1. 1.1. $\forall x(\neg P(x) \to (P(x) \land Q(x))) \models \forall x P(x)$

   The formula has an easy counterexample. To show this find an Interpretation I $\langle U, I, \{\}\rangle$ with $I \not\models \forall x P(x)$.

   U = ℕ, I(P) = (x == x), I(Q) = (x == x)

   It is easy to see, that under I $\neg P(x)$ becomes true and $(P(x) \land Q(x))$ false.

   1.2. $p \to q \models \neg q \to \neg p$

   Lets assume I = $\langle U, I, \{\}\rangle$ is a arbitrary model of the left side. This means $I \models p \to q$. The only way for the entailment to not hold is if $I \not\models \neg q \to \neg p$, which requires $I \not\models q$ and $I \models p$. But, if this were to be true,the left side would not be modeled by I, so the entailment always holds.

   1.3. $(p \land q) \to (p \lor q) \models \neg q$

   Lets assume I = $\langle U, I, \{\}\rangle$ is a arbitrary model of the left side. This means $I \models (p \land q) \to (p \lor q)$. This is always the case. If p or q are not modeled by I, the implication becomes true, and, if they are both modeled by I, $I \models (p \lor q)$. Now its easy to find the counterexample $I \models q$.

   1.4. $(p \to q) \land (p \lor r) \models q \lor r$

   Lets assume I = $\langle U, I, \{\}\rangle$ is a arbitrary model of the left side. This means $I \models (p \to q) \land (p \lor r)$. The only way for the entailment to not hold is if $I \not\models q \lor r$, which requires either $I \not\models q$ or $I \not\models r$ or both. For $I \models (p \to q)$, $I \not\models p$ is necessary. This means that $I \not\models (p \lor r)$. So it is impossible for I to be a model of the left side, while it is not one of the right.

2. 2.1. $\psi \to \phi$

   (i) tautological: ψ is a contradiction and stands on the left side of the implication. This makes the whole implication a tautology.

   2.2. $\chi \to (\psi \to \phi)$

   (i) tautological: The right side is a tautology. The only way for an implication to be false is for the left side to be true and the right to be false, which is impossible here.

   2.3. $\phi \land \chi$

   (iii) contingent and (iv) logical equivalent to χ: ϕ is a tautology, so the whole statement depends solely on χ.

   2.4. $\neg\psi \lor \chi$

   (i) tautological: $\neg\phi$ is a negated contradiction, which makes it a tautology. Because there is always one true value, the or statement is alwys true.

2.5. $\phi \rightarrow \psi$

(ii) contradictory: The only way for an implication to be false, is for the left side to be true and the right to be false. Which happens exactly in this example.

2.6. $\neg\chi \rightarrow (\neg\chi \rightarrow \psi)$

(iii) contingent and (iv) logical equivalent to χ: If χ is true, the $\neg$ makes it false and because it stands on the left side of an implication, the whole formula becomes true. If χ is false, the $\neg$ makes it true. The right side of the implication is false, because ψ is a contradiction. So the whole formula becomes false.

3. 3.1. Show the contraposition theorem directly from the definition of $\models$

Contraposition theorem: $W \cup \{\phi\} \models \neg\psi$ iff $W \cup \{\psi\} \models \neg\phi$

Definition of $\models$: Let W be a set of closed formulas. Then W entails ϕ, $W \models \phi$, if and only if Mod(W) $\subseteq$ Mod(ϕ)

We have to show that $(W \cup \{\psi\} \models \neg\phi) \models (W \cup \{\phi\} \models \neg\psi)$

Lets take a arbitrary model I $\langle U, I, \{\}\rangle$ for $W \cup \{\psi\} \models \neg\phi$. Every model that entails the left side should also entail the right. In order for I to entail $W \cup \{\psi\} \models \neg\phi$ either

I $\not\models$ W, in this case, the left side of the entailment is always false, which makes it so that I entails $W \cup \{\psi\} \models \neg\phi$ and $W \cup \{\phi\} \models \neg\psi$, regardless if I entails ψ and ϕ

For brevity I $\models$ W for the next cases, W is going to be excluded

Now we have to show that $(\psi \models \neg\phi) \models (\phi \models \neg\psi)$

There are 3 possibilities:

$I \models \psi$ and $I \not\models \phi$, through the negation $(\psi \models \neg\phi)$ is true, and because I does not entail ϕ, the right side is true by default

$I \not\models \psi$ and $I \not\models \phi$, two false propositions mean, that anything can be followed from them, which makes the entailment hold true

$I \not\models \psi$ and $I \models \phi$, $(\psi \models \neg\phi)$ is true, because of the false proposition. $(\phi \models \neg\psi)$ is also true because through the negation, follows that $I \models \phi$ and $I \models \psi$.

