Statistik

Beschreibende Statistik

Messniveau

- **Nominalskala** (klassifikatorische Skala)
- **Ordinal**- (Rang-)**skala**
- **Intervallskala**
- **Verhältnisskala**

Kenngrößen

- **Modalwert**
- **Quantile**: $n \cdot \alpha = \begin{cases} ganzzahlig \Rightarrow q_\alpha = \frac{x_{n\alpha}+x_{n\alpha+1}}{2} \\ nichtganzzahlig \Rightarrow q_\alpha = x_{\lfloor n\alpha \rfloor+1} \end{cases}$
- **Mittelwert**: $\bar{x} = \frac{1}{n}\sum_i^n x_i$
- **Varianz**: $s^2 = \frac{1}{n-1}\sum_i^n (x_i - \bar{x})^2 \approx (\sum_i^n x_i^2) - n(\bar{x})^2$
- **Variationskoeffizient**: $v = \frac{s}{\bar{x}}$
- **Schiefe**: $g_1 = \frac{1}{n}\sum_i^n (x_i - \bar{x})^3 / s^3$ (linksschief: $g_1 < 0$)
- **Kurtosis**: $g_2 = \frac{1}{n}\sum_i^n (x_i - \bar{x})^4 / s^4 - 3$

Wahrscheinlichkeitstheorie

Stichprobenraum: $\Omega = \{(x_1, \dots, x_n): x_i = \cdots\}$
Ereignisraum: $M = \mathcal{P}(\Omega)$

Rechenregeln

- $P(A) = 1 - P(A^C)$
- $P(A \cup B) = P(A) + P(\mathrm{B}) - P(A \cap B)$

bedingte Wahrscheinlichkeit: $P(\,A \mid B\,) = \frac{P(A \cap \backslash \mathrm{B})}{P(A)}$

Unabhängigkeit: $P(A \cap B) = P(A) \cdot P(B)$

Zufallsvariable

Wahrscheinlichkeitsfunktion: $P(X = x) = f(x)$
Verteilungsfunktion: $P(X <= x) = F(x) = \int_{-\infty}^{x} f(t)dt \mid \sum_{x_i \leq x} P(X = x_i)$

Erwartung: $E\big(h(X)\big) = \int_{-\infty}^{\infty} h(x)f(x)dx \mid \sum_i^{\infty} h(x_i)p_i$

Varianz: $\sigma^2 = Var(X) = E(X-\mu)^2 = E(X - EX)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Poissonverteilung

$$P(x = i) = \frac{\lambda^i}{i!}e^{-\lambda}$$

$\lambda = np$, für ganzzahlige Variablen sowie $n > 50$ und $p < 0.1$

mehrdimensionale Zufallsvariablen

Randverteilung: $f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy,\ f_{\mathrm{Y}}(x) = \int_{-\infty}^{\infty} f(x,y)dx$

$p_{X,\mathrm{i}} = \sum_j^{\infty} p_{ij},\ p_{Y,j} = \sum_i^{\infty} p_{ij}$

Erwartung: $E(h(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)f(x,y)dxdy$

Unabhängigkeit: $f(x,y) = f_X(x)f_Y(y)$
Kovarianz: $\sigma_{XY} = E\big((X-EX)(Y-EY)\big) = E(XY)-(EX)(EY)$
Korrelation: $\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y}$

Analytische Statistik

Schätzer

Erwartungstreue: Der Erwartungswert des Schätzers eines Parameters ist der Parameter selbst.

Most-Likelikhood: $l(\theta, x_1, \dots, x_n) = \prod_i^n f(x_i), \theta \mid \frac{\delta l}{\delta\theta} = 0$

Hypothesentest

- Formulierung der Voraussetzungen und der Hypothese
- Wahl des Signifikanzniveaus
- Wahl der Teststatistik und des kritischen Bereichs
- Präsentierung der Rechnungen
- vollständige Angabe von Schlussfolgerungen

Fehlerarten

		Hypothese	
		richtig	falsch
Entscheidung:	annehmen	$1-\alpha$	β
	ablehnen	α	$1-\beta$

Fehler 1. Art: α
Fehler 2. Art: β

Binomialverteilung

- $\mu = np$
- $\sigma^2 = np(1-p)$

Varianzanalyse

H_0	H_1	Entscheidung gegen H_0	kritische Werte
$\mu_1 = \mu_2 = \cdots = \mu_k$	$\neg H_0$	$F > c_1$	$c_1 = F_{k-1,n-k,1-\alpha}$

