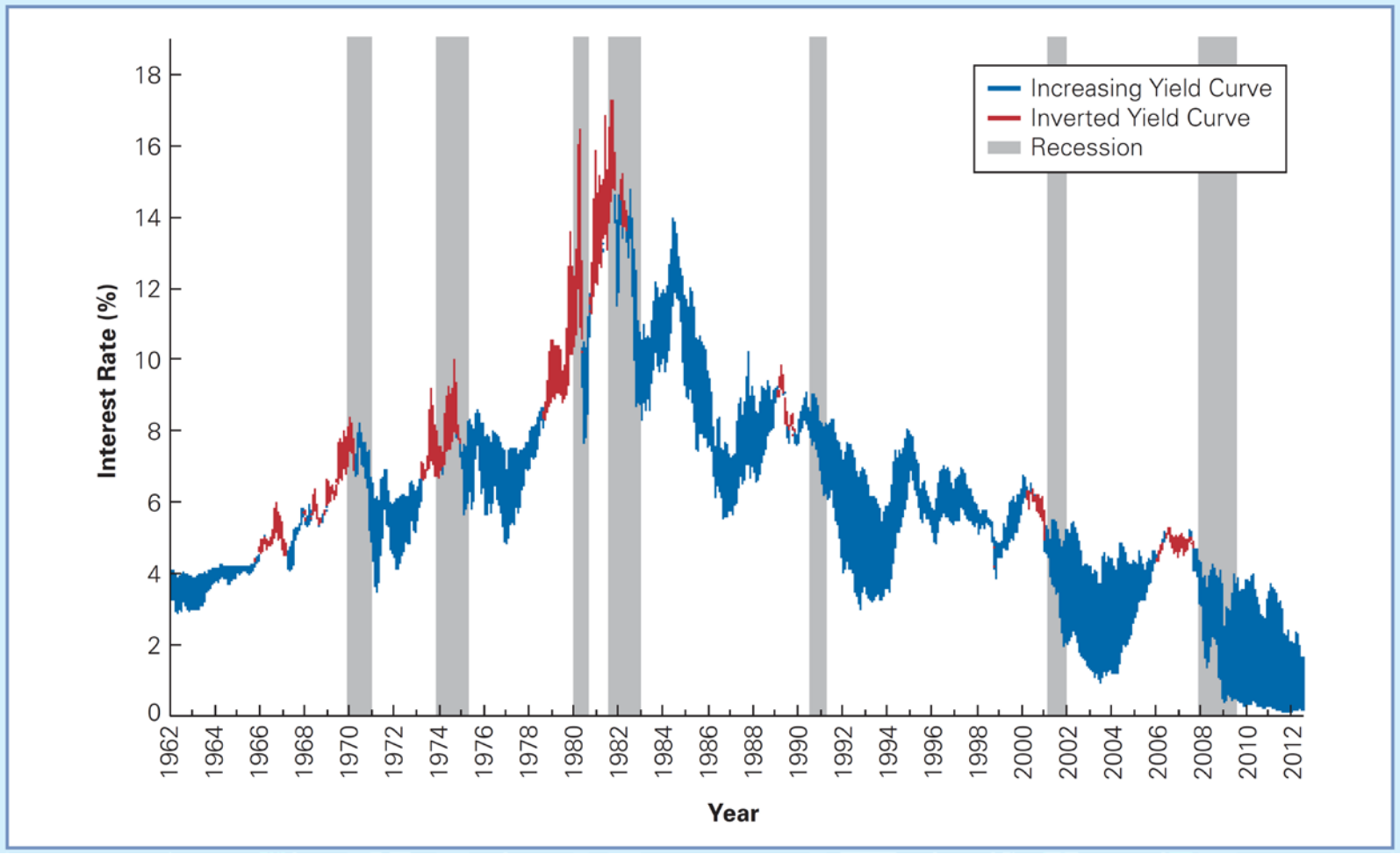
❍ Example: Short-Term vs. Long-Term U.S. Interest Rates[2]



[2] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 153.

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The above graph shows:

- 1-Year and 10-Year U.S. Treasury rates

- Spread between them:
  Blue if the shape of the yield curve is increasing (‘normal’ yield curve)
  Red if the shape of the yield curve is decreasing (‘inverted’yield curve)

- Recessions: Gray bars (National Bureau of Economic Research)

- Inverted yield curves tend to precede recessions by 12–18 months

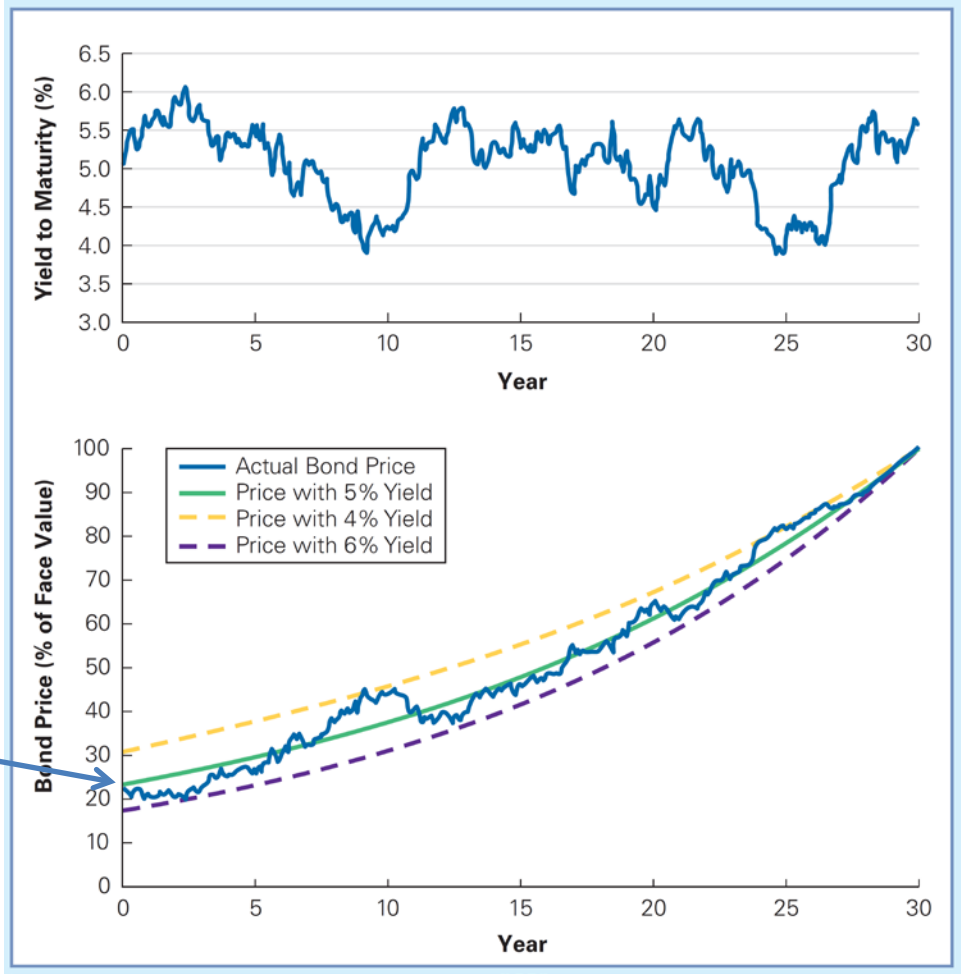
- In recessions: Interest rates tend to fall.
  Short-term rates are dropping more. Thus, the yield curve tends to be steep at the end of a recession.

❍ Yield to maturity and zerobond price development[3]

30 year Zero Bond:

$PV = 100 \cdot (1.05)^{-30}$

$= 23.14$



[3] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 180.

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The above graph shows:

- YTM and price changes for a 30-Y zerobond over its life time.
- Top graph: Changes in the bond's YTM over its life
- Bottom graph: Actual bond price in blue. As the YTM is not constant over then life of the zerobond, the zerobond price fluctuates as it converges to the face value over time.

  Price of the zerobond in case the YTM remains fixed at 4%, 5%, or 6%.

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1.2 Coupon Bond

- When a corporation (or government) wishes to **borrow money** from the **public** on a **long-term basis**
  - Issuing (selling) of debt securities (bonds)

- **Bond:** interest-only loan (the borrower will pay the **interest every period**, but none of the **principal** will be repaid until the **end** of the loan).

Example

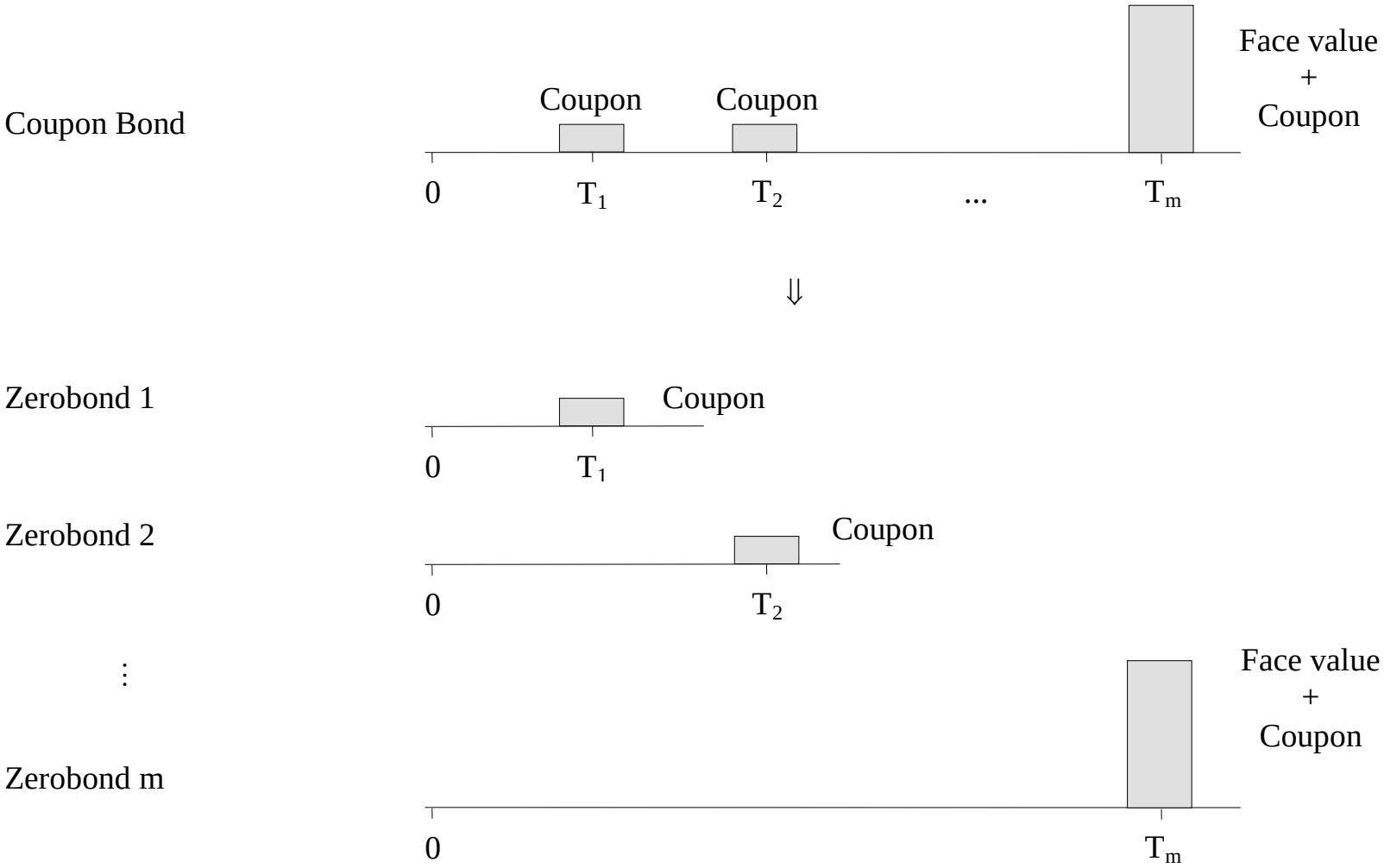
Beck Corporation wants to borrow €10 Mio for 10 years.

Current interest rate on similar debt issued by similar corporations: 10%.

Beck will pay: 0.1•€10 Mio = €1 Mio in interest every year for 10 years.

At the end of year 10, Beck will repay the €10 Mio.

- **Product Mapping** of a **coupon bond** (investor perspective, long-position) with <u>m cash flow components</u>:



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Example:

5% (risk free) €-Government bond, 2 year maturity, face value = €1,000

Cash flows: €50 of the face value in 1 year,
€1,050 of the face value in 2 years

Factors influencing the bond value: Current interest rates, prices of other bonds

❍ **Valuation if interest rates are given**

€-Spot Rates: For 1 Year (1Y): 4% p.a.
For 2 Years (2Y): 6% p.a.

Spot Rates ➔ **Investment/Borrowing** without interim cash in- or outflows

↳ **Duplication** (replication) of the bond.

- **Duplication (replication) strategy:** Nominal (face) value of €1,000

Actions	**today**	**1 Year**	**2 Years**
Investment 1Y (Zerobond)	- 48.08	50.00	
Investment 2Y (Zerobond)	- 934.50		1,050.00
Result	- 982.58	50.00	1,050.00

Thus:

$$48.08 \cdot (1+0.04)^1 = 50.00 \qquad 934.50 \cdot (1+0.06)^2 = 1{,}050.00$$

➔ **Fair Value** = 982.58 = amount of investment to generate the **identical payment flow**!

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Example

On **June 9th 2006**, we want to **value a 5%-coupon bond** (face value: 100). **Coupons** are paid annually on **September 15th** (day count convention: **actual/actual**) with a final maturity in 2008. The coupon bond can be decomposed into three zerobonds:

❍ Cash Flows

Date	Maturity *T* (years)	Cash Flow
15. Sept. 2006	0.2685 = (98/365)	5
15. Sept. 2007	1.2685	5
15. Sept. 2008	2.2685	5 + 100 = 105

❍ Valuation (June, 9th 2006)

To value the coupon bond we furthermore need discount factors or spot rates. Suppose we receive from information providers the relevant spot rates:

Maturity *T*	Cash Flow	Spot Rate	Discount Factor	Present Value
0.2685	5	4.900	0.98724	5·0.98724 = **4.94**
1.2685	5	5.150	0.93829	5·0.93829 = **4.69**
2.2685	105	5.225	0.89089	105·0.89089 = **93.54**
Present Value				**103.17**

103.17 = **dirty present value** (includes accrued interest)

To make this present value comparable with market prices we have to deduct the accrued interest for 9 months (since 15. Sep. 2005).

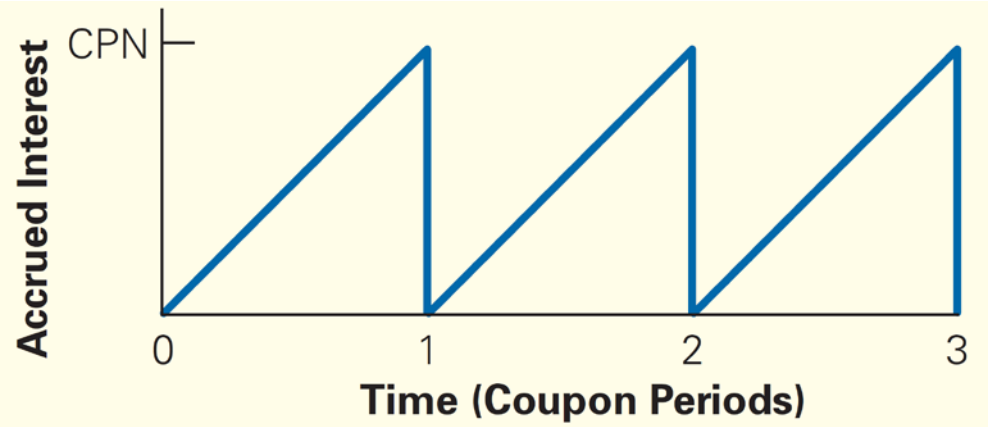
Clean present value: *103.17 - 0.7315·5 = 99.51*.