Everytime $I \models (\psi \models \neg\phi)$, I also entails $(\phi \models \neg\psi)$, which proofs the contraposition theorem.

4. 4.1. $\Gamma \cup \{\phi\} \models r$ implies $\Gamma \cup \{\phi \wedge \psi\} \models r$

Lets examine the statement under the arbitrary model I $\langle U, I, \{\}\rangle$.

If I $\not\models \Gamma$, the left side of both entailments is false, therefore the whole becomes true. Next we examine the statement if $\models \Gamma$. This acts like an $\wedge$, so we can omit the Γ.

$\phi \models r$ implies $(\phi \wedge \psi) \models r$

Now a counterexample is easy to see. If $I \models \phi$ and $I \not\models \psi$, the left side

The only way for an imply-statement to be not true, is for the left side to be true, while the right is false. In this example, this can only be achieved, if $I \models \phi$ and r. But if r has to be true, the right can never be false. Therefore, the statement holds.

5. 5.1. $(\forall x P(x) \rightarrow \exists y Q(y)) \rightarrow \exists x \forall y (P(x) \rightarrow Q(y))$

   Is equal to $(\forall x P(x) \rightarrow \exists y Q(y)) \rightarrow (\exists x P(x) \rightarrow \forall y Q(y))$

   The formula is no tautology. To show this find an Interpretation I $\langle U, I, \{\} \rangle$ with $I \models (\forall x P(x) \rightarrow \exists y Q(y))$ and $I \not\models (\exists x P(x) \rightarrow \forall y Q(y))$.

   U = $\mathbb{N}$, I(P) = (x is a Natural Number), I(Q) = (x is a prime number)

   For P we chose something that is always true and for Q something that can sometimes be true, but is not true for every number. This way, it is easy to see that we can make the left side true (e.g. y = 3). Then, let's examine the right side $(\exists x P(x) \rightarrow \forall y Q(y))$. A natural number that is a natrual number exists, but not every natural number is prime. Therefore, the statement is no tautology.

   5.2. $\forall x \exists y$ R(x, y) $\rightarrow \forall y \exists x$ R(x, y)

   The formula is no tautology. To show this find an Interpretation I $\langle U, I, \{\} \rangle$ with $I \models \forall x \exists y$ R(x, y) and $I \not\models \forall y \exists x (P(x) \rightarrow Q(y))$.

   U = $\mathbb{N}$, I(R) = (x < y)

   $I \models \forall x \exists y$ R(x, y) is fullfilled, because every natural number has a successor.

   $I \not\models \forall y \exists x (P(x) \rightarrow Q(y))$ is the case, because y could be 0 and there is no natural number smaller than 0.

   Therefore, the formula is no tautology.

1. $\forall x(P(x) \vee R(x))$, 1

   $\forall x(\neg Q(x) \rightarrow R(x))$, 2

   $P(a) \wedge R(b)$, 3

   $\neg Q(a)$, 4

   i)

   P(a): $T \models P(a)$ because 3.

   P(b): $T \not\models P(b)$ because 1 is an or statement and it could be true because $T \models R(b)$

   Q(a): $T \not\models Q(a)$ because of 4.

   Q(b): $T \not\models Q(b)$ because if $T \not\models Q(b)$ would be not possible, R(b) in 2 would not be entailed by T, which can not be, because of 3.

   R(a): $T \models R(a)$ because 2 and 4 makes $\neg Q(x)$ true.

   R(b): $T \models R(b)$ because 3.

   $\neg Q(b), \neg P(b), R(b)$ are in CWA(T).

   ii)

   CWA(T) is the logical closure of all assumptions (explicit and implicit ones).

   CWA(T) = $\forall x(P(x) \vee R(x)), \forall x(\neg Q(x) \rightarrow R(x)), P(a) \wedge R(b), \neg Q(a), \neg Q(b), \neg P(b), R(b), P(a), R$

   The only disjunction is $\forall x(P(x) \vee R(x))$.