$F = \frac{q_Z/(k-1)}{q_I/(n-k)}, q_I = \sum_{j=1}^{k}\sum_{i=1}^{n_j}\left(x_{ij}-\overline{x_j}\right)^2, q_Z = \sum_{j=1}^{k} n_j\left(\overline{x_j}-\bar{x}\right)^2$

Regression

Schätzer: $\hat{a} = \bar{y}, \hat{b} = \frac{s_{xy}}{s_x^2} = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2}, \hat{y} = \hat{a}+\hat{b}(x-\bar{x}), s^2 = \frac{1}{n-2}\sum(y_i-\hat{y_i})^2$

Kofidenzintervalle: $a = \bar{Y} \pm t_{n-2;1-\frac{\alpha}{2}}\frac{S}{\sqrt{n}}, b = \hat{b} \pm t_{n-2;1-\frac{\alpha}{2}}\frac{S}{s_x\sqrt{n-1}}$

$\mu_{y.x} = \widehat{y_x} \pm t_{n-2;1-\frac{\alpha}{2}} S\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{(n-1)s_x^2}}$

H_0	H_1	Entscheidung gegen H_0	kritische Werte
$b = 0$	$b \neq 0$	$\lvert T\rvert > c_1$	$c_1 = t_{n-2;1-\frac{\alpha}{2}}$

$T = \hat{b}\,\frac{s_x\sqrt{n-1}}{S}$

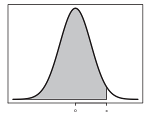
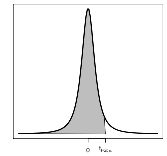


Table A.1: $N(0,1)$-Verteilung. $\alpha = P(Z \leq z) = G(z)$[1]

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

[1] R: `pnorm(`z`)`

Table A.2: Student-t-Verteilung. Quantile $t_{FG;\alpha}$; $\alpha = P(T \leq t_{FG;\alpha})$

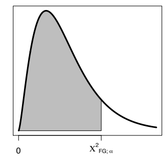


	$\boldsymbol{\alpha}$					
FG	**.75**	**.9**	**.95**	**.975**	**.99**	**.995**
1	1.000	3.078	6.314	12.706	31.824	63.659
2	.816	1.886	2.920	4.303	6.965	9.925
3	.765	1.638	2.353	3.182	4.541	5.841
4	.741	1.533	2.132	2.776	3.747	4.604
5	.727	1.476	2.015	2.571	3.365	4.032
6	.718	1.440	1.943	2.447	3.143	3.707
7	.711	1.415	1.895	2.365	2.998	3.499
8	.706	1.397	1.860	2.306	2.896	3.355
9	.703	1.383	1.833	2.262	2.821	3.250
10	.700	1.372	1.812	2.228	2.764	3.169
11	.697	1.363	1.796	2.201	2.718	3.106
12	.695	1.356	1.782	2.179	2.681	3.055
13	.694	1.350	1.771	2.160	2.650	3.012
14	.692	1.345	1.761	2.145	2.624	2.977
15	.691	1.341	1.753	2.131	2.602	2.947
16	.690	1.337	1.746	2.120	2.583	2.921
17	.689	1.333	1.740	2.110	2.567	2.898
18	.688	1.330	1.734	2.101	2.552	2.878
19	.688	1.328	1.729	2.093	2.539	2.861
20	.687	1.325	1.725	2.086	2.528	2.845
21	.686	1.323	1.721	2.080	2.518	2.831
22	.686	1.321	1.717	2.074	2.508	2.819
23	.685	1.319	1.714	2.069	2.500	2.807
24	.685	1.318	1.711	2.064	2.492	2.797
25	.684	1.316	1.708	2.060	2.485	2.787
26	.684	1.315	1.706	2.056	2.479	2.779
27	.684	1.314	1.703	2.052	2.473	2.771
28	.683	1.313	1.701	2.048	2.467	2.763
29	.683	1.311	1.699	2.045	2.462	2.756
30	.683	1.310	1.697	2.042	2.457	2.750
31	.682	1.309	1.696	2.040	2.453	2.744
32	.682	1.309	1.694	2.037	2.449	2.738
33	.682	1.308	1.692	2.035	2.445	2.733
34	.682	1.307	1.691	2.032	2.441	2.728
35	.682	1.306	1.690	2.030	2.438	2.724
40	.681	1.303	1.684	2.021	2.423	2.704
45	.680	1.301	1.679	2.014	2.412	2.690
50	.679	1.299	1.676	2.009	2.403	2.678
55	.679	1.297	1.673	2.004	2.396	2.668
60	.679	1.296	1.671	2.000	2.390	2.660
65	.678	1.295	1.669	1.997	2.385	2.654
70	.678	1.294	1.667	1.994	2.381	2.648
75	.678	1.293	1.665	1.992	2.377	2.643
80	.678	1.292	1.664	1.990	2.374	2.639
85	.677	1.292	1.663	1.988	2.371	2.635
90	.677	1.291	1.662	1.987	2.368	2.632
95	.677	1.291	1.661	1.985	2.366	2.629
100	.677	1.290	1.660	1.984	2.364	2.626