❍ In general:

- Clean price = Dirty price - accrued interest.

- $$\text{accrued interest} = \text{coupon} \cdot \frac{\text{days since last coupon payment}}{\text{days in current coupon period}}$$

❍ Sawtooth pattern of the accrued interest[4]



[4] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 179.

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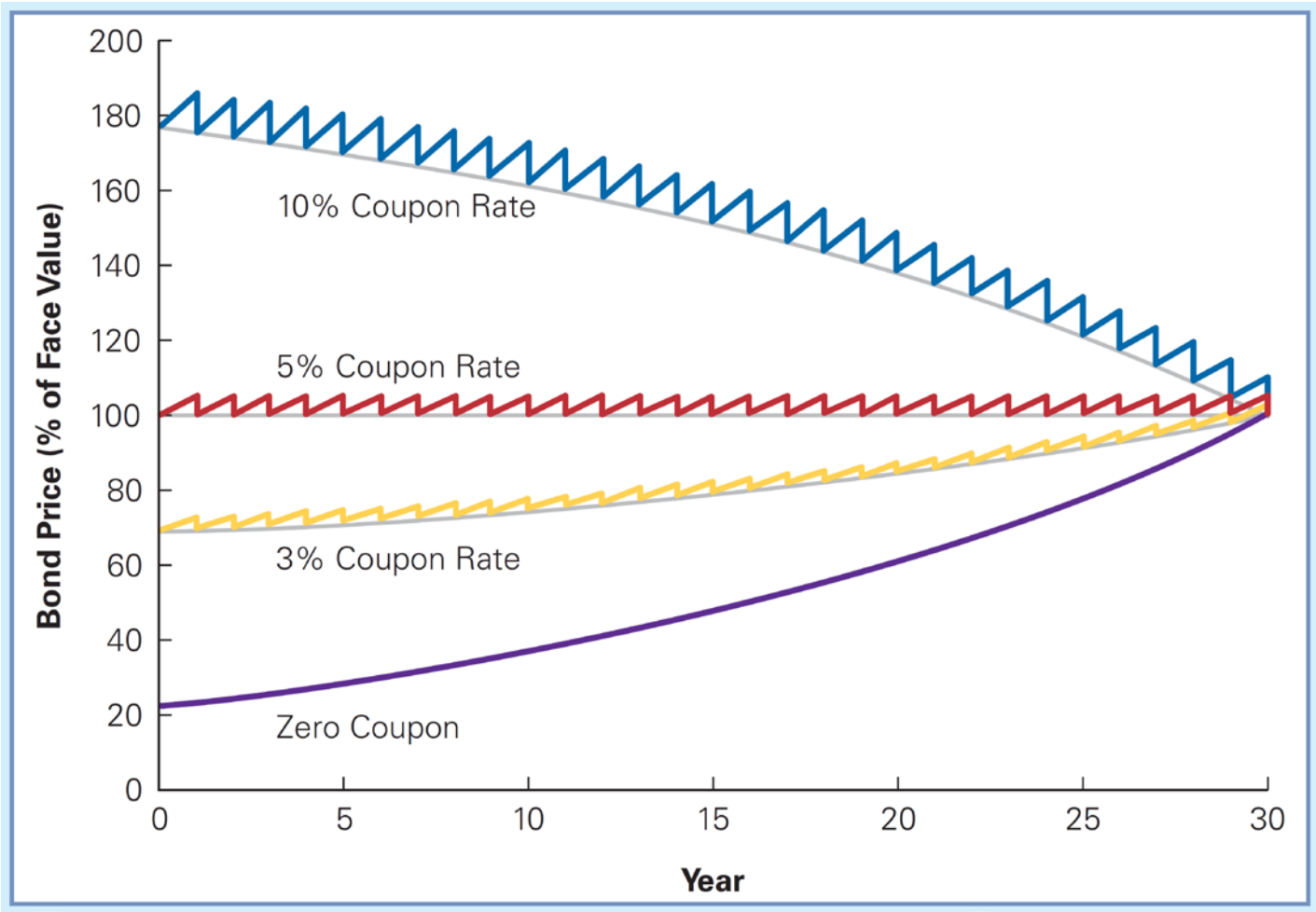
❍ Effect of Time on Bond prices[5]



[5] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 178.

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The above graph shows:

- The graph documents the effect of elapsing time on bond prices when the yield remains constant.
- First, the price of a zero-coupon bond rises smoothly.
- Second, the price of a coupon bond also rises between coupon payments. But it drops on the coupon date, reflecting the amount of the coupon payment.

- Therefore: **Quoted bond prices** in the market are always **clean prices** (to make bonds better comparable to each other).

- But:

  When buying/selling a bond, accrued interest have to be added to the clean price.

  The resulting ‘total price’ then equals the dirty price.

Valuation rule 2: Present Value of a Coupon Bond

The present value *PV(Bond)* of a bond with *m* outstanding cash flows (*Z*) is defined as:

$$PV(Bond) = \sum_{j=1}^{m} Z_j \cdot P(T_j) = \sum_{j=1}^{m} Z_j \cdot (1 + r(T_j))^{-T_j}$$

$$MV(Bond) = PV(Bond) - \text{accrued interest}$$

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- **Yield-to-Maturity**

  ➔ internal rate of return

Assumptions:

All coupon payments can be invested until maturity at the same interest rate.

- Yield-to-maturity (YTM): Numerical solution of the following equation

$$PV = \sum_{j=1}^{m} Z_j \cdot (1 + YTM)^{-T_j} \quad = \quad \textbf{Present value of a bond}$$

(quoted price + accrued interest = dirty price)

Nominal yield > YTM ➔ bond price > 100 (**above par** quotation)

Nominal yield < YTM ➔ bond price < 100 (**under par** quotation)

Nominal yield = YTM ➔ bond price = 100 (**at par** quotation)

- Consequences of the *yield-to-maturity concept*:

(1) *flat* interest rate term structure

(2) interest rate term structure is *not unique*

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Selection of Austrian Government Bonds

Vienna Stock Exchange, Official Market (Amtlicher Handel, AH), 21.2.2019

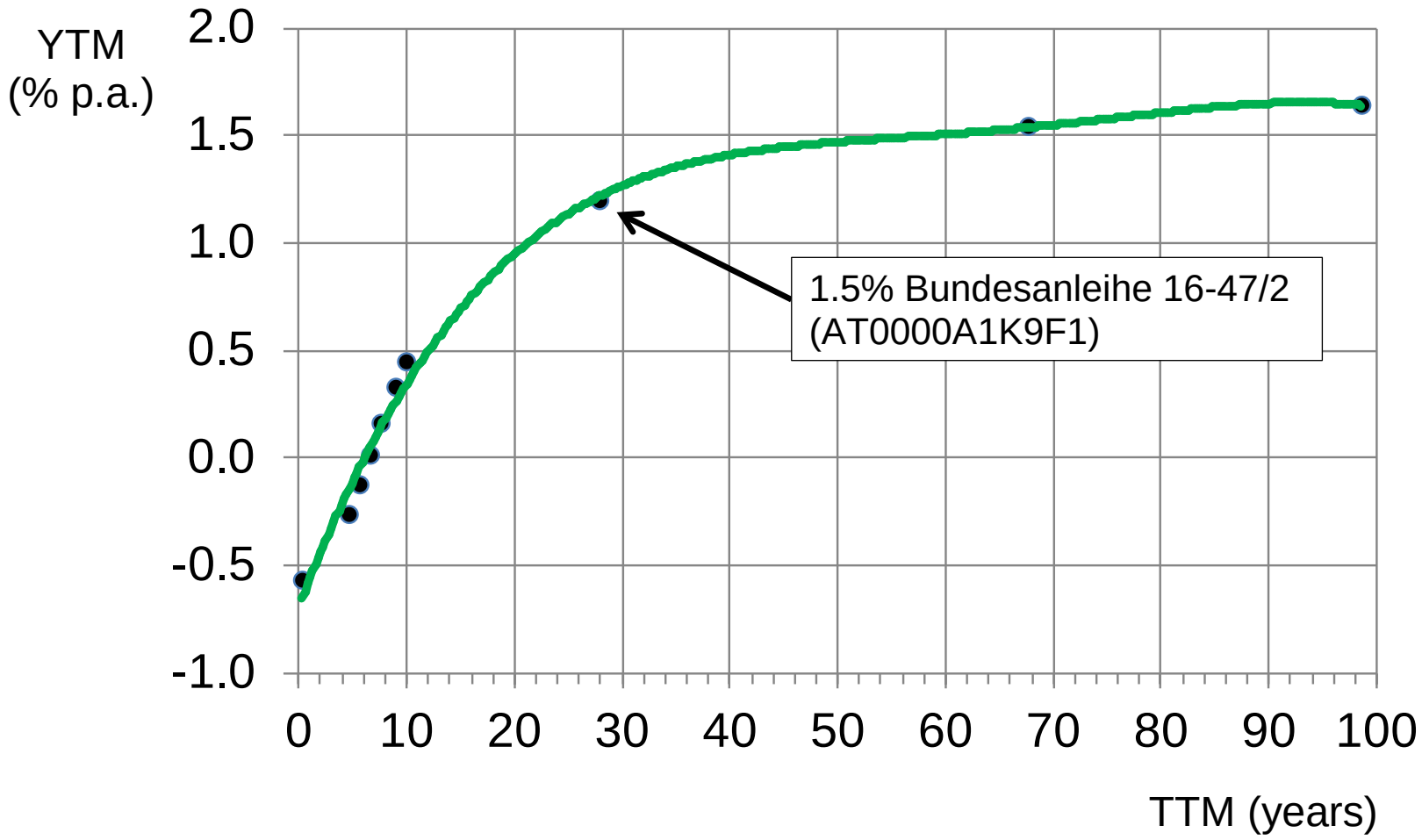
						Bid		Ask	
ISIN	**Name**	**Maturity**	**TTM**	**Last Price**	**YTM**	**Price**	**Vol-ume[1]**	**Price**	**Vol-ume[1]**
AT0000A0VRF9	1.95% Bundesanl. 12-19/3	18.06.2019	0.32	100.78	-0.57	100.7	50	100.9	50
AT0000A105W3	1.75% Bundesanleihe 13-23/2	20.10.2023	4.66	109.43	-0.26	109.3	50	109.3	50
AT0000A185T1	1.65% Bundesanleihe 14-24/1	21.10.2024	5.66	110.08	-0.12	109.9	50	110	50
AT0000A1FAP5	1.2% Bundesanleihe 15-25/1	20.10.2025	6.66	107.95	0.01	107.7	50	107.8	50
AT0000A1K9C8	0.75% Bundesanleihe 16-26/1	20.10.2026	7.66	104.48	0.16	104.3	300	104.4	50
AT0000A1ZGE4	0.75% fix. Bundesanl 18-28/1	20.02.2028	8.99	103.70	0.33	103.4	300	103.6	50
AT0000A269M8	0.5% Bundesanleihe 19-29/1	20.02.2029	9.99	100.48	0.45	99.11	10	101.3	10
AT0000A1K9F1	1.5% Bundesanleihe 16-47/2	23.02.2047	27.99	107.10	1.20	106.6	300	106.8	50
AT0000A1PEF7	1.5% Bundesanleihe 16-86/4	02.11.2086	67.69	98.68	1.54	96.85	300	98.55	300
AT0000A1XML2	2.1% Bundesanl. 17-2117/3	20.09.2117	98.58	122.88	1.64	121	300	122.9	50

[1] Bid and Ask Volume in th €

YTM: Yield to Maturity (% p.a.), **TTM**: Time to Maturity (years)

ISIN: International Securities Identification Number

Interest Rate Term Structure: Yield to Maturities (YTM), 21.2.2019



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Example: 1.5% Bundesanleihe 16-47/2 (AT0000A1K9F1)

Reference Data

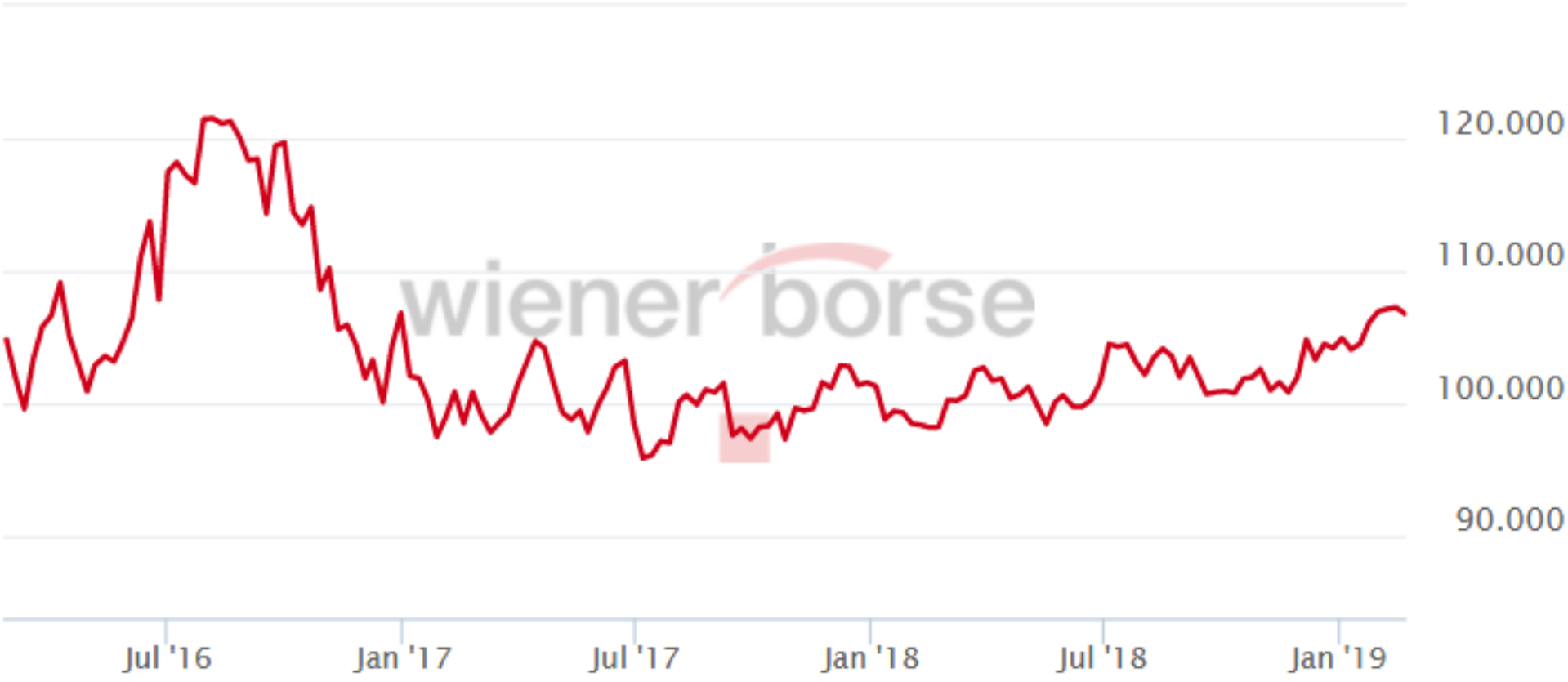
Issuer	Republik Österreich
Market	Official Market
Market Segment	public sector
Type Of Security	Bond
Nominal Currency	EUR
Issue Volume (for unit trading: units)	5,997,044,000.00
Denomination	1,000.00
Minimum Trading Amount	-

First Day Of Trading	02/23/2016
Begin of Maturity	02/23/2016
Maturity	02/20/2047
Time to Maturity	27.99 years
Type of Coupon	Fixed
Interest Rate	1.500%
Coupon Frequency in Months	12
Date Of Coupon	02/20/2020
Interest Rate Adjustment Frequency in Months	-
Next Interest Rate Adjustment	-

Ranking Of The Bond	Senior
Eligible as Trustee Security	Yes
Clearing by CCP.A	Yes
Type Of Quotation	Percentage Quotation
Accrued Interest	exclusive
Trading Hours	09:00 - 17:30
Trading Procedure	Continuous Trading
Time Of Opening Auction	09:00
Time Of Intraday Auction	-
Time Of Closing Auction	17:25

Link - Vienna Stock Exchange – Government Bonds:
https://www.wienerborse.at/marktdaten/anleihen/public-sector/

Price Development: 1.5% Bundesanleihe 16-47/2 (AT0000A1K9F1)



Link: https://www.wienerborse.at/marktdaten/anleihen/public-sector/

Price Development: 2.1% Bundesanl. 17-2117/3 (AT0000A1XML2)

Link: https://www.wienerborse.at/marktdaten/anleihen/public-sector/

❍ Rating and Probability of Default

- Cost of debt are influenced

  (1) by the general **level** of **interest rates** (i.e. a risk-free interest rate term structure, like, e.g., interest rates for German government debt), and

  (2) by the **rating** of the firm. The lower the rating is, the higher the cost of debt.