   P(a) and R(a) are in CWA(T), P(b) is not, but R(b) is. Therefore, CWA(T) is consistent.

   iii)

   $(P(a) \wedge P(b)) \notin CWA(T)$ because P(b) is not in CWA(T).

   $(\exists x(P(x) \wedge R(x))) \in CWA(T)$ because P(a) and R(a) are in CWA(a).

   $(\neg Q(a) \rightarrow R(a)) \in CWA(T)$ because of 2.

2: a) ~~(a)~~ First prove $T_{asm} \neq \emptyset$

Assume $T_{asm} = \emptyset$, $CWA(T)$ is inconsistent

$CWA(T) = \{ \varphi \mid T \cup T_{asm} \models \varphi, \varphi \text{ closed} \} = \{ \varphi \mid T \models \varphi, \varphi \text{ closed} \}$

$CWA(T) = T$, Contradiction T should be consistent

Compactness Theorem CT: A set of formulas is satisfiable, if every finite Subset is ~~sa~~ satisfiable.

Through negation of both sides, we get if a finite Subset is not satisfiable, the whole set of Formulas is not satisfiable.

Show: There is a finite Subset ~~of~~ T^0_{asm} such that $Cn(T \cup T^0_{asm})$ is inconsistent and $|T^0_{asm}| > 1$, $T^0_{asm} \subseteq T_{asm}$

First $|T^0_{asm}| \neq 1$, assume $|T^0_{asm}| = 1$

$T^0_{asm} = \neg A_1$ $\quad T \cup T^0_{asm}$ is inconsistent, Therefore $T \models A_1$, which Contradicts the definition of T_{asm}.

(T_{asm} only holds $\neg$formulas, if $T \not\models$ the formula)

T is consistent, T_{asm} ~~unsatisfiable~~ consistent

$T \cup T_{asm}$ is inconsistent, $T^0_{asm} \subseteq T_{asm}$, $T \cup T^0_{asm}$ inconsistent

$T \cup T^0_{asm}$ is unsatisfiable

$T \models \neg T^0_{asm}$

$T \models \neg(\neg A_1 \wedge \ldots \wedge \neg A_n)$ de Morgan

$T \models (A_1 \vee \ldots \vee A_n)$

b) A if B : $A \Leftarrow B$: $B \rightarrow A$

B = There are ground Atoms $A_1 \dots A_n$ such that
$T \models (A_1 \vee \dots \vee A_n)$ but $T \not\models A_i$ for all $i = 1 \dots n$

A = T is inconsistent

T_{asm} contains $\{\neg A_1, \dots, \neg A_n\}$

$CWA(T) = Cn(T \cup T_{asm})$

$T \cup T_{asm} \models (\neg A_1 \wedge \dots \wedge \neg A_n)$ de Morgan

~~$T \cup T_{asm} \models (\neg A_1 \vee \dots \vee \neg A_n)$~~

$T \cup T_{asm} \models \neg(A_1 \vee \dots \vee A_n)$

inconsistent because $T \models (A_1 \vee \dots \vee A_n)$

2.3 A closure of T is given by $\overline{T} = (\overline{W}, \overline{\Delta})$, where

$$\overline{W} = \{ \forall x \forall y ((Q(x) \vee P(y)) \to R(x,y)), R(a,a) \vee Q(a) \}$$

$$\overline{\Delta} = \left\{ \frac{\top : \neg Q(a)}{Q(a)}, \frac{R(a,a) : \neg P(a)}{\neg P(a)}, \frac{Q(a) : \neg P(a)}{\neg R(a,a)} \right.$$

$E_1 = Cn(\overline{W})$

$E_2 = Cn(\overline{W} \cup \{Q(a)\})$

$E_3 = Cn(\overline{W} \cup \{\neg P(a)\})$

$E_4 = Cn(\overline{W} \cup \{\neg R(a,a)\})$ inconsistent

$E_5 = Cn(\overline{W} \cup \{Q(a), \neg P(a)\})$

$E_6 = Cn(\overline{W} \cup \{Q(a), \neg R(a,a)\})$ inconsistent

$E_7 = Cn(\overline{W} \cup \{\neg P(a), \neg R(a,a)\})$ inconsistent

$E_8 = Cn(\overline{W} \cup \{Q(a), \neg P(a), \neg R(a,a)\})$ inconsistent

Classical Reducts:

$E_1 = Cn(\overline{W})$

- $\Delta_{E_1} = \{\top / Q(a), R(a,a)/\neg P(a), Q(a)/\neg R(a,a)\}$
- $\Gamma_{\overline{T}}(E_1) = Cn(\overline{W} \cup \{Q(a), \neg R(a,a)\}) = E_6$

$E_2 = Cn(\overline{W} \cup \{Q(a)\}$

- $\Delta_{E_2} = \{R(a,a)/\neg P(a), Q(a)/\neg R(a,a)\}$
- ~~$\Gamma_{\overline{T}}(E_2) = Cn(\overline{W} \cup \{$~~
- $\Gamma_{\overline{T}}(E_2) = Cn(\overline{W}) = E_1$

$E_3 = Cn(\overline{W} \cup \{\neg P(a)\}$

- $\Delta_{E_3} = \{\top/Q(a), R(a,a)/\neg P(a), Q(a)/\neg R(a,a)\}$
- $\Gamma_{\overline{T}} = Cn(\overline{W} \cup \{Q(a), \neg R(a,a)\} = E_6$

$E_4 = Cn(\overline{W} \cup \{\neg R(a,a)\}$

- $\Delta_{E_4} = \{\emptyset\}$
- $\Gamma_{\overline{T}} = Cn(\overline{W}) = E_1$

$E_5 = Cn(\overline{W} \cup \{Q(a), \neg P(a)\})$

- $\Delta E_5 = R(a,a) / \neg P(a), Q(a) / \neg R(a,a)$
- ~~$\neg E_5$~~ - $\neg\mp(E_5) = Cn(\overline{W}) = \cancel{E_1}\ E_1$

$E_6 = Cn(\overline{W} \cup \{Q(a), \neg R(a,a)\})$

- $\Delta E_6 = \cancel{R(a,a)}\ \{\emptyset\}$
- $\neg\mp(E_6) = Cn(\overline{W}) = E_7$

$E_7 = Cn(\overline{W} \cup \{\cancel{\emptyset}\ \neg P(a), \neg R(a,a)\})$

- $\Delta E_7 = \{\emptyset\}$
- $\cancel{\Delta}\ \neg\mp(E_7) = Cn(\overline{W}) = E_7$

$E_8 = Cn\ \overline{W} \cup \{Q(a), \neg P(a), \neg R(a,a)\}$

- $\Delta E_8 = \{\emptyset\}$
- $\neg\mp(E_8) = Cn(\overline{W}) = E_1$

2.4 Prove closed normal default theory $T = (W, \Delta)$, Extensions E, E', then $E \cup E'$ is inconsistent

Default is normal if $\frac{A : B}{B}$

$E_0 = W$

$E_i = Cn(E_{i-1}) \cup \{ C \mid \frac{A : B_1, \ldots, B_n}{C} \in \Delta, E_{i-1} \models A$ and $\neg B_1, \ldots, \neg B_n \notin E \}$

E is an Extension of $T = (W, \Delta)$ iff $E = \bigcup_{i \geq 0} E_i$

T has two distinct Extensions E and E', $E \neq E'$

There has to be a point i in the recursion, where E and E' start to differ.

There is ~~a~~ ~~def~~ default $\frac{A : B_1, \ldots, B_n}{B_1, \ldots, B_n} \in \Delta$ with $A \in E_{i-1}$ and $A \in E'_{i-1}$

Because they have to differ, $B_1, \ldots, B_n \in E_i$ ~~A~~ and $\neg B_i, \ldots, \neg B_n \in E'_i$

$E \cup E'$ is inconsistent

2.5 Are $\frac{A:\emptyset}{B}$ and $\frac{A:T}{B}$ interchangable, such that the Extensions do not change?

Counterexample $W = \{A, \neg B\}$, $\Delta = \{\frac{A:\emptyset}{B}\}$

$W' = \{A, \neg B\}$ $\Delta' = \{\frac{A:T}{B}\}$

$E_1 = Cn(W)$

- $\Delta_{E_1} = \{A/B\}$
- $\Gamma_T(E_1) = Cn(W \cup \{B\}) \neq E_1$

$E_2 = Cn(W \cup \{B\})$

- $\Delta_{E_2} = \{A/B\}$
- $\Gamma_T(E_2) = Cn(W \cup \{B\}) = E_2$ Extension!