[2]

[2] R: `qt(alpha,FG)`

Table A.3: Chi-Quadrat-Verteilung. $\chi^2_{FG;\alpha}$; $\alpha = P(\chi^2 \leq \chi^2_{FG;\alpha})$



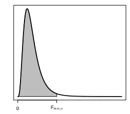
	α										
FG	**.005**	**.01**	**.025**	**.05**	**.1**	**.5**	**.9**	**.95**	**.975**	**.99**	**.995**
1	.000	.000	.001	.004	.016	.455	2.706	3.841	5.024	6.635	7.879
2	.010	.020	.051	.103	.211	1.386	4.605	5.991	7.378	9.210	10.597
3	.072	.115	.216	.352	.584	2.366	6.251	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	1.064	3.357	7.779	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	1.610	4.351	9.236	11.070	12.832	15.086	16.750
6	.676	.872	1.237	1.635	2.204	5.348	10.645	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	2.833	6.346	12.017	14.067	16.013	18.475	20.278
8	1.344	1.647	2.180	2.733	3.490	7.344	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	8.343	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	9.342	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	10.341	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	11.340	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	12.340	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	13.339	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	14.339	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	15.338	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	16.338	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	17.338	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	18.338	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	19.337	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	20.337	29.615	32.671	35.479	38.932	41.401
22	8.643	9.543	10.982	12.338	14.041	21.337	30.813	33.924	36.781	40.289	42.796
23	9.261	10.196	11.689	13.091	14.848	22.337	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	23.337	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	24.337	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	25.336	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	26.336	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	27.336	37.916	41.337	44.461	48.278	50.994
29	13.121	14.257	16.047	17.708	19.768	28.336	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	29.336	40.256	43.773	46.979	50.892	53.672
31	14.458	15.655	17.539	19.281	21.434	30.336	41.422	44.985	48.232	52.191	55.003
32	15.134	16.362	18.291	20.072	22.271	31.336	42.585	46.194	49.480	53.486	56.328
33	15.815	17.074	19.047	20.867	23.110	32.336	43.745	47.400	50.725	54.776	57.648
34	16.501	17.789	19.806	21.664	23.952	33.336	44.903	48.602	51.966	56.061	58.964
35	17.192	18.509	20.569	22.465	24.797	34.336	46.059	49.802	53.203	57.342	60.275
40	20.707	22.164	24.433	26.509	29.051	39.335	51.805	55.758	59.342	63.691	66.766
45	24.311	25.901	28.366	30.612	33.350	44.335	57.505	61.656	65.410	69.957	73.166
50	27.991	29.707	32.357	34.764	37.689	49.335	63.167	67.505	71.420	76.154	79.490
55	31.735	33.570	36.398	38.958	42.060	54.335	68.796	73.311	77.380	82.292	85.749
60	35.534	37.485	40.482	43.188	46.459	59.335	74.397	79.082	83.298	88.379	91.952
65	39.383	41.444	44.603	47.450	50.883	64.335	79.973	84.821	89.177	94.422	98.105
70	43.275	45.442	48.758	51.739	55.329	69.334	85.527	90.531	95.023	100.425	104.215
75	47.206	49.475	52.942	56.054	59.795	74.334	91.061	96.217	100.839	106.393	110.286
80	51.172	53.540	57.153	60.391	64.278	79.334	96.578	101.879	106.629	112.329	116.321
85	55.170	57.634	61.389	64.749	68.777	84.334	102.079	107.522	112.393	118.236	122.325
90	59.196	61.754	65.647	69.126	73.291	89.334	107.565	113.145	118.136	124.116	128.299
95	63.250	65.898	69.925	73.520	77.818	94.334	113.038	118.752	123.858	129.973	134.247
100	67.328	70.065	74.222	77.929	82.358	99.334	118.498	124.342	129.561	135.807	140.169

