

Examination for “Logic and Computability”
March 17, 2021 — second Exam for WS20/21

Matrikelnummer	FAMILY NAME	First Name
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Task 1:

Formalize the following sentences:

- (a) For every unfortunate there is some goat that bites him.
- (b) Alf is unfortunate.
- (c) Bic is an unfortunate goat.
- (d) Bic bites Alf.

Does $(a), (b), (c) \models (d)$? Motivate your answer.

Task 2:

Describe the interpretations that are models for the formula

$$\forall x \exists y (\neg(x = y) \wedge A(x, y))$$

where $=$ is the equality predicate.

Task 3:

Is the countable union of recursive set recursive? (Provide a proof or a counterexample)

Task 4:

Let Φ_i be a partial computable function and consider the following sets:

- 1. $K = \{x \mid x \notin \text{Ran}(\Phi_i)\}$
- 2. $J = \{x \mid x \in \text{Dom}(\Phi_i) \wedge x \in \text{Ran}(\Phi_i)\}$

Are they recursive, r.e., or neither?

($\text{Ran} = \text{Range}$ and $\text{Dom} = \text{Domain} (= \{x \mid \Phi_i(x) \downarrow\})$)

Task 5:

Show by Robinson-resolution that the clause set $\{C, D, E\}$ is unsatisfiable, where

$$C = \neg p(f(x), y) \vee p(f(y), f(x))$$

$$D = p(x, a) \vee p(f(y), a),$$

$$E = \neg p(f(f(x)), f(a)).$$

Specify all used factors, MGUs, and unified literals. (a is a constant, x, y are variables.)

Task 6:

Let G be the modal formula $\neg \diamond \Box A \vee A$. Prove or refute:

- (1) G is valid in every symmetric frame.
- (2) If $\mathcal{F} \models G$ for a frame \mathcal{F} , then \mathcal{F} is symmetric.

Task 7:

Use the proof of the ADRF theorem to prove that the function $f(x + 2, g(x, x))$ is arithmetically definable, if f and g are arithmetically definable. *Hint:* Specify the witnessing formula, following slide 23 of the last set of slides (on incompleteness).