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Begonnen am Mittwoch, 22. Februar 2023, 21:02**Status** Beendet**Beendet am** Mittwoch, 22. Februar 2023, 21:09**Verbrauchte Zeit** 6 Minuten 51 Sekunden**Bewertung** 8 von 10 (80%)**Feedback** Congratulations! You have successfully passed the test!**Frage 1**

Vollständig

Erreichte
Punkte 1 von 1

Suppose you toss a fair coin ten times and it comes up heads every time. Which one of the following statements is **true** ?

- ☐ a. The law of large numbers tells how many tosses will be necessary before the percentages of heads and tails are again in balance.
- ☐ b. By the law of large numbers, the next toss is more likely to be tails than another heads.
- ☒ c. The probability that the next toss will again be heads is 0.5.
- ☐ d. By the properties of conditional probability, the next toss is more likely to be heads given that ten tosses in a row have been heads.

Frage 2

Vollständig

Erreichte
Punkte 1 von 1

Assume the given distribution is normal. Pumpkins grown on a certain farm have weights with a mean of 10 kilograms. What is the standard deviation (in kilograms) of the weight if 37% of the pumpkins weigh more than 13 kilograms?

The values given in the table below should be used.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051

Table 1: Cumulative distribution function of the standard normal distribution

- ☒ a. 8.82
- ☐ b. 1.22
- ☐ c. 2.97
- ☐ d. 4.76

Frage 3

Vollständig

Erreichte
Punkte 0 von 1

Let X_1, \dots, X_{64} be a random sample from a distribution with the expectation 1.2 and variance 4. Let $\bar{X} = \frac{1}{64} \sum_{i=1}^{64} X_i$ be the sample mean.

Determine the approximate value of $P(\bar{X} > 0.72)$ using the Central limit theorem and express it in terms of a suitable R-function.

- ☐ a. `pnorm(-0.72, 1.2, 0.25)`
- ☐ b. `pnorm(1.92)`
- ☒ c. `pnorm(0.72, 1.2, 0.25)`
- ☐ d. `pnorm(-1.92)`

Frage 4

Vollständig

Erreichte

Punkte 1 von 1

Let X_1, \dots, X_{100} be a random sample from an exponential distribution

with the expectation $\frac{1}{2}$.

Determine the approximate value of

$$P\left(\sum_{i=1}^{100} X_i > 53\right)$$

using the Central limit theorem.

Recall, a random variable with probability density function

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x \leq 0 \end{cases}$$

is exponential with parameter $\lambda = 2$ and its expectation equals $\frac{1}{2}$.

Use the values given in the table below.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051

Table 1: Cumulative distribution function of the standard normal distribution

- ☐ a. 0.309
- ☒ b. 0.274
- ☐ c. 0.345
- ☐ d. 0.271

Frage 5

Vollständig

Erreichte

Punkte 1 von 1

Let X_1, X_2, \dots, X_6 be ten independent standard normal random variables. Let $S = \max\{X_1, \dots, X_6\}$. Express the probability that $S \leq 1.2$ in terms of the cumulative distribution function Φ of the standard normal random variable

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt, \quad x \in \mathbb{R}.$$

- ☒ a. $\Phi(1.2)^6$
- ☐ b. $1 - (1 - \Phi(1.2))^6$
- ☐ c. $1 - \Phi(1.2)^6$
- ☐ d. $(1 - \Phi(1.2))^6$

Frage 6

Vollständig

Erreichte

Punkte 1 von 1

The distribution of cholesterol levels for patients in a cardiology practice follows a normal distribution with a mean of 220 and a standard deviation of 40. In this practice, the probability that a patient has a cholesterol level less than 140 is the same as the probability that a patient has a cholesterol level reading more than:

- ☐ a. 340
- ☒ b. 300
- ☐ c. 320
- ☐ d. 400

Frage 7

Vollständig

Erreichte

Punkte 1 von 1

Let X_1, X_2, \dots, X_{81} be an i.i.d. sample from a population with population mean $\mu = 5$ and population variance $\sigma^2 = 4$ and let $S = X_1 + X_2 + \dots + X_{81}$. Approximate the probability $P(S \notin [387, 423])$ using the Central limit theorem.

- ☒ a. 32%
- ☐ b. 68%
- ☐ c. 78%
- ☐ d. 45%

Frage 8

Vollständig

Erreichte
Punkte 1 von 1

Assume that a baseball team has an average pitcher, that is, one whose probability of winning any decision is 0.5. If this pitcher has 30 decisions in a season, what is the probability that he will win at least 23 games?

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Table 1: Cumulative distribution function of the standard normal distribution

- ☒ a. less than 0.5%
- ☐ b. between 0.5% and 1%
- ☐ c. more than 5%
- ☐ d. between 1% and 5%

Frage 9

Vollständig

Erreichte
Punkte 0 von 1

Assume the given distribution is normal. The mean income per household in a certain state is \$9500 with a standard deviation of \$1750. The middle 95% of incomes are between which two values?

The following table for z_α -values should be used. Here, z_α denotes the number that satisfies $P(z \leq z_\alpha) = \alpha$, for $\alpha \in [0, 1]$.

z_α	$\alpha=0.05$	$\alpha=0.25$	$\alpha=0.50$	$\alpha=0.75$	$\alpha=0.90$	$\alpha=0.95$	$\alpha=0.975$	$\alpha=0.99$	$\alpha=0.995$
	-1.645	-0.674	0.000	0.674	1.282	1.645	1.960	2.326	2.575

Table: z_α -values

- ☐ a. \$5422 and \$13,578
- ☐ b. \$8049 and \$10,951
- ☒ c. \$6621 and \$12,379
- ☐ d. \$5422 and \$13,578

Frage 10

Vollständig

Erreichte
Punkte 1 von 1

Assume the given distributions are normal. The average score on a college placement exam is 500 with a standard deviation of 100. Ninety-five percent of the test takers score approximately above what score?

The following table for z_α -values is useful. Here, z_α denotes the number that satisfies $P(z \leq z_\alpha) = \alpha$, for $\alpha \in [0, 1]$.

z_α	$\alpha=0.05$	$\alpha=0.25$	$\alpha=0.50$	$\alpha=0.75$	$\alpha=0.90$	$\alpha=0.95$	$\alpha=0.975$	$\alpha=0.99$	$\alpha=0.995$
	-1.645	-0.674	0.000	0.674	1.282	1.645	1.960	2.326	2.575

Table: z_α -values

- ☐ a. 405
- ☐ b. 484
- ☐ c. 664
- ☒ d. 336