Die Tabelle enthält die α-Quantile der χ^2-Verteilung mit FG Freiheitsgraden. Die Zeilen sind die FG, die Spalten beziehen sich auf α. Z.B. wird für $\alpha = 0.975$ und 10 FG ein Wert $\chi^2_{10;0.025} = 20.483$ abgelesen. [3]

[3] R: `qchisq(alpha,FG)`

Table A.4: F-Verteilung. Quantile $F_{m,n;\alpha}$; $\alpha = P(F \leq F_{m,n;\alpha})$

$\boldsymbol{\alpha = .9}$



FG	**n = 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
m = 1	39.863	8.526	5.538	4.545	4.060	3.776	3.589	3.458	3.360
2	49.500	9.000	5.462	4.325	3.780	3.463	3.257	3.113	3.006
3	53.593	9.162	5.391	4.191	3.619	3.289	3.074	2.924	2.813
4	55.833	9.243	5.343	4.107	3.520	3.181	2.961	2.806	2.693
5	57.240	9.293	5.309	4.051	3.453	3.108	2.883	2.726	2.611
6	58.204	9.326	5.285	4.010	3.405	3.055	2.827	2.668	2.551
7	58.906	9.349	5.266	3.979	3.368	3.014	2.785	2.624	2.505
8	59.439	9.367	5.252	3.955	3.339	2.983	2.752	2.589	2.469
9	59.858	9.380	5.240	3.936	3.316	2.958	2.725	2.561	2.440
10	60.195	9.392	5.230	3.920	3.297	2.937	2.703	2.538	2.416
12	60.706	9.408	5.216	3.896	3.268	2.905	2.668	2.502	2.379
15	61.220	9.425	5.200	3.870	3.238	2.871	2.632	2.464	2.340
20	61.740	9.441	5.184	3.844	3.207	2.836	2.595	2.425	2.298
30	62.265	9.458	5.168	3.817	3.174	2.800	2.555	2.383	2.255
60	62.794	9.475	5.151	3.790	3.140	2.762	2.514	2.339	2.208
120	63.061	9.483	5.143	3.775	3.123	2.742	2.493	2.316	2.184
200	63.168	9.486	5.139	3.769	3.116	2.734	2.484	2.307	2.174
500	63.265	9.489	5.136	3.764	3.109	2.727	2.476	2.298	2.165

FG	**n = 10**	**12**	**15**	**20**	**30**	**60**	**120**	**200**	**500**
m = 1	3.285	3.177	3.073	2.975	2.881	2.791	2.748	2.731	2.716
2	2.924	2.807	2.695	2.589	2.489	2.393	2.347	2.329	2.313
3	2.728	2.606	2.490	2.380	2.276	2.177	2.130	2.111	2.095
4	2.605	2.480	2.361	2.249	2.142	2.041	1.992	1.973	1.956
5	2.522	2.394	2.273	2.158	2.049	1.946	1.896	1.876	1.859
6	2.461	2.331	2.208	2.091	1.980	1.875	1.824	1.804	1.786
7	2.414	2.283	2.158	2.040	1.927	1.819	1.767	1.747	1.729
8	2.377	2.245	2.119	1.999	1.884	1.775	1.722	1.701	1.683
9	2.347	2.214	2.086	1.965	1.849	1.738	1.684	1.663	1.644
10	2.323	2.188	2.059	1.937	1.819	1.707	1.652	1.631	1.612
12	2.284	2.147	2.017	1.892	1.773	1.657	1.601	1.579	1.559
15	2.244	2.105	1.972	1.845	1.722	1.603	1.545	1.522	1.501
20	2.201	2.060	1.924	1.794	1.667	1.543	1.482	1.458	1.435
30	2.155	2.011	1.873	1.738	1.606	1.476	1.409	1.383	1.358
60	2.107	1.960	1.817	1.677	1.538	1.395	1.320	1.289	1.260
120	2.082	1.932	1.787	1.643	1.499	1.348	1.265	1.228	1.194
200	2.071	1.921	1.774	1.629	1.482	1.326	1.239	1.199	1.160
500	2.062	1.911	1.763	1.616	1.467	1.306	1.212	1.168	1.122

