Knowledge Based Systems, 4.0 VU, 184.730

Exercise Sheet 2 – Description Logics and Truth Maintenance Systems

This exercise sheet serves as a **preparation for the mandatory exercise test**, which covers the exercises and related background knowledge. You do *not* need to submit solutions.

For questions regarding exercises please visit the tutoring sessions (times are announced in **TUWEL**). You can find them in Conference room Hahn (room HG 03 06), Favoritenstr. 9-11, Stiege 3, 3rd floor, Institute of Information Systems. For questions of general interest, please use the **TISS** Forum or contact us on `kbsci-2018s@kr.tuwien.ac.at`.

Exercise 2.1: Consider the interpretation $\mathcal{I}$ with

$$\begin{aligned} \Delta^{\mathcal{I}} &= \{a, b, c, d\} \\ P^{\mathcal{I}} &= \{b, c, d\} \\ Q^{\mathcal{I}} &= \{b\} \\ R^{\mathcal{I}} &= \{(d, c), (c, a)\} \\ S^{\mathcal{I}} &= \{(a, a), (a, d), (c, b)\} \end{aligned}$$

For each of the following concept expressions C list all elements $x \in \Delta^{\mathcal{I}}$ such that $x \in C$ (follow the definitions of semantics of $\mathcal{ALC}$ from the slides).

a) $(\exists R.\neg Q) \sqcap (\forall S.\neg P)$

b) $\forall R.\exists S.\exists R.\exists S.P$

c) $\neg\exists R.(\neg P \sqcap \neg Q)$

d) $(\exists S.(P \sqcap \forall S.\neg Q)) \sqcap (\neg\forall R.\exists R.(P \sqcup \neg P))$

Exercise 2.2: Prove or disprove the following statements (for the description logic $\mathcal{ALC}$):

a) There is a TBox that has no models at all.

b) There is a TBox that has only finite models.

c) Every TBox has either no models at all or infinitely many models.

d) For every TBox $\mathcal{T}$, there is an equivalent TBox $\mathcal{T}'$ that contains only a single general concept inclusion (GCI), where two TBoxes are equivalent if they have the same models.

Exercise 2.3: Consider the $\mathcal{ALC}$-TBox

$$\mathcal{T} = \{\texttt{Species} \sqsubseteq \exists\texttt{feedsOn}.\texttt{Species}\}$$

Show that the concept `Species` is satisfiable w.r.t. $\mathcal{T}$ by defining an interpretation $\mathcal{I}$ satisfying $\mathcal{T}$ such that $\texttt{Species}^{\mathcal{I}} \neq \emptyset$. Using this example, explain why the naive $\mathcal{ALC}$ tableau algorithm with the expansion rules that have been introduced in the lecture does not always terminate without blocking.

Exercise 2.4: Let $G = \langle V, E, \mathcal{L} \rangle$ be a complete and clash-free completion graph. Prove that for every v and every concept C we have

$$C \in \mathcal{L}(v) \implies v \in C^{\mathcal{I}_G}.$$

Exercise 2.5: Let R and S be role names and C, D be concept names. Using the $\mathcal{ALC}$ tableau algorithm for concept satisfiability, show that the GCI

$$\forall S.\exists R.(C \sqcup D) \sqcap D \sqsubseteq \forall S.C \sqcup \forall S.\exists R.D$$

is not satisfied by each interpretation.

Exercise 2.6: Construct a $\mathcal{SROIQ}$ knowledge base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ (we do not need an RBox here) expressing the following knowledge concerning spies and their operations.

- Every person is an agent, a target, or a source.
- Every agent spies on someone and works for a country.
- Every target is spied upon by some agent.
- No one spies on themselves.
- Korolev and Jones spy on each other.
- Müller works for Germany, and Jones works for America.
- Korolev is an agent, but not a target.
- Müller and Jones have a common target that they spy upon (*Hint:* use nominal concepts).
- No German agent spies on American agents.
- Double agents are agents who work for more than one country.
- Müller, Korolev and Jones are all different persons. Germany and America are different countries. No entity is both a person and a country.

Use the class names

$$\mathsf{Person}, \mathsf{Agent}, \mathsf{Target}, \mathsf{Source}, \mathsf{Country}, \mathsf{DoubleAgent},$$

the role names $\mathsf{spiesOn}$ and $\mathsf{worksFor}$, and the individual names

$$\mathsf{Jones}, \mathsf{Korolev}, \mathsf{Müller}, \mathsf{Germany}, \mathsf{America}.$$

Exercise 2.7:

a) Consider the TMN $\mathcal{T}_1 = (\{A\}, \{\langle \emptyset | A \to A \rangle\})$. Does $\mathcal{T}_1$ have a non-empty founded model?

b) Compute all founded models for the TMN $\mathcal{T}_2 = (\{A, B\}, \{\langle \emptyset | B \to A \rangle, \langle \emptyset | A \to B \rangle\})$.

Exercise 2.8: Prove that the JTMS inference relation does not satisfy cumulativity. That is, find a counterexample to the following statement:

> Let $\mathcal{T} = (N, \mathcal{J})$ be a TMN, let $A \subseteq N$, and let $m, n \in N$.
> Then $A \mathrel{|\!\sim}_{\mathcal{J}} n$ implies $(A \cup \{n\} \mathrel{|\!\sim}_{\mathcal{J}} m$ iff $A \mathrel{|\!\sim}_{\mathcal{J}} m)$.