- Firms that are borrowing money from a bank are **rated by** that **bank**.

- Firms that issue a bond are typically **rated by** rating **agencies**.

- **4 main rating agencies:** Moody's, Standard & Poor's (S&P), Duff & Phelps (D&P) und Fitch.

- **Rating Classes by Rating Agencies**

Moody's	S&P	Fitch	Duff & Phelps	Brief Definition
Aaa	AAA	AAA	AAA	Prime, maximum safty
Aa1	AA+	AA+	AA+	Very high grade, high quality
Aa2	AA	AA	AA	
Aa3	AA-	AA-	AA-	
A1	A+	A+	A+	Upper medium grade
A2	A	A	A	
A3	A-	A-	A-	
Baa1	BBB+	BBB+	BBB+	Lower medium grade
Baa2	BBB	BBB	BBB	
Baa3	BBB-	BBB-	BBB-	
Ba1	BB+	BB+	BB+	Upper low grade, higher risk
Ba2	BB	BB	BB	
Ba3	BB-	BB-	BB-	
B1	B+	B+	B+	Lower low grade, high risk
B2	B	B	B	
B3	B-	B-	B-	
	CCC+			Substantial risk, poor quality
Caa	CCC	CCC	CCC	
	CCC-			
Ca	CC	CC		Extremely risky, nearly in default
C	C	C		
	CI			
		DDD		Default
		DD	DD	
	D	D		

- **Annual Mean Issuer-Weighted Corporate Default Rates (%), Moody's[6]**

Period	Length	Aaa	Aa	A	Baa	Ba	B	Caa-C
1920-2014	95 Years	0.00	0.06	0.10	0.27	1.06	3.36	13.08
1982-2014	33 Years	0.00	0.04	0.05	0.20	1.16	4.77	22.14
1995-2014	20 Years	0.00	0.04	0.08	0.20	0.67	2.77	15.14
2005-2014	10 Years	0.00	0.07	0.11	0.20	0.48	1.44	11.81
		Investmentgrade				Non-Investmentgrade		

[6] Source: Moody's Investors Services (2015): Annual Default Study: Corporate Default and Recovery Rates, 1920-2014, p. 29, Exhibit 30.

- **Annual Mean Credit Loss Rates (%), Moody's[7]**

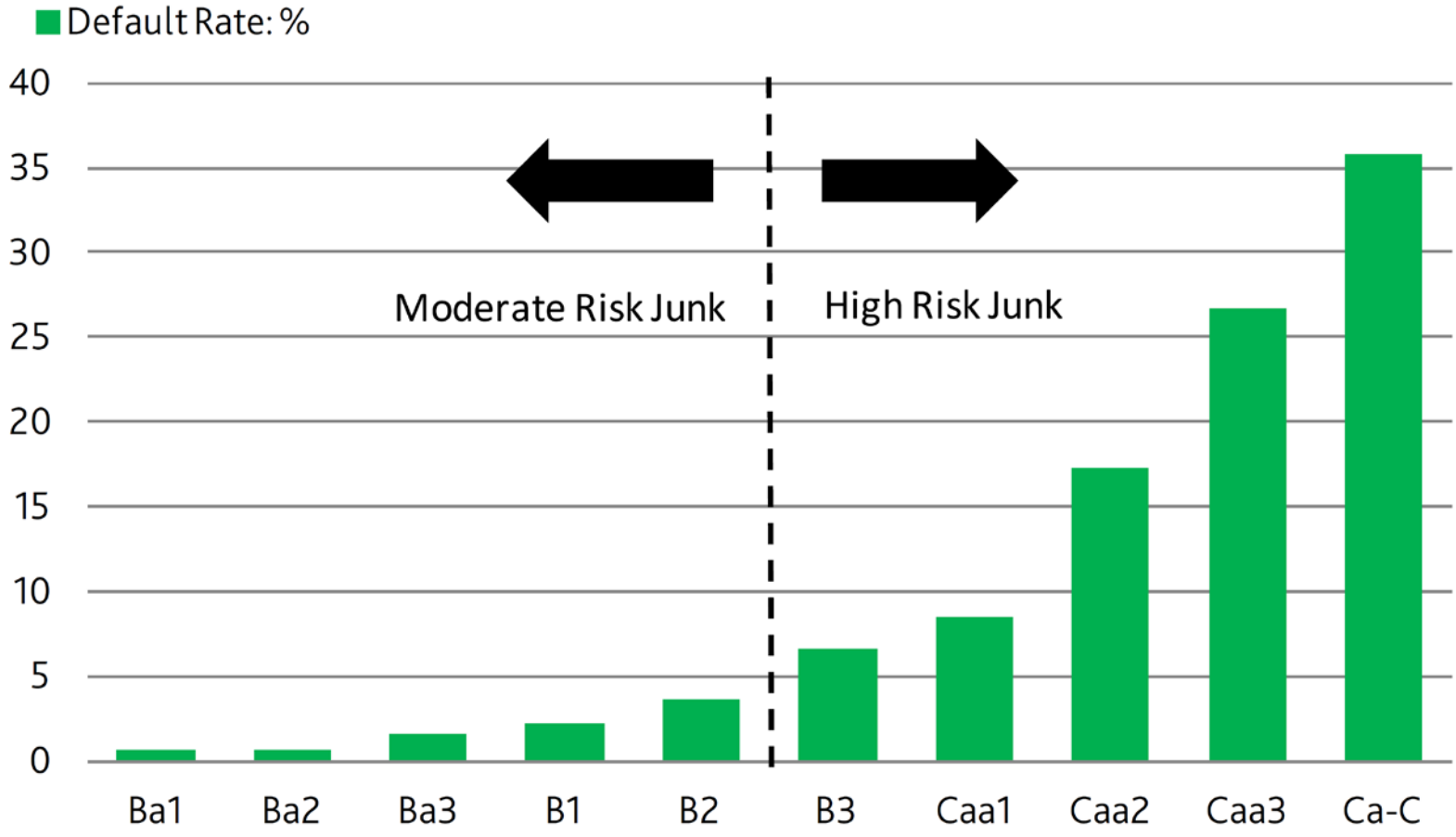
Period	Length	Aaa	Aa	A	Baa	Ba	B	Caa-C
1982-2014	33 Years	0.00	0.03	0.03	0.12	0.67	2.72	12.61

Credit Loss Rate = Default Rate · Recovery Rate

Average Recovery Rate (all bonds) ≈ 30 to 60%

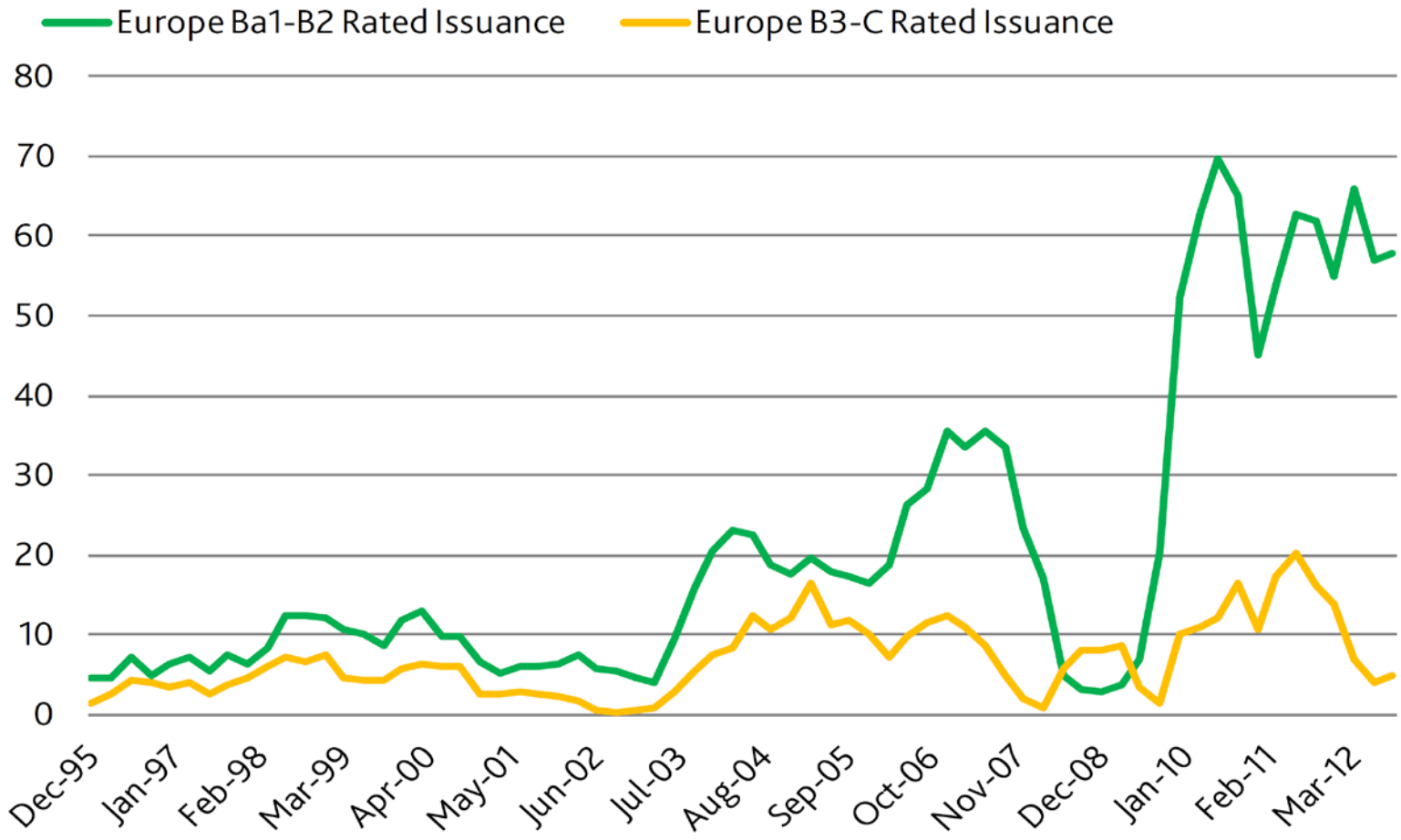
[7] Source: Moody's Investors Services (2015): Annual Default Study: Corporate Default and Recovery Rates, 1920-2014, p. 25, Exhibit 23.

- **High Yield Market: Quality Junk vs. Real Junk, Moody's[8]**



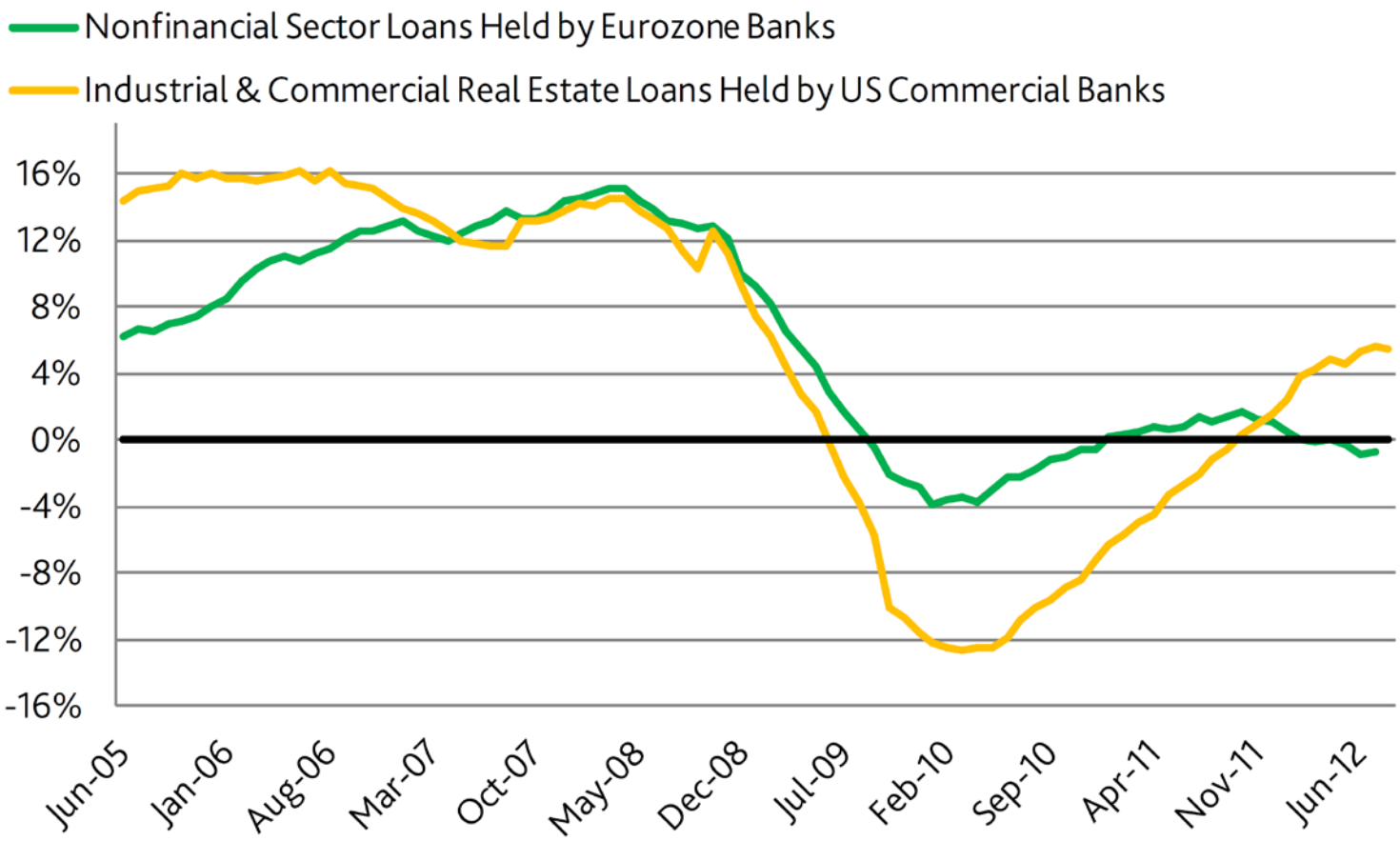
[8] Source: Moody's Analytics (2012): Moody's Capital market Research: Weekly market Outlook, 20.9.2012, p. 8, Figure 4.