Die Tabellen enthalten die rechten α-Quantile der F-Verteilung mit m (Zeilen) und n (Spalten) Freiheitsgraden (FG). Die beiden obigen Tabellen sind für $\alpha = 0.9$, nachfolgende Tabellen für andere Werte von α. Z.B. wird für $\alpha = 0.9$ und $m = 10$ und $n = 12$ FG ein Wert $F_{10,12;0.9} = 2.188$ abgelesen.

R: `qf(alpha,m,n)`

Tabelle A.4: F-Verteilung. Fortsetzung

α = **.95**

FG	**n = 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
m = 1	161.449	18.513	10.128	7.709	6.608	5.987	5.591	5.318	5.117
2	199.501	19.000	9.552	6.944	5.786	5.143	4.737	4.459	4.256
3	215.708	19.164	9.277	6.591	5.409	4.757	4.347	4.066	3.863
4	224.583	19.247	9.117	6.388	5.192	4.534	4.120	3.838	3.633
5	230.162	19.296	9.013	6.256	5.050	4.387	3.972	3.687	3.482
6	233.987	19.330	8.941	6.163	4.950	4.284	3.866	3.581	3.374
7	236.769	19.353	8.887	6.094	4.876	4.207	3.787	3.500	3.293
8	238.883	19.371	8.845	6.041	4.818	4.147	3.726	3.438	3.230
9	240.544	19.385	8.812	5.999	4.772	4.099	3.677	3.388	3.179
10	241.882	19.396	8.786	5.964	4.735	4.060	3.637	3.347	3.137
12	243.906	19.413	8.745	5.912	4.678	4.000	3.575	3.284	3.073
15	245.950	19.429	8.703	5.858	4.619	3.938	3.511	3.218	3.006
20	248.014	19.446	8.660	5.803	4.558	3.874	3.445	3.150	2.936
30	250.096	19.462	8.617	5.746	4.496	3.808	3.376	3.079	2.864
60	252.196	19.479	8.572	5.688	4.431	3.740	3.304	3.005	2.787
120	253.253	19.487	8.549	5.658	4.398	3.705	3.267	2.967	2.748
200	253.678	19.491	8.540	5.646	4.385	3.690	3.252	2.951	2.731
500	254.060	19.494	8.532	5.635	4.373	3.678	3.239	2.937	2.717

FG	**n = 10**	**12**	**15**	**20**	**30**	**60**	**120**	**200**	**500**
m = 1	4.965	4.747	4.543	4.351	4.171	4.001	3.920	3.888	3.860
2	4.103	3.885	3.682	3.493	3.316	3.150	3.072	3.041	3.014
3	3.708	3.490	3.287	3.098	2.922	2.758	2.680	2.650	2.623
4	3.478	3.259	3.056	2.866	2.690	2.525	2.447	2.417	2.390
5	3.326	3.106	2.901	2.711	2.534	2.368	2.290	2.259	2.232
6	3.217	2.996	2.790	2.599	2.421	2.254	2.175	2.144	2.117
7	3.135	2.913	2.707	2.514	2.334	2.167	2.087	2.056	2.028
8	3.072	2.849	2.641	2.447	2.266	2.097	2.016	1.985	1.957
9	3.020	2.796	2.588	2.393	2.211	2.040	1.959	1.927	1.899
10	2.978	2.753	2.544	2.348	2.165	1.993	1.910	1.878	1.850
12	2.913	2.687	2.475	2.278	2.092	1.917	1.834	1.801	1.772
15	2.845	2.617	2.403	2.203	2.015	1.836	1.750	1.717	1.686
20	2.774	2.544	2.328	2.124	1.932	1.748	1.659	1.623	1.592
30	2.700	2.466	2.247	2.039	1.841	1.649	1.554	1.516	1.482
60	2.621	2.384	2.160	1.946	1.740	1.534	1.429	1.386	1.345
120	2.580	2.341	2.114	1.896	1.683	1.467	1.352	1.302	1.255
200	2.563	2.323	2.095	1.875	1.660	1.438	1.316	1.263	1.210
500	2.548	2.307	2.078	1.856	1.637	1.409	1.280	1.221	1.159

