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Grade	**10.00** out of 10.00 (**100**%)

Question 1

Correct

Mark 1.00 out of 1.00

Assume that $l(d) \wedge \forall i(m(j(i), e(d, i)) \vee o(d))$ is a well-formed formula. Classify each symbol as either a function or a predicate by drag-and-dropping it to the correct location.

Important: The symbols should be specified in alphabetical order. For instance, in $a(b) \wedge c(d)$, the predicate symbols should be specified as a, c and the function symbols as b, d. Furthermore, note that constants are 0-ary functions.

- The predicate symbols are l, m, o.
- The function symbols are d, e, j.

Question 2

Correct

Mark 1.00 out of 1.00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{a\ (arity\ 0), b\ (arity\ 0), f\ (arity\ 2), g\ (arity\ 1)\} \subseteq \mathcal{F}$ and $\{x, y\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. Which formulas are **not** well-formed formulas in First-Order Logic?

Important: Note that there can be more than one correct answer.

- [x] a. $(P(a, a) \wedge Q(g(b))) \rightarrow \forall(x \wedge y)P(f(x, b), g(f(x, b)))$
- [x] b. $(P(f(a), a) \rightarrow Q(g(a))) \rightarrow \exists x Q(f(x, b), g(x))$
- [x] c. $\exists x Q(f(x, b), g(x)) \wedge \neg(f(a, b) \rightarrow Q(g(b)))$
- [ ] d. $P(f(a, b), g(a)) \vee \forall x(Q(a, b) \rightarrow P(x, b))$

Question 3

Correct

Mark 1.00 out of 1.00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{f\ (arity\ 2), a\ (arity\ 0), b\ (arity\ 0)\} \subseteq \mathcal{F}$ and $\{x\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. How many **different terms** occur in the formula $P(f(a, b), f(a, a)) \vee \exists x(P(a, x) \rightarrow Q(a, x))$? Note that if a term occurs twice, it only counts as one.

Example: The formula $P(f(x), g(x), f(a))$ contains 4 unique terms - x, $f(x)$, $g(x)$, $f(a)$.

Answer: 5

Question 4

Correct

Mark 1.00 out of 1.00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{f\ (arity\ 2)\} \subseteq \mathcal{F}$ and $\{z, x, y\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. Which variables have free occurrences in the formula $\forall x(P(f(x,x), f(x,y)) \land \exists y Q(f(y,z), f(x,y)))$? Note that variables that have both free and bound occurrences should also be listed.

Important: In order for your answer to be properly validated, input your variables comma-separated and with no white-spaces in between. For instance, for the formula $\forall y P(x,y) \land P(x,y)$ your answer should be: *x,y*

Answer: y,z

Question 5

Correct

Mark 1.00 out of 1.00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{a\ (arity\ 0), f\ (arity\ 2), b\ (arity\ 0), g\ (arity\ 1)\} \subseteq \mathcal{F}$ and $\{x\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. How many **subformulas** does the formula $(\forall x P(a, f(x,b)) \to P(f(a,b), f(a,a))) \land Q(f(a,b), g(a))$ have? Note that if a subformula occurs twice, it only counts as one.

Answer: 6

Question 6

Correct

Mark 1.00 out of 1.00

How is the correct formalization of the natural language sentence "All Viennese people are friendly."?

- a. $\forall x(Viennese(x) \land Friendly(x))$
- b. $\forall x\ Viennese(x) \to Friendly(x)$
- c. $Viennese(x) \to Friendly(x)$
- d. $\forall x(Viennese(x) \to Friendly(x))$ (selected, correct)

Question 7

Correct

Mark 1.00 out of 1.00

Which formula is equivalent to the **negation** of $\neg\forall x\ P(x)$?

- a. $\exists x\ P(x)$
- b. $\exists x\ \neg P(x)$
- c. $\neg\exists x\ P(x)$
- d. $\neg\exists x\ \neg P(x)$ (selected, correct)

Question 8

Correct

Mark 1.00 out of 1.00

Which one of the following entailments holds?

- a. $\forall x(P(x) \rightarrow Q(x)), P(a) \models \forall y\, Q(y)$
- b. $\forall x\, P(x) \models \exists y\, P(y)$ (selected, correct)
- c. $\exists x(P(x) \wedge Q(x)), \forall y\, P(y) \models Q(b)$
- d. $\forall x(P(x) \rightarrow Q(x)), Q(a) \models \forall x\, P(x)$

Question 9

Correct

Mark 1.00 out of 1.00

Consider the formula $\varphi := \exists x \forall y(\neg P(x,y) \vee P(y,y)) \wedge \exists x \neg P(x,x)$. Given the structure $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with domain $D_{\mathcal{M}} = \{a,b\}$, your task is to specify the relation $P^{\mathcal{M}}$ associated to the predicate symbol P by $I_{\mathcal{M}}$, such that the formula φ is satisfied by $\mathcal{M}$.

Note: List the tuples in $P^{\mathcal{M}}$ comma-separated, with no spaces in between. In case $P^{\mathcal{M}}$ should not contain any elements, input "empty" (without the quotes) in the field below. Furthermore, note that multiple correct answers may be possible.

Example: For $\exists x P(x,x)$ and $\mathcal{M} = (\{a,b\}, I_{\mathcal{M}})$, a possible input could be: *(a,a),(a,b)*

Answer: (b,b) (correct)

Question 10

Correct

Mark 1.00 out of 1.00

Which of the following interpretations is a model of the formula $\forall x \exists y P(x,y)$? Note that more than one answer may be correct.

- a. $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with $D_{\mathcal{M}} = \{a,b,c\}$ and $P^{\mathcal{M}} = \{(a,a),(a,b),(a,c)\}$
- b. $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with $D_{\mathcal{M}} = \{a,b,c\}$ and $P^{\mathcal{M}} = \emptyset$
- c. $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with $D_{\mathcal{M}} = \{a,b,c\}$ and $P^{\mathcal{M}} = \{(b,a),(a,b)\}$
- d. $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with $D_{\mathcal{M}} = \{a,b,c\}$ and $P^{\mathcal{M}} = \{(a,a),(b,b),(c,c)\}$ (selected, correct)