- **Issuing Volume* in the European High Yield Market, Moody's[9]**



[9] Source: Moody's Analytics (2012): Moody's Capital market Research: Weekly market Outlook, 20.9.2012, p. 7, Figure 3.

- **Loan Growth*, Commercial Sector by Region, Moody's[10]**



*YoY% Changes, sources: ECB, Federal Reserve

[10] Source: Moody's Analytics (2012): Moody's Capital market Research: Weekly market Outlook, 20.9.2012, p. 8, Fig. 5.

- **High Yield Corporate Bond Market in Europe**[11]
  - Issuers with Non-Investment Grade rating:

    2009: 17%
    2014: 42% (US: 53%)

  - **New** issuers with Non-Investment Grade rating in 2014: 74%

[11] See also Standard and Poor's (2015: Default, Transition, and Recovery: 2014 Annual Global Corporate Default Study And Rating Transitions, 30.4.2015, p. 46-47 and Figures 23-24.

❍ Bonds <u>with</u> credit risk

- Especially corporate bonds
- Also government bonds (to some extent)
- Non risk-free: credit risk $\neq$ 0
  - ↳ Risk of default
  - ↳ A bond's cash flows are not certain!

• Effect of credit risk on bond prices and yields

↳ A lower price and a higher yield for bonds with credit risk compared to an otherwise identical default risk-free bonds

- **Example:**

(i) The yield of a 1-year, zero coupon, default free **government bond** is 4%.

(ii) What is the **price** and the **yield** of a 1-year, zero coupon, **corporate bond**, with a face value of €1,000 and with:

(a) a default probability of zero (i.e., a default is not possible), PD = 0%

(b) a certain default in 1 year with a certain recovery rate of 90%, PD = 100%

(c) a default probability of 50% with a certain recovery rate of 90%, PD = 50%

Ad (a): PD = 0%

Investors receive for a face value of €1,000 in 1 year with certainty €1,000.

The law of one price (see duplication (replication) portfolio) guarantees that the corporate bond will have the same price and yield as the 1-year, zero coupon, default risk-free government bond:

$$P = \frac{1{,}000}{1 + YTM_1} = \frac{1{,}000}{1.04} = €961.54$$

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Ad (b): PD = 100%, recovery rate = 90%

Now the default is certain! Investors know that they will receive only €900 (for a face value of €1,000) at year-end (with certainty). Thus, the payment of €900 is risk-free:

$$P = \frac{900}{1 + YTM_1} = \frac{900}{1.04} = €865.38$$

Remember: The company defaults with certainty but there is a 90% recovery rate!

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Based on the face value (FV) of €1,000 we can calculate the implied YTM:

$$\text{YTM} = \frac{\text{FV}}{\text{P}} - 1 = \frac{1{,}000}{865.38} - 1 = 15.56\% >> 4\%$$

But as there is a certain default, the expected YTM is still:

$$\frac{900}{865.38} - 1 = 4\%$$

There are no future scenarios where investors can receive the FV of €1,000.

In reality: ad (a) with PD = 0%: not so realistic

ad (b) with PD = 100%: not so realistic

Ad (c): PD = 50%, recovery rate = 90%

- *Scenario A*: No default, face value of €1,000 will be repayed
- *Scenario B*: Default, recovery rate = 90%, the company will only repay €900

➔ Expected repayment at t = 0: €950

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This corporate bond can be valued using the yield of other (zero) bonds with equivalent risk (i.e., bond in the <u>same rating</u> class).

As the bond is risky, investors will require a risk premium. Let us assume that an appropriate risk premium is 1%. Then:

$$P = \frac{950}{1 + 0.04 + 0.01} = \frac{950}{1.05} = €904.76$$

Yield to Maturity:

$$YTM_A = \frac{FV}{P} - 1 = \frac{1{,}000}{904.76} - 1 = 10.53\%$$

This is equal to the return in **scenario A**!

But investors receive much less in **scenario B**:

$$YTM_B = \frac{TV}{P} - 1 = \frac{900}{904.76} - 1 = -0.53\%$$

And the average (expected) return is:

$$0.5 \cdot 10.53\% + 0.5 \cdot (-0.53\%) = 5\%$$

$$= 4\% \text{ risk free} + 1\% \text{ risk premium}$$

The take away: The **higher** the probability of default (**PD**),

- the **lower** will be the **bond price**, and
- the **higher** will be the (traditional) **YTM**

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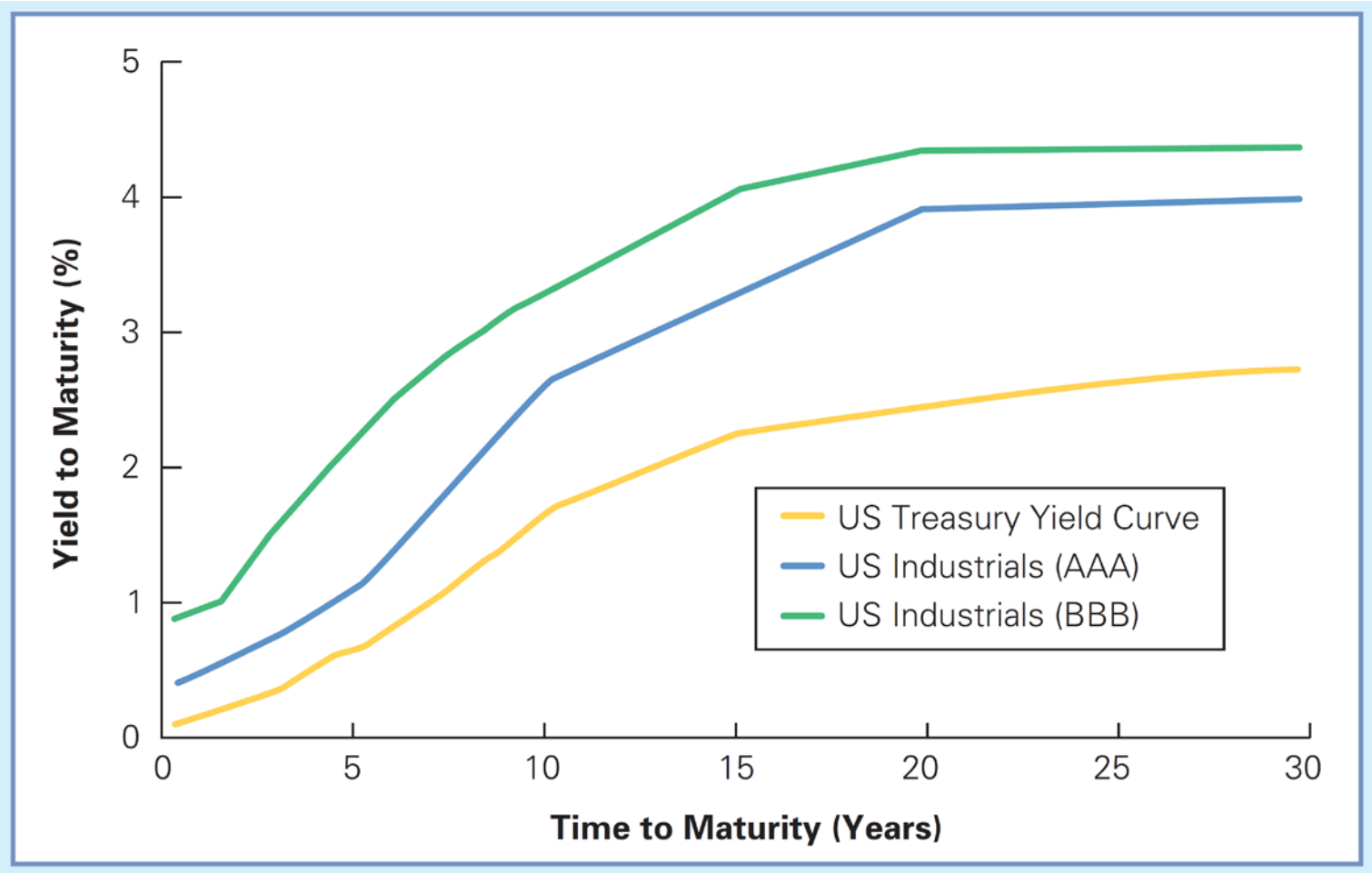
❍ Corporate Yield Curves for varies Ratings (June 2012)[12]



[12] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 188.

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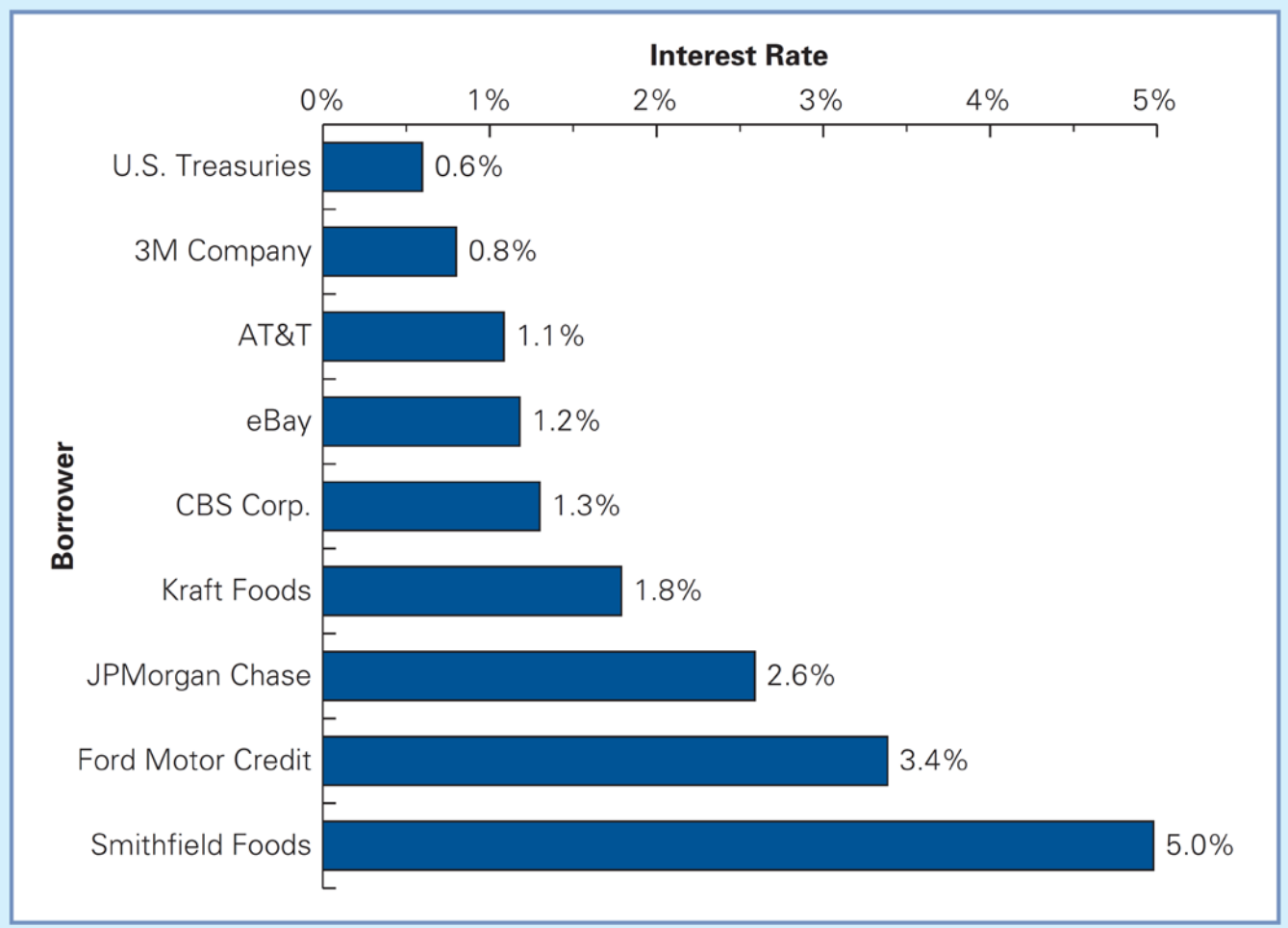
❍ Interest Rates, 5-Year Bonds for various Borrowers (July 2012)[13]



[13] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 156.