Tabelle A.4: F-Verteilung. Fortsetzung

α = **.975**

FG	**n = 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
m = 1	647.789	38.506	17.443	12.218	10.007	8.813	8.073	7.571	7.209
2	799.500	39.000	16.044	10.649	8.433	7.260	6.542	6.059	5.715
3	864.163	39.166	15.439	9.979	7.764	6.599	5.890	5.416	5.078
4	899.584	39.248	15.101	9.604	7.388	6.227	5.523	5.053	4.718
5	921.811	39.298	14.885	9.364	7.146	5.988	5.285	4.817	4.484
6	937.111	39.331	14.735	9.197	6.978	5.820	5.119	4.652	4.320
7	948.217	39.355	14.624	9.074	6.853	5.695	4.995	4.529	4.197
8	956.656	39.373	14.540	8.980	6.757	5.600	4.899	4.433	4.102
9	963.217	39.387	14.473	8.905	6.681	5.523	4.823	4.357	4.026
10	968.628	39.398	14.419	8.844	6.619	5.461	4.761	4.295	3.964
12	976.708	39.415	14.337	8.751	6.525	5.366	4.666	4.200	3.868
15	984.867	39.431	14.253	8.657	6.428	5.269	4.568	4.101	3.769
20	993.103	39.448	14.167	8.560	6.329	5.168	4.467	3.999	3.667
30	1001.415	39.466	14.080	8.461	6.227	5.065	4.362	3.894	3.560
60	1009.800	39.481	13.992	8.360	6.123	4.959	4.254	3.784	3.449
120	1014.020	39.490	13.947	8.309	6.069	4.904	4.199	3.728	3.392
200	1015.713	39.493	13.929	8.289	6.048	4.882	4.176	3.705	3.368
500	1017.254	39.496	13.913	8.270	6.028	4.862	4.156	3.684	3.347

FG	**n = 10**	**12**	**15**	**20**	**30**	**60**	**120**	**200**	**500**
m = 1	6.937	6.554	6.200	5.871	5.568	5.286	5.152	5.100	5.054
2	5.456	5.096	4.765	4.461	4.182	3.925	3.805	3.758	3.716
3	4.826	4.474	4.153	3.859	3.589	3.343	3.227	3.182	3.142
4	4.468	4.121	3.804	3.515	3.250	3.008	2.894	2.850	2.811
5	4.236	3.891	3.576	3.289	3.026	2.786	2.674	2.630	2.592
6	4.072	3.728	3.415	3.128	2.867	2.627	2.515	2.472	2.434
7	3.950	3.607	3.293	3.007	2.746	2.507	2.395	2.351	2.313
8	3.855	3.512	3.199	2.913	2.651	2.412	2.299	2.256	2.217
9	3.779	3.436	3.123	2.837	2.575	2.334	2.222	2.178	2.139
10	3.717	3.374	3.060	2.774	2.511	2.270	2.157	2.113	2.074
12	3.621	3.277	2.963	2.676	2.412	2.169	2.055	2.010	1.971
15	3.522	3.177	2.862	2.573	2.307	2.061	1.945	1.900	1.859
20	3.419	3.073	2.756	2.464	2.195	1.944	1.825	1.778	1.736
30	3.311	2.963	2.644	2.349	2.074	1.815	1.690	1.640	1.596
60	3.198	2.848	2.524	2.223	1.940	1.667	1.530	1.474	1.423
120	3.140	2.787	2.461	2.156	1.866	1.581	1.433	1.370	1.311
200	3.116	2.763	2.435	2.128	1.835	1.543	1.388	1.320	1.254
500	3.094	2.740	2.411	2.103	1.806	1.507	1.343	1.269	1.192