ΔE_2 is A/B, because $\Delta_E = \{\varphi/\chi \mid (\varphi : \psi_1, \dots, \psi_n / \chi) \in \Delta$ and $\{\neg\psi_1, \dots, \neg\psi_n\} \cap E = \emptyset\}$

$\psi_1 = \emptyset$, and $\emptyset \cap E$ is ~~also~~ always $\emptyset$

$E_1' = Cn(W')$

- $\Delta'_{E_1} = \{A/B\}$
- $\Gamma_{T'}(E_1') = Cn(W' \cup \{B\}) \neq E_1'$

$E_2' = Cn(W' \cup \{B\})$

- $\Delta'_{E_2} = \emptyset$, $\perp \cap E_2'$ because A, B and $\neg B$ are in E_2'
- $\Gamma_{T'}(E_2) = Cn(W') \neq E_2'$

$\Delta = \{\frac{A:T}{B}\}$ has no Extension

2.6 $W = \{\forall_x (P(x) \vee R(x)), Q(a), \forall x (Q(x) \to R(x))\}$

$\Delta = \left\{ \frac{P(x) : \neg Q(x)}{\neg Q(x)}, \frac{Q(x) : \neg P(x)}{\neg P(x)} \right\}$

T is normal, we have to add a non-normal default

$T' = (W, \Delta')$

$\Delta' = \left\{ \frac{P(x) : \neg Q(x)}{\neg Q(x)}, \frac{Q(x) : \neg P(x)}{\neg P(x)}, \frac{\top : \cancel{\neg Q(x)}\ R(x)}{\neg Q(x)} \right\}$

Then we build the closure of $T' = \overline{T}' = \{\overline{W}, \overline{\Delta}'\}$, $\overline{W} = W$

$\overline{\Delta}' = \left\{ \frac{P(a) : \neg Q(a)}{\neg Q(a)}, \frac{Q(a) : \neg P(a)}{\neg P(a)}, \frac{\top : \cancel{\neg Q(a)}\ R(a)}{\neg Q(a)} \right\}$

$E_1 = Cn(\overline{W})$

$E_2 = Cn(\overline{W} \cup \{\neg Q(a)\})$ inconsistent

$E_3 = Cn(\overline{W} \cup \{\neg P(a)\})$ ~~inconsistent~~

$E_4 = Cn(\overline{W} \cup \{\neg Q(a), \neg P(a)\})$ inconsistent

$\cancel{\Delta_{E_1} = \{Q(a) / \neg P(a)\}}$

$\cancel{\Gamma_{\overline{T}'}(E_1) = Cn(\overline{W} \cup \{\neg P(a)\}) = E_3}$

$\cancel{\Delta_{E_2} = \emptyset}$

$\cancel{\Gamma_{\overline{T}'}(E_2) = Cn(\overline{W}) = E_1}$

$\cancel{\Delta_{E_3} = \{Q(a) / \neg P(a)\}}$

$\Delta_{E_1} = \{Q(a) / \neg P(a), \top / \neg Q(a)\}$

$\Gamma_{\overline{T}'}(E_1) = Cn(\overline{W} \cup \{\neg P(a), \neg Q(a)\}) = E_4$

$\Delta_{E_2} = \emptyset$

$\Gamma_{\overline{T}'}(E_2) = Cn(\overline{W}) = E_1$

$\Delta_{E_3} = \{Q(a) / \neg P(a), \top / \neg Q(a)\}$

$\Gamma_{\overline{T}'}(E_3) = Cn(\overline{W} \cup \{\neg P(a), \neg Q(a)\}) = E_4$

$\Delta_{E_4} = \emptyset$

$\Gamma_{\overline{T}'}(E_4) = Cn(\overline{W}) = E_1$

2.7 $W = \{T(d), P(d)\}$

$$\Delta = \left\{ \frac{P(x) \wedge R(x) : C(x)}{C(x)}, \frac{P(x) : R(x)}{R(x)}, \frac{T(x) : \neg C(x)}{\neg C(x)} \right\}$$

Closure $\bar{T} = (\bar{W}, \bar{\Delta}) = (W', \Delta')$ of T

$W' = \{ \cancel{\;} T(d), P(d)\} = W$

$$\Delta' = \left\{ \frac{P(d) \wedge R(d) : C(d)}{C(d)}, \frac{P(d) : R(d)}{R(d)}, \frac{T(d) : \neg C(d)}{\neg C(d)} \right\}$$