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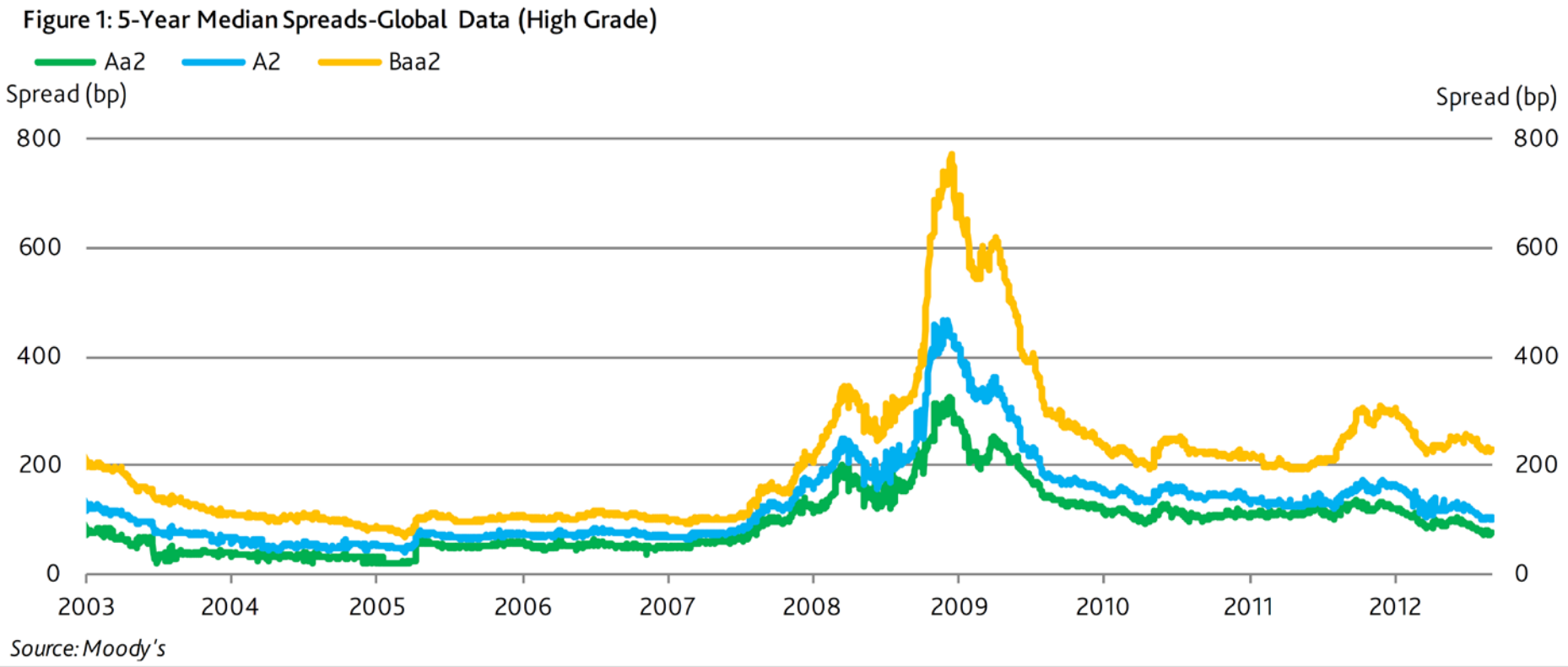
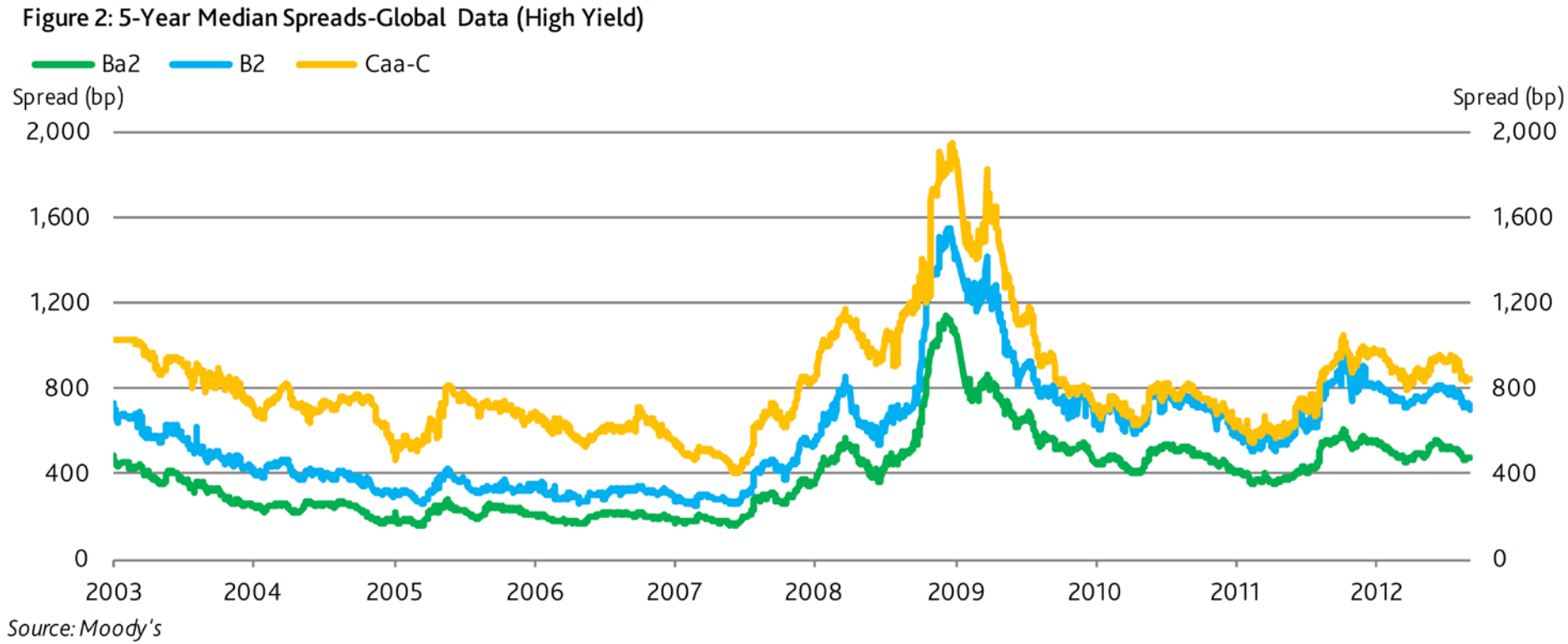
❍ Yield Spreads of Investment Grade Bonds[14]



[14] Source: Moody's Analytics (2012): Moody's Capital market Research: Weekly market Outlook, 20.9.2012, p. 25.

❍ Yield Spreads of Non-Investment Grade Bonds[15]

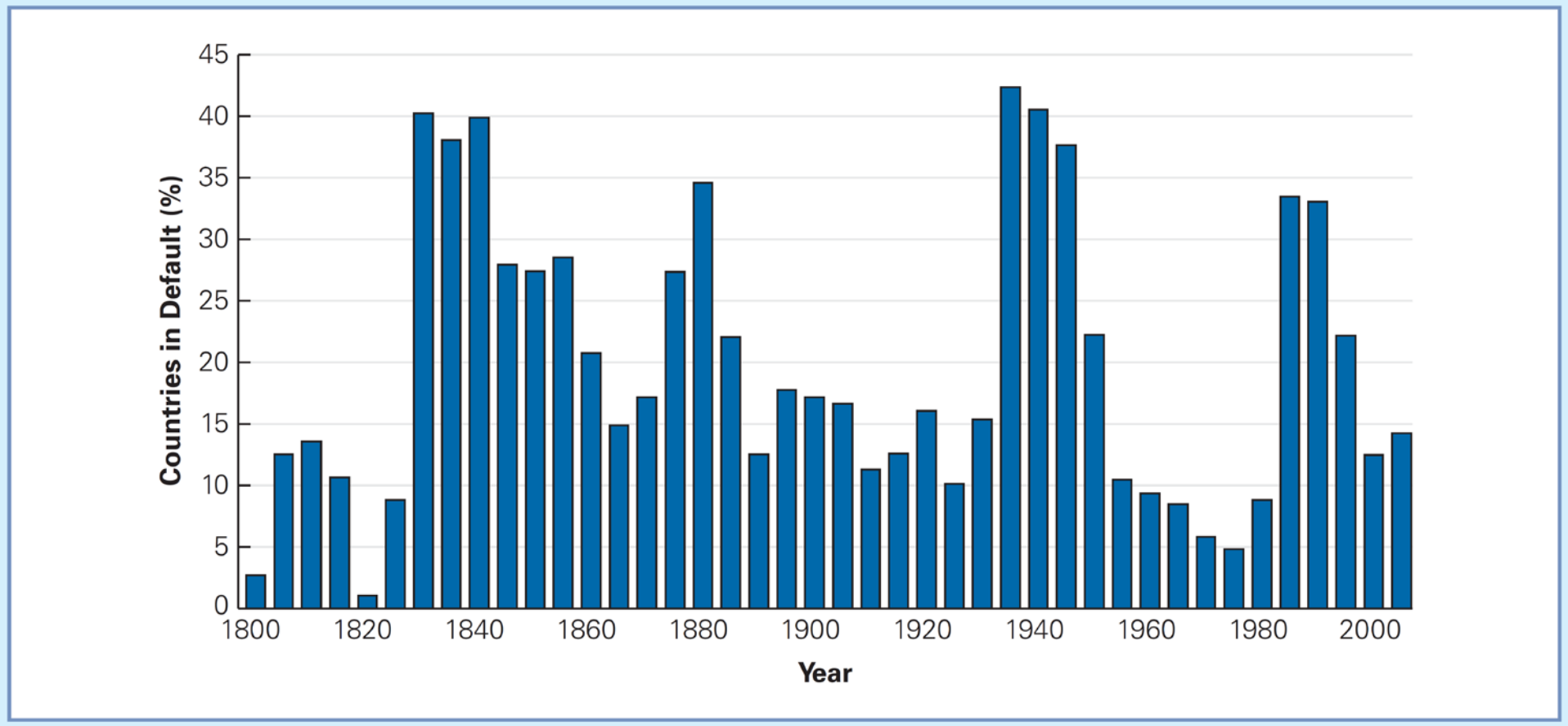


[15] Source: Moody's Analytics (2012): Moody's Capital market Research: Weekly market Outlook, 20.9.2012, p. 25.

Sovereign (Government) Bonds

- These are bonds issued by national governments.
- Bonds of not all countries are considered to be default free.
- Debtor countries in default or in restructuring is far from unique.

❍ Fraction of Debtor Countries in Default or Restructuring[16]



[16] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 190.

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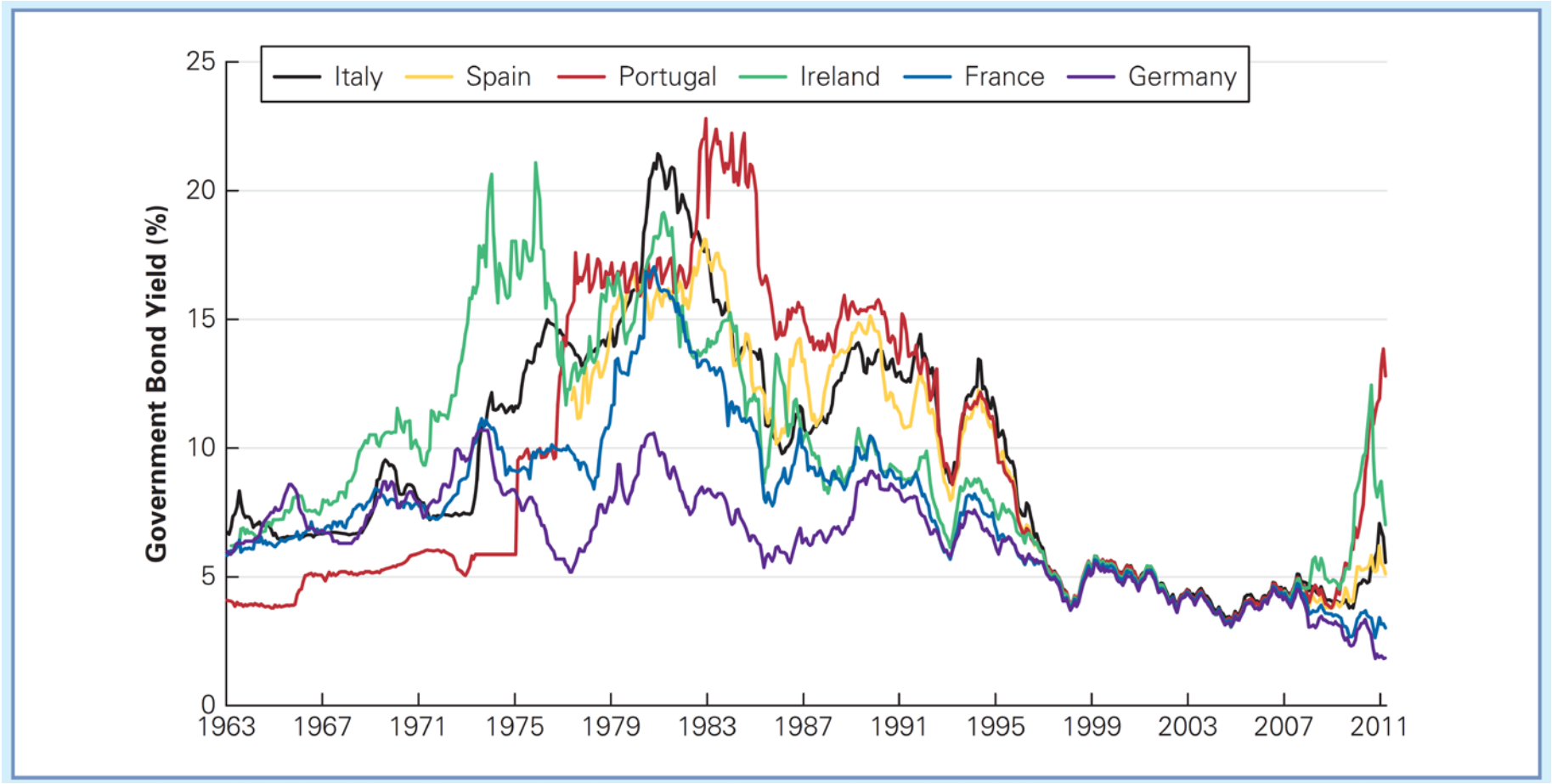
European Corporate Bond Yields[17]



[17] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 191.

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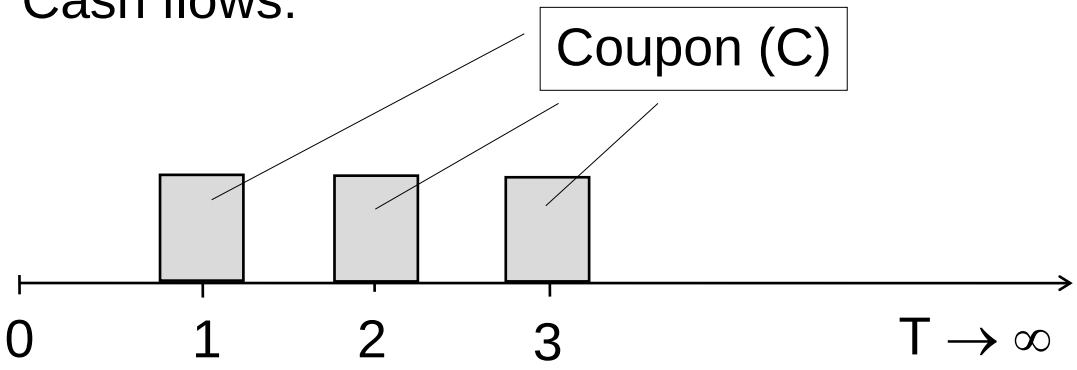
❍ Special case: Perpetual Bond (Consol Bond)

- Coupon payments during maturity period (C > 0)
- No maturity ($T \to \infty$)
- Cash flows:



- **Valuation with flat term structure:**

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots\ldots\ldots$$

$$a = \frac{C}{1+r} \qquad x = \frac{1}{1+r}$$

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$$PV = a\cdot(1 + x + x^2 + \ldots\ldots)$$
$$x\cdot PV = a\cdot x + a\cdot x^2 + \ldots\ldots$$
$$PV - x\cdot PV = a$$

$$PV - \frac{PV}{1+r} = \frac{C}{1+r} \quad \Rightarrow \quad C = (1+r)\cdot PV - PV$$

$$PV = \frac{C}{r}$$

- **Valuation with non-flat term structure:**

$$PV = C \cdot \sum_{t=1}^{T} P(t) + \frac{C}{r} \cdot P(T)$$

e.g.: T = 30 years; P(t) = discount factor

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2. Floater (Floating-Rate Bond)

- Coupon payments are **adjustable**
- Most common: **Money-market floaters**

  ➔ Coupon is tied to a money-market rate (e.g. 1-month EURIBOR rate)

- EURIBOR: EUR Interbank Offered Rate
- The **simplest form** of a floater is a **perfectly indexed floater**

❍ Perfectly Indexed Floater

(a) Valuation at the adjustment date (coupon date):

- **Coupon c** = money-market rate = benchmark rate
- Valuation based on a revolving investment strategy
  ➔ Par value is always invested at the benchmark rate (over coupon period).