Tabelle A.4: F-Verteilung. Fortsetzung

$\alpha = .99$

FG	**n = 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
m = 1	4052.192	98.505	34.116	21.198	16.258	13.745	12.246	11.259	10.561
2	4998.686	99.002	30.817	18.000	13.274	10.925	9.546	8.649	8.022
3	5402.648	99.169	29.457	16.694	12.060	9.779	8.451	7.591	6.992
4	5623.821	99.252	28.710	15.977	11.392	9.148	7.847	7.006	6.422
5	5763.357	99.300	28.237	15.522	10.967	8.746	7.460	6.632	6.057
6	5858.054	99.335	27.911	15.207	10.672	8.466	7.191	6.371	5.802
7	5927.838	99.359	27.672	14.976	10.455	8.260	6.993	6.178	5.613
8	5980.675	99.376	27.489	14.799	10.289	8.101	6.840	6.029	5.467
9	6021.547	99.389	27.347	14.659	10.157	7.976	6.719	5.911	5.351
10	6055.443	99.400	27.229	14.546	10.051	7.874	6.620	5.814	5.257
12	6105.356	99.416	27.052	14.374	9.888	7.718	6.469	5.667	5.111
15	6156.220	99.434	26.872	14.198	9.722	7.559	6.314	5.515	4.962
20	6208.075	99.452	26.690	14.020	9.552	7.396	6.155	5.359	4.808
30	6259.915	99.468	26.506	13.838	9.379	7.228	5.992	5.198	4.649
60	6312.735	99.484	26.316	13.652	9.202	7.056	5.824	5.032	4.483
120	6338.517	99.491	26.221	13.558	9.111	6.969	5.737	4.946	4.398
200	6349.377	99.495	26.183	13.520	9.075	6.934	5.702	4.911	4.363
500	6358.308	99.499	26.148	13.486	9.042	6.902	5.671	4.880	4.332

FG	**n = 10**	**12**	**15**	**20**	**30**	**60**	**120**	**200**	**500**
m = 1	10.044	9.330	8.683	8.096	7.562	7.077	6.851	6.763	6.686
2	7.559	6.927	6.359	5.849	5.390	4.978	4.787	4.713	4.648
3	6.552	5.953	5.417	4.938	4.510	4.126	3.949	3.881	3.821
4	5.994	5.412	4.893	4.431	4.018	3.649	3.480	3.414	3.357
5	5.636	5.064	4.556	4.103	3.699	3.339	3.174	3.110	3.054
6	5.386	4.821	4.318	3.871	3.473	3.119	2.956	2.893	2.838
7	5.200	4.640	4.142	3.699	3.304	2.953	2.792	2.730	2.675
8	5.057	4.499	4.004	3.564	3.173	2.823	2.663	2.601	2.547
9	4.942	4.388	3.895	3.457	3.067	2.718	2.559	2.497	2.443
10	4.849	4.296	3.805	3.368	2.979	2.632	2.472	2.411	2.356
12	4.706	4.155	3.666	3.231	2.843	2.496	2.336	2.275	2.220
15	4.558	4.010	3.522	3.088	2.700	2.352	2.192	2.129	2.075
20	4.405	3.858	3.372	2.938	2.549	2.198	2.035	1.971	1.915
30	4.247	3.701	3.214	2.778	2.386	2.028	1.860	1.794	1.735
60	4.082	3.535	3.047	2.608	2.208	1.836	1.656	1.583	1.517
120	3.996	3.449	2.959	2.517	2.111	1.726	1.533	1.453	1.377
200	3.962	3.414	2.923	2.479	2.070	1.678	1.477	1.391	1.308
500	3.930	3.382	2.891	2.445	2.032	1.633	1.421	1.328	1.232

Tabelle A.4: F-Verteilung. Fortsetzung

α = .995

FG	**n = 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
m = 1	16205.232	198.502	55.553	31.333	22.785	18.635	16.235	14.688	13.614
2	19991.950	199.000	49.803	26.284	18.314	14.544	12.404	11.042	10.107
3	21606.355	199.167	47.473	24.259	16.530	12.916	10.882	9.596	8.717
4	22491.330	199.250	46.196	23.157	15.556	12.027	10.050	8.805	7.956
5	23046.763	199.301	45.394	22.456	14.940	11.463	9.522	8.301	7.471
6	23428.396	199.333	44.838	21.975	14.513	11.073	9.155	7.952	7.134
7	23705.137	199.358	44.436	21.622	14.200	10.786	8.885	7.694	6.885
8	23915.941	199.376	44.131	21.352	13.961	10.565	8.678	7.496	6.693
9	24081.789	199.390	43.882	21.139	13.772	10.391	8.514	7.339	6.541
10	24215.665	199.401	43.692	20.967	13.618	10.250	8.380	7.211	6.417
12	24417.562	199.417	43.388	20.705	13.384	10.034	8.176	7.015	6.227
15	24620.402	199.402	43.085	20.438	13.146	9.814	7.967	6.814	6.032
20	24826.230	199.450	42.777	20.167	12.903	9.588	7.754	6.608	5.832
30	25034.044	199.467	42.467	19.891	12.656	9.358	7.534	6.396	5.625
60	25243.835	199.484	42.149	19.611	12.402	9.122	7.309	6.177	5.410
120	25348.584	199.492	41.989	19.468	12.274	9.001	7.193	6.065	5.300
200	25391.425	199.495	41.925	19.411	12.222	8.952	7.147	6.019	5.255
500	25429.346	199.498	41.867	19.359	12.175	8.908	7.104	5.978	5.215

