1	2	3	4	5	Σ	Grade

Examination for "Logic and Reasoning in Computer Science"

June 21st, 2024 **1st Exam for SS 2024**

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This exam sheet consists of five problems, yielding a total of 100 points. Good luck!

Problem 1. (25 points) Consider the formula:

$$(p \leftrightarrow \neg q) \rightarrow ((p \vee q) \wedge \neg(p \vee r))$$

(a) Which atoms are pure in the above formula?

(b) Compute a clausal normal form C of the above formula by applying the CNF transformation algorithm with naming and optimization based on polarities of subformulas;

(c) Decide the satisfiability of the computed CNF formula C by applying the DPLL method to C. If C is satisfiable, give an interpretation which satisfies it.

Problem 2. (15 points) Suppose we know that:

1. If Paolo is thin, then Carlo is not blonde or Roberta is not tall.
2. If Roberta is tall, then Sandra is lovely.
3. If Sandra is lovely and Carlo is blonde, then Paolo is thin.
4. Carlo is blonde.

Can we then conclude that *"Roberta is not tall"*?

Formalize this problem in logic and argue whether the entailment holds.

Problem 3. (20 points) Let A be a propositional, well-formed formula using $n \geq 1$ propositional variables such that

- A is not a propositional atom, and
- A is built from propositional atoms using only $\neg, \vee$ and $\rightarrow$.

Let S be the set of clauses obtained from A by applying the standard conjunctive normal form transformation (that is, without naming clauses). Let p be an atom occurring in A. Show that p is pure in A if and only if one of the literals p or $\neg p$ is pure in S.

Problem 4 (15 points) Provide either a proof or a counterexample for the following statement:

$$\exists x(A(x) \rightarrow \forall x A(x)) \models \exists x A(x)$$

If you provide a counterexample, you have to show that it is in fact a counterexample.

Problem 5. (25 points) Consider the formula:

$$a = b + 5 \wedge f(b + 2) = c \wedge (read(write(A, a + 1, 4), b + 6) = 2 \vee f(a - 3) \neq c)$$

where a, b, c are integer constants, A is an array constants, f is a unary function, $read, write$ are interpreted in the array theory, and $+, -, 1, 2, 3, ...$ are interpreted in the standard way over the integers.

Use the Nelson-Oppen decision procedure for reasoning in the combination of the theories. Use the decision procedures for the theory of arrays and the theory of uninterpreted functions, and use simple mathematical reasoning for deriving new equalities among the constants in the theory of linear integer arithmetic. If the formula is satisfiable, give an interpretation that satisfies the formula.