Example: 5-year floater, tied to the 12M-Euribor rate (= coupon C), par value = 100

	0	1	2	3	4	5
Long Floater	**-F**	**C(0)**	**C(1)**	**C(2)**	**C(3)**	**100 + C(4)**
Duplication	-100	100 + C(0)	-100	100 + C(2)	-100	100 + C(4)
		-100	100 + C(1)	-100	100 + C(3)	
Result	**-100**	**C(0)**	**C(1)**	**C(2)**	**C(3)**	**100C(4)**

(i) Value at t=0: Price of duplication strategy = 100. Value of the floater (F) is therefore 100.

(ii) Value at t=1, 2, … :

❍ **Last adjustment date** (arbitrage free pricing):

	Adjustment date (t = 4)	Maturity (T = 5)
Long Floater	- F	+ 100 + C(4)
Borrowing (short money market)	+ 100	- 100 - C(4)
	100 – F = 0	0

➔ Arbitrage free price (at the adjustment date): ***F = 100***

A recursive procedure leads to the following statement:

At the <u>start of each coupon period</u> (adjustment date), the arbitrage free price of a perfectly indexed money-market floater is always <u>equal to</u> the <u>par (face) value</u> (often 100).

Valuation rule 3: Present value of a perfectly indexed floater

At **all coupon dates,** the **present value** of a perfectly indexed floater equals the **par (face) value**. Accordingly, a perfectly indexed floater can be **valued *like a zerobond*** with a **maturity until the next coupon date** with the following face value:

Face Value (Zerobond) = Face Value (Floater) + Current Coupon

Terms for a perfectly indexed floater:

(1) The coupon adjustment takes place exactly at the coupon dates.
(2) The duration of the money market rate used to adjust the coupon equals the coupon period (matching maturities).
(3) The coupon payment at the end of the coupon period equals the money market rate at the beginning of this period.
(4) No further agreements (like, e.g., Caps or Floors) exist.

(b) Valuation at arbitrary dates (i.e., between coupon dates):

If we want to value a floater **between two coupon dates**, we have to consider two facts:

(1) The **next coupon C_{t1}** is already **known**.

(2) The **value** of the floater **at the next coupon** (t2) date equals the **face value**.

(t2 – t1) time units **before the next coupon date** the present value at t1 (PV_{t1}) of a perfectly indexed floater can be calculated as:

$$PV_{t1} = 100 \cdot \left(1 + C_{t1}\right) \cdot P_{t1,t2}$$

$P_{t1,\,t2}$: Discount factor for the period from t1 to t2.

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3. Convertible Bond, Warrant Bond

❍ Convertible Bond (Wandelanleihe)

= a bond that can be **converted into** a fixed number of **shares** (before the maturity of the bond **at the holder's option**).

Advantages for investors:

- **Increasing stock prices**, as the conversion rate is fixed in advance.
  - ➔ He/she can wait and **monitor the stock price**.
  - ➔ He/she has a **right but no obligation** to convert the bond into stocks.
- If a conversion is not favorable, he/she will not convert the bond and receive interest from holding the bond.

Disadvantages for investors:

- **Lower coupon rate** compared to "normal" bonds with similar features
  - ➔ Due to the **option** included in a convertible bond

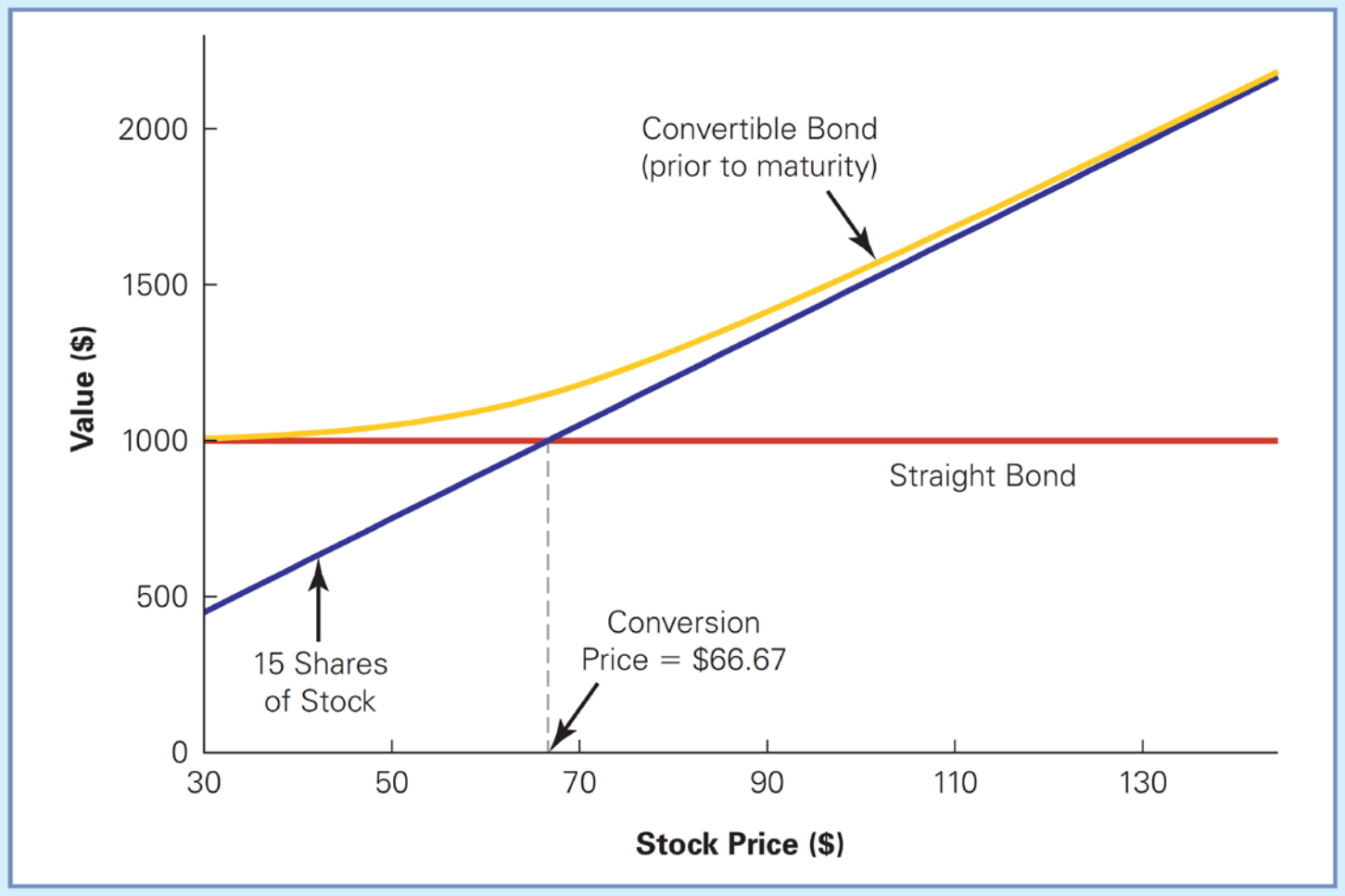
- **Example:**

Let us consider a convertible bond with a face value of $1,000, a conversion rate of 15 (i.e. for a face value of $1,000 an investor receives 15 shares, in case he/she is converting the convertible bond into shares). If he/she is not converting the convertible bond, he/she will receive the face value of $1,000.

Thus, the implied price per share is $1,000/15 = $66.67.

If the share price in the market > $66.67 (= conversion price), the investor will choose to convert, otherwise he/she will take the $1,000.

- **Convertible Bond Value**[18]



[18] Source: Berk and DeMarzo (2014), Corporate Finance, 3rd edition, p. 853.

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❍ Warrant Bond (Optionsanleihen)

= a bond that includes an option. The owner of the (call) option has the right to buy a particular amount of stocks for a prespecified (exercise) price within or at the end of a specified period. These kind of options are also called warrants (Optionsscheine).

Advantages for investors:

- The price of the warrant bond will increase, when the stock price increases. Typically, the warrant is separated from the bond after the issue. In this case bond and option trade separately on the market.

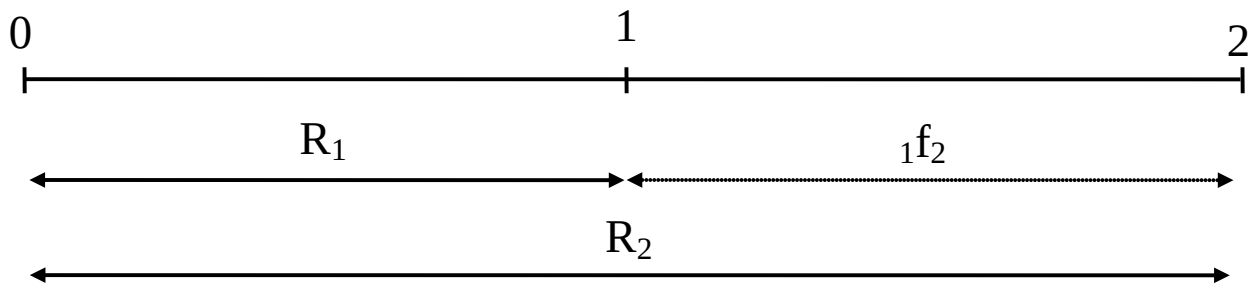
Disadvantages for investors:

- When the stock price drops then also the value of the warrant will decrease. If the stock price falls below the exercise price until maturity of the warrant, then the warrant will be worthless.

4. Spot and Forward Rates, Forward Rate Agreement

❍ Spot and Forward Rates

Example: Forward Rate for year 2



$$(1+R_1)\cdot(1+{}_1f_2)=(1+R_2)^2,\quad (1+{}_1f_2)=\frac{(1+R_2)^2}{1+R_1}\qquad {}_1f_2=\frac{(1+R_2)^2}{1+R_1}-1$$

R_1: Spot rate for one year (first year)
R_2: Spot rate for two years (first two years)
${}_1f_2$: Forward rate: interest rate for the second year (from t = 1 to t = 2)

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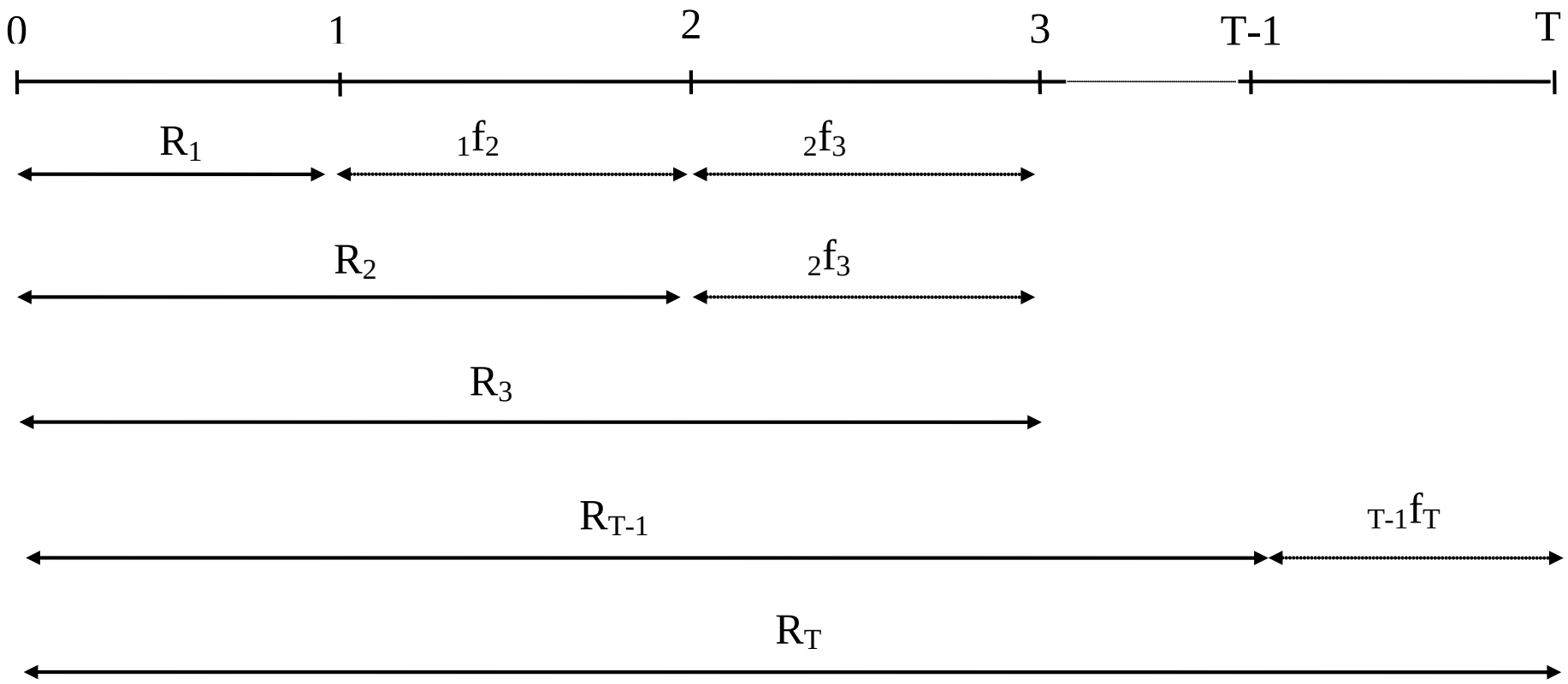
In general:

$$(1+R_t)^T = (1+R_1)\cdot(1+{}_1f_2)\cdot\ldots\cdot(1+{}_{T-1}f_T)$$



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❍ Forward Rate Agreement (FRA)

Aim: Protection of the investment conditions for a future period

Example: 12x24 FRA, 8.04% p.a., F = 100 (face value)

Spot Rate 1Y = 4% p.a., Spot Rate 2Y = 6% p.a.

➔ **Future period**: from 12 months (t=1 year) until 24 months (t=2 years)

➔ **Agreed forward rate f = 8.04% p.a. (**from t=1 to t=2 years)

➔ Can be used to **protect against <u>rate increases</u>**, in case we want to **borrow money** (from t=1 to t=2); → **FRA long**

➔ Can be used to **protect against <u>rate decreases</u>**, in case we want to **lend (invest) money** (from t=1 to t=2); → **FRA short**

Economic Cash flows of this FRA:

Transactions	**t=0**	**t=1 Y**	**t = 2 Y**
FRA short (lending, investing in the future period (t=1 to t=2))	0	-100	+108.04
FRA long (borrowing in the future period (t=1 to t=2))	0	+100	-108.04

The **value of the FRA** is **zero at t=0** (today).

Until the start of the future period, **rates may change** and this may generate an **advantage** or a **disadvantage** for the lending (FRA short) or the borrowing (FRA long) position.

At the **beginning of the future period** (at t=1 in the example):

To compensate for rate increases (in case of a borrowing position (FRA long)) or rate decreases (in case of a lending position (FRA short)) a compensatory payment (CP) is exchanged between the FRA long and the FRA short position.

Compensatory payment (CP):

Discounted difference between the (at t=0) agreed **forward rate f** and the **spot rate r** for the future period at the beginning of the future period (at t=1 year in the example):

$$CP = \frac{F \cdot (f - r)}{1 + r}$$

Lets assume that in the above example r = 7% p.a. at t=1 (for a period of 1 year). The CP is:

$$CP = \frac{100 \cdot (0.0804 - 0.07)}{1 + 0.07} = 0.971963$$

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- **As r (7%) < f (8.04%):**

The investor (**lending** position (FRA short)) **receives the CP** from the borrower (**borrowing** position (FRA long)). The **CP compensates** the investor for having now to invest from t=1 to t=2 at the lower market rate (r = 7%). The overall return for the investor is 8.04% (i.e. equal to f).