$E_1 = Cn(\bar{W})$

$E_2 = Cn(\bar{W}' \cup \{C(d)\}$

$E_3 = Cn(\bar{W} \cup \{R(d)\}$

$E_4 = Cn(\bar{W} \cup \{\neg C(d)\}$

$E_5 = \cancel{Cn(C(d) \cup \{R(d)}$ $Cn(\bar{W} \cup \{C(d), R(d)\})$

$E_6 = Cn(\bar{W} \cup \{C(d), \neg C(d)\})$ inconsistent

$E_7 = Cn(\bar{W} \cup \{R(d), \neg C(d)\})$

$E_8 = Cn(\{R(d), C(d), \neg C(d)\} \cup \bar{W})$ inconsistent

$-\Delta_2 = \{ P(d) \wedge R(d) / C(d), P(d) / R(d)\}$

$\neg\bar{T}(E_2) = \cancel{R(d)}\ Cn(\bar{W} \cup \{R(d)\}) = E_3$

$-\Delta_3 = \{P(d) \wedge R(d) / C(d), P(d) / R(d), T(d) / \neg C(d)\}$

$\neg\bar{T}(E_3) = \cancel{\;} Cn(\bar{W} \cup \{C(d), R(d), \neg C(d)\}, = E_8$

$-\Delta_6 = \{P(d) / R(d)\}$

$\neg\bar{T}(E_6) = Cn(\bar{W} \cup \{R(d)\} = E_3$

$-\Delta_7 = \{P(d) / R(d), T(d) / \neg C(d)\}$

$\neg\bar{T}(E_7) = Cn(\bar{W} \cup \{R(d), \neg C(d)\} = E_7$ ✓

E_7 is an Extension

There are 7 unique candidates ($E_6 = E_8$)

3.1 facts if: only the head
ground if: no variables
non-disjunktive: if it has no disjunction in head
normal: non disjunctive and no strong negation
basic: if no default negation and no non-empty head
Horn: normal and basic

	ground	Horn	normal	non-disj.	basic	facts
P1	✓			✓		
P2	✓				✓	
P3					✓	
P4	✓			✓	✓	
P5	✓	✓	✓	✓	✓	✓
P6				✓		

$a \leftarrow b_1, b_2, \text{not } c_1, \text{not } c_2$ becomes $\dfrac{b_1 \wedge b_2 : \neg c_1, \neg c_2}{a}$

$\cancel{B(P_1) = (\emptyset, \{\frac{T : \neg B(a)}{P(a)} \wedge \frac{T : \emptyset}{\neg B(a)}}$

$P_1 := \{P(a) \leftarrow \text{not } B(a).\ B(a).\}.$

$\cancel{B(P_1) = (\emptyset, \{}$ $B(P_1) = (\emptyset \{ \frac{T : \neg B(a)}{P(a)}, \frac{T : \emptyset}{B(a)} \})$

$\cancel{\Delta E_1 = E_1 = Cn(W)},$

$E_1 = Cn(\emptyset)$, $\Delta E_1 = \{T/P(a), T/B(a)\}$, $\Gamma(E_1) = Cn(P(a), B(a))$

$E_2 = Cn(P(a))$, $\Delta E_2 = \{T/P(a), T/B(a)\}$, $\Gamma(E_2) = Cn(P(a), B(a))$

$E_3 = Cn(B(a))$, $\Delta E_3 = \{T/B(a)\}$, $\Gamma(E_3) = Cn(B(a))$ ✓

$E_4 = Cn(P(a), B(a))$, $\Delta E_4 = \{T/B(a)\}$, $\Gamma(E_4) = Cn(B(a))$ ✗

The only answer set is $\{B(a)\}$ which corresponds to the only Extension $Cn(B(a))$

$P_2 := \{ P(b) \vee P(a).\ \ B(a). \}.$

$\delta(P_2) = (\emptyset, \{ \frac{T : \emptyset}{P(b) \vee P(a)} ; \frac{T : \emptyset}{B(a)} \})$

The only extension is $Cn(P(b) \vee P(a), B(a))$, the two answer sets are $\{P(b), B(a)\}$ and $\{P(a), B(a)\}$, there is no correspondence.

$P_3 := \{ B(a).,\ \neg P(x) \vee Z(x) \leftarrow B(x). \}.$

$\delta(P_3) = (\emptyset, \{ \frac{T : \cancel{B(a)} \emptyset}{B(a)} , \frac{B(x) : \emptyset}{\neg P(x) \vee Z(x)} \})$

The only extension is $Cn(B(a), \neg P(a) \vee Z(a))$.