The borrower (**borrowing** position (FRA long)) **pays the CP** to the investor (**lending** position (FRA short)). This is a disadvantage, but as she can now borrow at the lower market rate (r = 7%), the toal borrowing costs are 8.04% (i.e. equal to f).

- **If r > f:**

In this case, the investor has a higher lending rate and the borrower a higher borrowing rate. Thus:

The investor (**lending** position (FRA short)) **pays the CP** to the borrower (**borrowing** position (FRA long)). Overall, his investment return is again equal to the forward rate f.

The borrower (**borrowing** position (FRA long)) **receives the CP** from the investor (**lending** position (FRA short)). This compensates for the higher borrowing rate r. Overall, the borrowing cost are again equal to the forward rate f.

Summary of CPs:

	Lending (FRA short)	**Borrowing (FRA long)**
Agreed forward rate **f > r**:	CP > 0	CP < 0
Agreed forward rate **f < r**:	CP < 0	CP > 0

- **Duplication** of the **FRA short** in the above example via spot market transactions (zero bonds):

Transactions	**Today**	**1 Year**	**2 Years**
Short zero bond 1Y (borrowing)	96.15	-100	
Long zero bond 2Y (investment)	-96.15		108.04
Total	0	-100	108.04

↳ value of FRA at t=0

In addition: $100 \cdot (1.04)^{-1} = 96.15$, $96.15 \cdot (1.06)^{2} = 108.04$

❍ Arbitrage free forward rate ***f*** for the FRA:

$$f = \frac{(1+0.06)^2}{(1+0.04)} - 1 = 8.04\% = \textbf{Forward Rate}$$

➔ Protection of 8.04% (1Y - 2Y) even without a FRA

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- **In case FRA-rate ≠ Forward Rate: Arbitrage**

e.g. FRA-rate =7%:

Transactions	Today	1 Year	2 Years
Long FRA		+100	-107
Borrowing 1Y	$96.154 = 100 \cdot (1.04)^{-1}$	-100	
Investment 2Y	$-95.230 = 107 \cdot (1.06)^{-2}$		107
Total	+0.924	0	0

Free Lunch

➔ **Forward Rate is determinated by the no-arbitrage condition**

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- If **spot rates** are indicated **in BEY (bond equivalent yield)**, then the following equation is relevant to calculate the annualized **Forward Rate *f(T₁, T₂)*** for an investment from T_1 to T_2:

$$f(T_1, T_2)^{BEY} = \left[\frac{\left(1 + R(T_2)^{BEY}\right)^{T_2}}{\left(1 + R(T_1)^{BEY}\right)^{T_1}}\right]^{\frac{1}{T_2 - T_1}} - 1 = \left[\frac{P(T_1)}{P(T_2)}\right]^{\frac{1}{T_2 - T_1}} - 1$$

Example:

$$f(1,2)^{BEY} = \left[\frac{(1 + 0.06)^2}{(1 + 0.04)^1}\right]^{\frac{1}{2-1}} - 1 = \left[\frac{0.9615}{0.8900}\right]^{\frac{1}{2-1}} - 1 = 0.0804$$

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- Real markets for FRAs: Typically money market yield (MMY) quotation

Forward Rate in case of MMY for the period T_1 to T_2:

$$f(T_1,T_2)^{MMY} = \frac{1}{T_2 - T_1} \cdot \left[\frac{1 + R(T_2)^{MMY} \cdot T_2}{1 + R(T_1)^{MMY} \cdot T_1} - 1 \right]$$

Example:

If $R(2)^{MMY}$ = 6.18% p.a. and $R(1)^{MMY}$ = 4% p.a.:

$$f(1,2)^{MMY} = \frac{1}{2-1} \cdot \left[\frac{1 + 0.0618 \cdot 2}{1 + 0.04 \cdot 1} - 1 \right] = 8.04\%$$

Valuation principle: Valuation and Mapping of FRAs

FRA-**Short** position:
(lending, investment)

- **Short** Zerobond, Notional = Notional of the FRA
  Maturity at the **start** of the future period
- **Long** Zerobond, Notional = "amount paid out" of the FRA
  Maturity at the **end** of the future period

FRA-**Long** position:
(borrowing)

- **Long** Zerobond, maturity at the **start** of the future period
- **Short** Zerobond, maturity at the **end** of the future period

After the issue date, the **market value** of an FRA equals the **difference** between the market value of the **two zero bonds**.

- Generally: **Forward Rate** (MMY) for the period from T_1 to T_2:

$$f(T_1, T_2) = \frac{1}{T_2 - T_1} \cdot \left(\frac{1 + R(T_2) \cdot T_2}{1 + R(T_1) \cdot T_1} - 1 \right)$$

- **For the example above: Forward Rate 1 month after the issue date**

Spot rates have changed over 1 month to: R(11/12) = 3.8% p.a. (MMY)

R(23/12) = 5.5% p.a. (MMY)

New forward rate: $$f\left(\frac{11}{12}, \frac{23}{12}\right) = 1 \cdot \left(\frac{1 + \frac{23}{12} \cdot 0.055}{1 + \frac{11}{12} \cdot 0.038} - 1 \right) = 6.82074\ \%$$

Thus, a new FRA with the same remaining time to maturity (of 23 months) has to have an annual forward rate of 6.82074% (otherwise an arbitrage opportunity would exist).

❍ Market value of the FRA (1 month after the issue date):

The rate difference is **1.00·(8.04% - 6.82074%) = 121.926** basis points (bp); discounted over 23 months gives exactly the market value of the FRA (1.103 currency units):

- Present Value (PV) of 121.926 bp:

$$\mathrm{PV}(121.926) = 121.926 \frac{1}{1 + \frac{23}{12} \cdot 0.055} = 110.30 \text{ bp}$$

- Market value of the FRA:

$$\frac{108.04}{\left(1 + \frac{23}{12} \cdot 0.055\right)} - \frac{100}{\left(1 + \frac{11}{12} \cdot 0.038\right)} = 97.7369 - 96.6339 = 1.1030 \text{ \% points}$$

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5. Interest Rate Swaps

A swap is an **agreement** to **exchange cash flows** at specified future times according to certain specified **rules**.

Typical use of swaps: to **convert** an investment/liability:

from **fixed** rate to **floating** rate

or

from **floating** rate to **fixed** rate

- **2 types** of interest rate swaps:
  - **Plain Vanilla Swaps** (Coupon Swaps, Generic Swap, Fixed-to-Floating IRS)
    - ↳ Exchange of **fixed** against **floating** interest rates
  - **Basis Rate Swaps** (Basis Swap, Index Swap)
    - ↳ Exchange of **floating** interest rates (e.g. 3M-Euribor against 6M-Euribor)

❍ Plain Vanilla Swaps

Quotation:

Fixed rate	against	**floating rate without agio or disagio**

Example: EUR IRS Quotation, 20.3.2009 (Source: Reuters Xtra 3000)

EURIRS EUR IRS FOCUS LINKED DISPLAYS SWAP/1

IRS V EURIBOR<EURIRS>V LIBOR<EURIRSL>V 1M<EUR1MIRS>V 3M<EUR3MIRS>

Fixed rate

Floating rate

	EUR	AB/6M EURIBOR	DEALING			
1Y/3	1.4650	1.4950	POHJOLA BANK	HEL		08:16
1Y	1.6850	1.7050	HSBC	PAR		19:54
18M	1.7550	1.7750	HSBC	PAR		19:58
2Y	1.8600	1.8900	POHJOLA BANK	HEL		08:16
3Y	2.1950	2.2250	RBS	LON	RBSL	13:03
4Y	2.4550	2.4850	POHJOLA BANK	HEL		08:16
5Y	2.6800	2.7100	POHJOLA BANK	HEL		08:16
6Y	2.8910	2.9110	HSBC	PAR		20:00
7Y	3.0490	3.0690	HSBC	PAR		20:00
8Y	3.1600	3.1900	POHJOLA BANK	HEL		08:16
9Y	3.2700	3.3000	POHJOLA BANK	HEL		08:16
10Y	3.3700	3.4000	POHJOLA BANK	HEL		08:16
11Y	3.4750	3.4950	BROKER	GFX		12:45
12Y	3.5520	3.5720	BROKER	GFX		12:45
13Y	3.6050	3.6350	POHJOLA BANK	HEL		08:16
14Y	3.6650	3.6950	POHJOLA BANK	HEL		08:16
15Y	3.7350	3.7550	BROKER	GFX		12:45
20Y	3.8550	3.8750	BROKER	GFX		12:45
25Y	3.7740	3.7940	BROKER	GFX		12:45
30Y	3.6760	3.6960	BROKER	GFX		12:45

Receiver Payer

For each maturity: 2 fixed rates

Bid-interest rate (2. column) = Fixed rate, the **seller** of an IRS **receives** (receiver)

Ask-interest rate (3. column) = Fixed rate, the **buyer** of an IRS **pays**

The fixed rate corresponds to a medium/long term capital market rate.

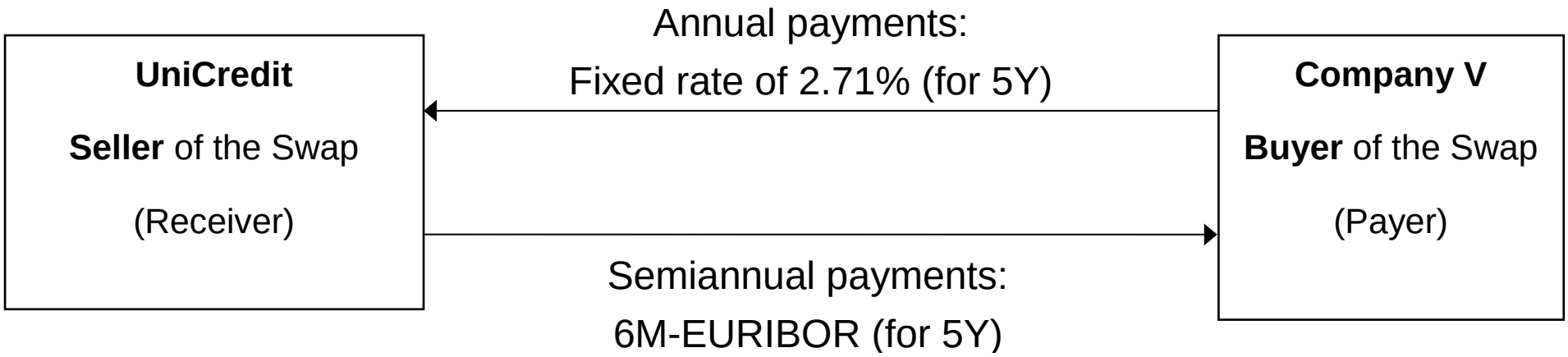
The floating leg of a swap equals normally only the floating rate (e.g. EURIBOR, LIBOR) without agio or disagio.

Example

UniCredit and company V are arranging a 5-year IRS

Principal amount = €10 Mio

Company V pays the fixed rate and receives the **6M-EURIBOR** from UniCredit



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Characteristics of a Plain Vanilla Swap:

- Maturity of the fixed rate = Maturity of the Swap
  Fixed rate (Swap rate): constant over the total maturity
  oriented towards bonds

- Floating rate: Valid for a short period of time (e.g. 6 months)
  Seller (Receiver) pays to the buyer (Payer)

- Fixed rate: is payed annually
  Floading rate: payment based on the maturity of the money market yield

- Normally an interest rate netting is arranged: at the interest rate payment dates floating and fixed rates are offset against each other. Only the difference is exchanged.

- Only interest rates are exchanged.

Example:

Microsoft (MSFT) has negotiated with JP Morgan an interest rate swap to **receive** 6-month LIBOR and **pays** a fixed rate of 5% p.a. every 6 months for 3 years on a notional principal of $100 million. (**pay fix** position)

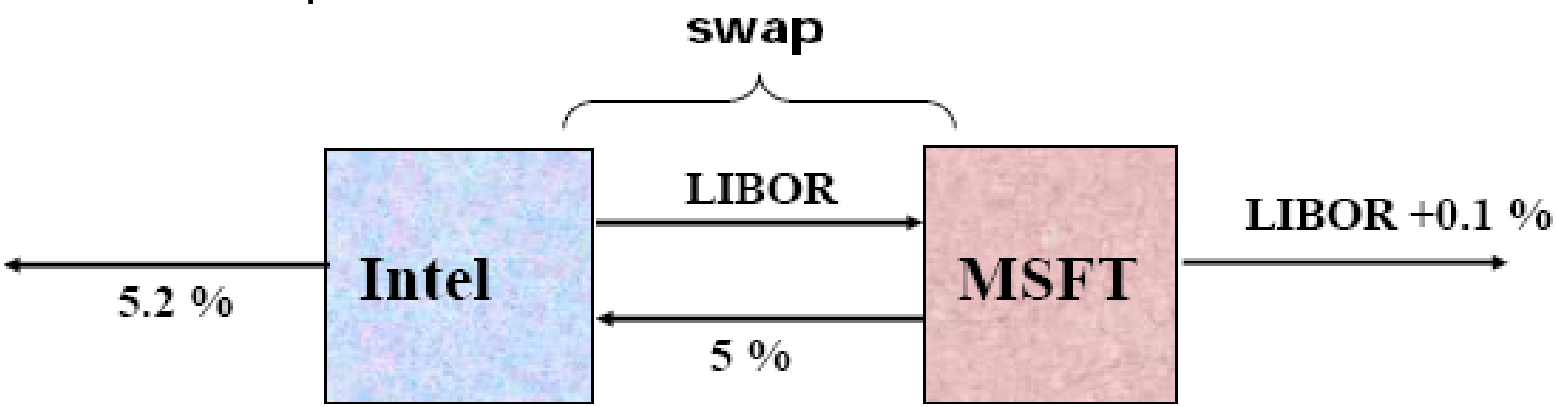
Cash Flows to MSFT:

Date	LIBOR Rate	---------Millions of Dollars--------- *FLOATING* Cash Flow	*FIXED* Cash Flow	Net Cash Flow
Mar.5, 2006	4.2%			
Sept. 5, 2006	4.8%	+2.10	–2.50	–0.40
Mar.5, 2007	5.3%	+2.40	–2.50	–0.10
Sept. 5, 2007	5.5%	+2.65	–2.50	+0.15
Mar.5, 2008	5.6%	+2.75	–2.50	+0.25
Sept. 5, 2008	5.9%	+2.80	–2.50	+0.30
Mar.5, 2009	6.4%	+2.95	–2.50	+0.45

Example:

MSFT has a liability with a floating rate of LIBOR+0.1%. The financial manager of MSFT wants to change the floating-rate credit into a fixed-rate payments.

(a) Interest rate swap with Intel



Net interest rate payment: LIBOR+0.2 %

Net interest rate payment: 5.1 %

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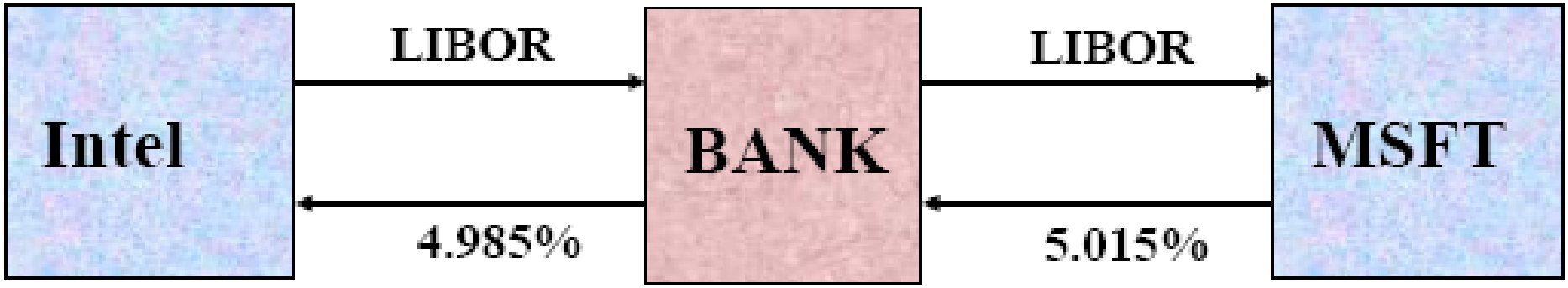
(b) Interest rate swap with an intermediary



➔ Bank acts as intermediary, simultaneously borrows and lends at LIBOR, and borrows at a fixed rate but lends at this fixed rate plus **3 basis points**.

➔ In exchange for the spread, the **bank absorbs the default risk** of the counterparties.

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- **Valuation of an interest rate swap**

A swap has a value of zero initially.

At a future date, its value is either positive or negative.

➔ Simplification: valuation with a (fictional) redemption payment

➔ Then: IRS = floating instrument **+** fixed rate instrument

Duplication idea:

Floater: Characteristics as the **variable side** (short leg)

Coupon bond: Characteristics as the **fixed side** (long leg)

Issue date: Both sides (legs) have a value of **100%**

Market value(IRS) = Market value(receiver leg) - Market value (pay leg)

Swap short (Verkäufer, receiver): long-Coupon bond (receiver leg)
short-Floater (pay leg)

Swap long (Käufer, payer): short-Coupon bond (pay leg)
Long-Floater (receiver leg)

6. Duration

Duration = mathematically exact **average residual time to maturity** of a cash flow

= sensitivity and **risk measure**

6.1 Rate sensitivity

Impact of a rate change on the market value of a bond

Simplification: **flat term structure** (equal rate for all maturities)

$$PV = \sum_{j=1}^{J} Z_j \cdot (1+r)^{-T_j}$$

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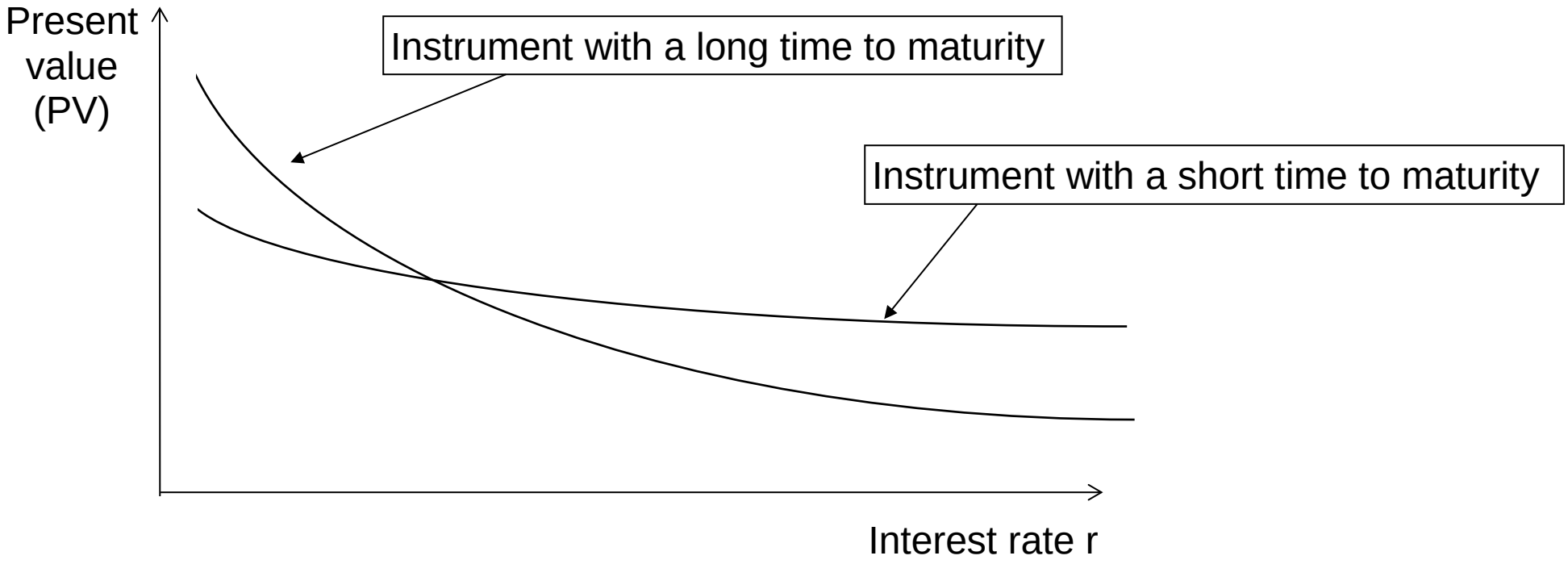
The price reaction of an instrument to rate changes depends on 2 parameters: (1) the cash flow structure and (2) the (remaining) time to maturity.

Present value as a function of interest rate level and maturity:



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- How reacts the present value (PV) on changes in the ‚market rate' (flat term structure)?

Consequences of a rate change - **1. derivative:**

$$PV = \frac{Z_1}{(1+r)^{T_1}} + \frac{Z_2}{(1+r)^{T_2}} + ... = \sum_{j=1}^{J} \frac{Z_j}{(1+r)^{T_j}}$$

$$\frac{dPV}{dr} = \sum_{j=1}^{J} Z_j \cdot (-T_j) \cdot (1+r)^{-T_j-1} = -\sum_{j=1}^{J} Z_j \cdot T_j \cdot (1+r)^{-T_j} \cdot (1+r)^{-1}$$

$$\frac{dPV}{dr} = -\frac{1}{1+r} \cdot \sum_{j=1}^{J} Z_j \cdot T_j \cdot (1+r)^{-T_j} \quad = \quad \text{Slope of the PV} - \text{Function}$$

Slope of the PV-Function:

Indicates how intense the PV ↓↑, when r ↓↑ marginally

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Via transformation one gets:

$$\frac{dPV}{PV} = \underbrace{\frac{\sum_{j=1}^{J} T_j \cdot Z_j \cdot (1+r)^{-T_j}}{\sum_{j=1}^{J} Z_j \cdot (1+r)^{-T_j}}}_{\text{Duration (D)}} \cdot \left(-\frac{1}{1+r}\right) dr$$

$$D = \frac{\sum_{j=1}^{J} T_j \cdot Z_j \cdot (1+r)^{-T_j}}{\sum_{j=1}^{J} Z_j \cdot (1+r)^{-T_j}}$$

$$D = \frac{PV(Z_1) \cdot T_1 + PV(Z_2) \cdot T_2 + \ldots\ldots + PV(Z_J) \cdot T_J}{PV}$$

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- **Approximative PV change:**

$$\Delta PV \approx -\frac{1}{1+r} \cdot D \cdot PV \cdot \Delta r$$

- **Features of the measure duration:**

♦ time measure

♦ risk measure

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- **Duration of a Zero Bond:**

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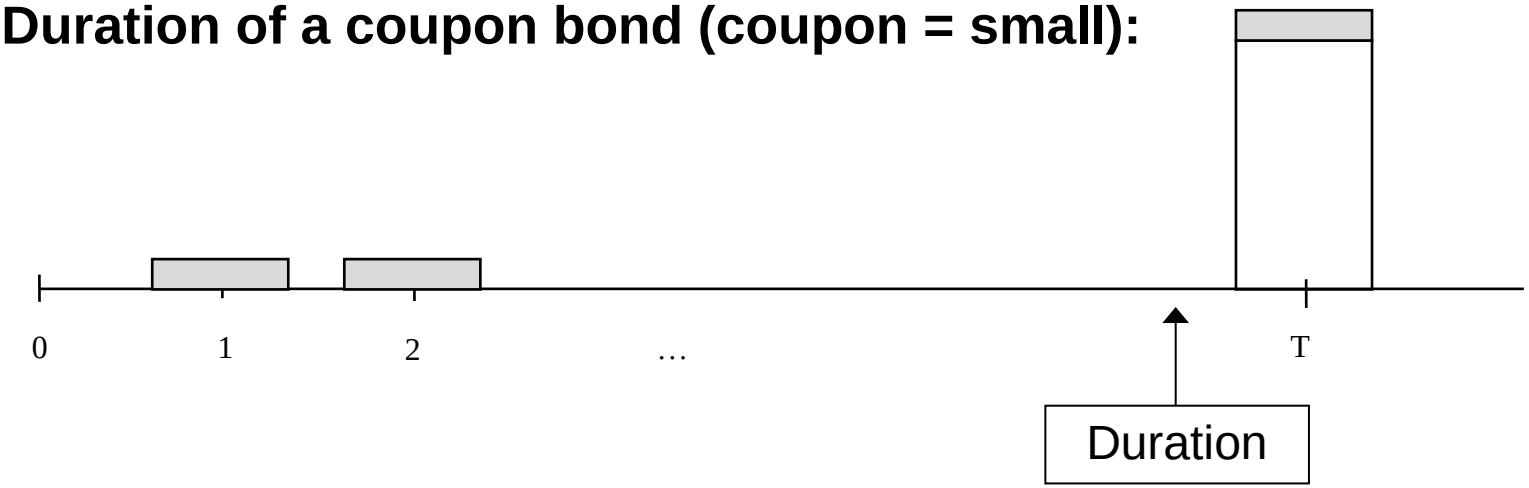
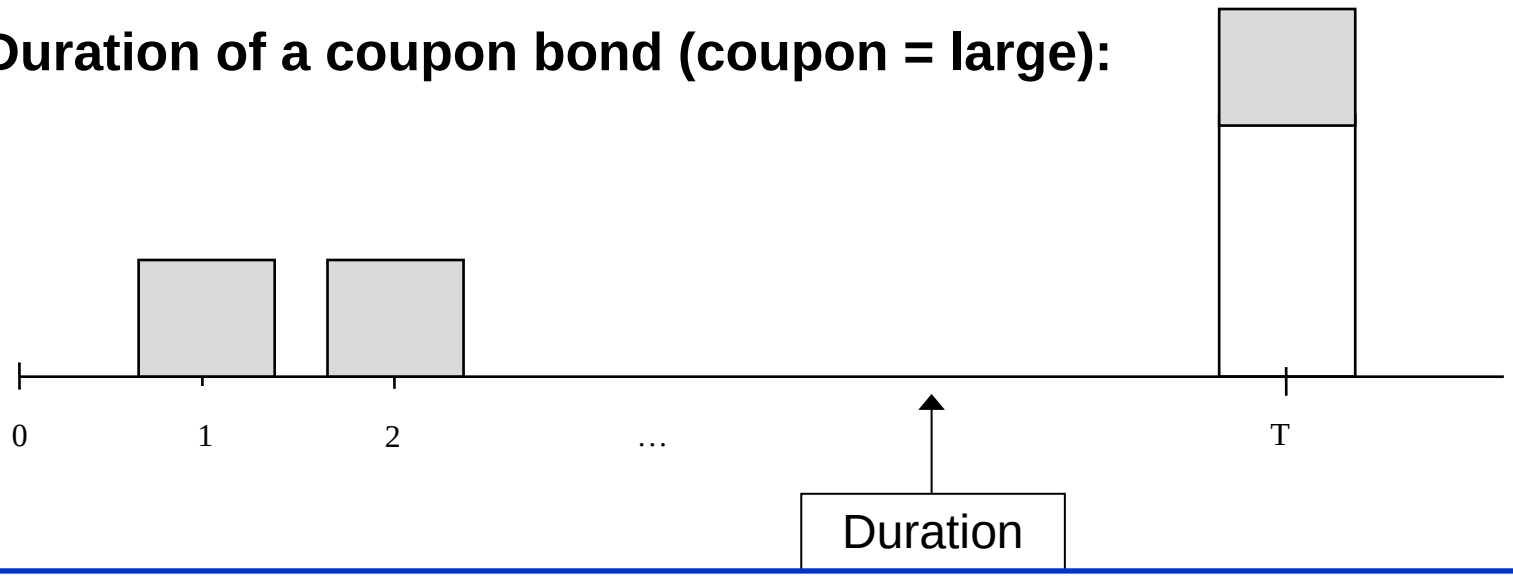
- **Duration of a coupon bond (coupon = small):**



- **Duration of a coupon bond (coupon = large):**



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6.2 Modified and Dollar Duration

(a) Modified Duration (MD): $$MD = D \cdot \frac{1}{1+r}$$

$$\frac{\Delta PV}{PV} \approx -MD \cdot \Delta r$$

MD: (approximative) percentage change of the PV, if the rate level changes by plus/minus 100 basis points.

(b) Dollar Duration (DD): $$DD = MD \cdot PV = D \cdot \frac{PV}{1+r}$$

$$\Delta PV \approx -DD \cdot \Delta r$$

DD: (approximative) change of PV in currency units, if the rate level changes by plus/minus 100 basis points.

Example:

8%-Coupon bond, maturity = 3 years, interest rate level = 8%.

Present value:	PV = 100
Dollar Duration:	DD = 257.71 BP
Modified Duration:	MD = 2.5771 %

Let's assume r decrease to 6%:

New present value (exact):	PV_{new} = 105.3460
Change in %:	5.3460 %
Approximation via DD:	5.1542 %
Approximation via MD:	5.1542 %

The approximation underestimates always the price change (improvement: Convexity).

6.3 Special cases

- Duration of a zero bond: $D = n$
- Duration of an annuity bond: $D = \frac{1+r}{r} - \frac{n}{(1+r)^n - 1}$
- Duration of a perpetual bond: $D = \frac{1+r}{r}$

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6.4 Duration of a portfolio

Duration of a portfolio consisting of the **2 bonds** A and B:

$$D_{PF} = x \cdot D_A + (1 - x) \cdot D_B$$

D_{PF}	Duration of the portfolio
D_A, D_B	Duration of bonds A and B
x	Fraction of bond A

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Or more general:

Duration of a portfolio consisting of *U* components:

$$D_{PF} = \frac{\sum_{u=1}^{U} PV_u \cdot D_u}{\sum_{u=1}^{U} PV_u} = \sum_{u=1}^{U} x_u \cdot D_u$$

with $\sum_{u=1}^{U} x_u = 1$

The weights x_u represent the present value fraction of portfolio component *u* on the total present value.

In addition: $MD_{PF} = \sum_{u=1}^{U} x_u \cdot MD_u$ $\qquad DD_{PF} = \sum_{u=1}^{U} DD_u$

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Example:

An investor owns two bonds, bond A_1 (8, 108) with a maturity of 2 years and a face value of €10,000 and bond A_2 (6, 6, 6, 106) with a maturity of 4 years and a face value of €10,000. The relevant discount rate is 7%.

$PV_1 = 101.81$ $D_1 = 1.93$

$PV_2 = 96.61$ $D_2 = 3.67$

Calculate the duration of the bond portfolio.