The two answer sets are $\{B(a), \neg P(a)\}$ and $\{B(a), Z(a)\}$

No correspondence.

$P_4 := \{ B(a).\ \neg F(a) \leftarrow B(a),\ P(a) \leftarrow F(a). \}.$

$\delta(P_4) = (\emptyset, \{ \frac{T : \emptyset}{B(a)} , \frac{B(a) : \emptyset}{\neg F(a)} , \frac{F(a) : \emptyset}{P(a)} \})$

$\frac{B(a) : \emptyset}{\neg F(a)}$ cancels $\frac{F(a) : \emptyset}{P(a)}$ out

the only extension is $Cn(B(a), \neg F(a))$, which corresponds to the only answer set $\{B(a), \neg F(a)\}$

$P_5 := \{ B(a).\ P(a). \}$

$\delta(P_5) = (\emptyset, \{ \frac{T : \emptyset}{B(a)} , \frac{T : \emptyset}{P(a)} \})$

Really obvious correspondence, $Cn(B(a), P(a)) \hat{=} \{B(a), P(a)\}$

$P_6 := \{ F(a).\ F(x) \leftarrow B(x), \neg P(x).\ P(x) \leftarrow B(x), not\ F(x). \}.$

~~$\delta(P_6) = (\emptyset, \{ \frac{T : \emptyset}{F(a)} , \frac{B(x) : \emptyset}{F(x)} \})$~~

$\delta(P_6) = (\emptyset, \{ \frac{T : \emptyset}{F(a)} , \frac{B(x) \wedge \neg P(x) : \emptyset}{F(x)} , \frac{B(x) : \neg F(x)}{P(x)})$

Because $B(x)$ can't be derived, the last two rules can never be true.

Therefore, the only extension $Cn(F(a))$ corresponds with the only answer set $(F(a))$.

3.2 P_1 has no answer sets and its rules contain no negation.

$$2 = \{ p(X) \} :- p(X).$$

$p(1).$

A Choice rule, where there is only 1 answer set, but 2 are needed, is unsatisfiable.

P_2 has 5 answer sets, each of size 2, and its rules contain only facts.

a | b | c | d | f.

e.

There are 5 possibilities to combine the rules.

P_3 has an answer set and it contains the rule

a :- not a.

a.

a :- not a.

A killing clause only kills the ~~rules~~ answer sets of new atoms.

3.3.

1) $P = \left\{ \begin{array}{l} a \vee c \leftarrow \text{not } c. \\ a \leftarrow \text{not } c. \\ b \leftarrow \text{not } c. \end{array} \right\}$

answer set = {a, b} ASP - Core 2, also easy to see

Gelfond - Lifschitz reduct: take candidate set

1) delete all rules where negative body is in conflict with candidate.

nothing has to be deleted

2) delete from remaining rules all negative literals

$\left\{ \begin{array}{l} a \vee c. \\ a. \\ b. \end{array} \right\}$ There is no superset from {a, b} that is an answer set. {a, b, c} would not be minimal

ii) E is a or b, the first part of the sum, ~~count~~ ~~evaluates~~ counts S of p(S, T, Q), where T is not the E currently worked with. The second part counts the first number ~~in~~ D in qu(D, G), where ~~D~~ G = G in ~~G~~ o(G, F)

E = a , X = ({3, 6, 4}, {5, 7}, {4, 7}, {6, 7})
E = b , X = ({7, a, 3}, {7, a, 4}, {5, 7}, {4, 7, {6, 7})

r(a, 18), r(b, 17)

3.4 If S_1 and S_2 are both answer sets of program P, then $S_1 \subseteq S_2$ implies $S_1 = S_2$.

An Interpretation M is an answer set of a ground program P, iff it is a minimal model of P^M, i.e. there is no $N \subset M$ which is also a model of P^M.

$S_1 = S_2$ we want to show

So we have to rule out $S_1 \subset S_2$.

Assume $S_1 \subset S_2$

P^{S_2} is the reduct of S_2, that means S_2 is a minimal model of P^{S_2}.

If $S_1 \subset S_2$ would be true, S_2 couldn't be a minimal model.

$S_1 \not\subset S_2$

$S_1 = S_2$ is true, if $S_1 \subseteq S_2$