FG	**n = 10**	**12**	**15**	**20**	**30**	**60**	**120**	**200**	**500**
m = 1	12.826	11.754	10.798	9.944	9.180	8.495	8.179	8.057	7.950
2	9.426	8.510	7.701	6.986	6.355	5.795	5.539	5.441	5.355
3	8.081	7.226	6.476	5.818	5.239	4.729	4.497	4.408	4.330
4	7.343	6.521	5.803	5.174	4.623	4.140	3.921	3.837	3.763
5	6.872	6.071	5.372	4.762	4.228	3.760	3.548	3.467	3.396
6	6.545	5.757	5.071	4.472	3.949	3.492	3.285	3.206	3.137
7	6.303	5.525	4.847	4.257	3.742	3.291	3.087	3.010	2.941
8	6.116	5.345	4.675	4.090	3.580	3.134	2.933	2.856	2.789
9	5.968	5.202	4.536	3.956	3.450	3.008	2.808	2.732	2.665
10	5.847	5.086	4.424	3.847	3.344	2.904	2.705	2.629	2.562
12	5.661	4.906	4.250	3.678	3.179	2.742	2.544	2.468	2.402
15	5.471	4.721	4.070	3.502	3.006	2.570	2.373	2.297	2.230
20	5.274	4.530	3.883	3.318	2.823	2.387	2.188	2.112	2.044
30	5.071	4.331	3.687	3.123	2.628	2.187	1.984	1.905	1.835
60	4.859	4.123	3.480	2.916	2.415	1.962	1.747	1.661	1.584
120	4.750	4.015	3.372	2.806	2.300	1.834	1.605	1.512	1.425
200	4.706	3.971	3.328	2.760	2.251	1.779	1.541	1.442	1.346
500	4.666	3.931	3.287	2.719	2.207	1.726	1.478	1.369	1.260

Tabelle A.5: Binomialverteilung. $F(x) = P(X \leq x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$.

		p									
n	**x**	**0.01**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**
8	**0**	.9227	.6634	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084
	1	.9973	.9428	.8131	.6572	.5033	.3671	.2553	.1691	.1064	.0632
	2	.9999	.9942	.9619	.8948	.7969	.6785	.5518	.4278	.3154	.2201
	3	1.0000	.9996	.9950	.9786	.9437	.8862	.8059	.7064	.5941	.4770
	4	1.0000	1.0000	.9996	.9971	.9896	.9727	.9420	.8939	.8263	.7396
	5	1.0000	1.0000	1.0000	.9998	.9988	.9958	.9887	.9747	.9502	.9115
	6	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9987	.9964	.9915	.9819
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9998	.9993	.9983
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	**0**	.9135	.6302	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046
	1	.9966	.9288	.7748	.5995	.4362	.3003	.1960	.1211	.0705	.0385
	2	.9999	.9916	.9470	.8591	.7382	.6007	.4628	.3373	.2318	.1495
	3	1.0000	.9994	.9917	.9661	.9144	.8343	.7297	.6089	.4826	.3614
	4	1.0000	1.0000	.9991	.9944	.9804	.9511	.9012	.8283	.7334	.6214
	5	1.0000	1.0000	.9999	.9994	.9969	.9900	.9747	.9464	.9006	.8342
	6	1.0000	1.0000	1.0000	1.0000	.9997	.9987	.9957	.9888	.9750	.9502
	7	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9986	.9962	.9909
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997	.9992
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	**0**	.9044	.5987	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025
	1	.9957	.9139	.7361	.5443	.3758	.2440	.1493	.0860	.0464	.0233
	2	.9999	.9885	.9298	.8202	.6778	.5256	.3828	.2616	.1673	.0996
	3	1.0000	.9990	.9872	.9500	.8791	.7759	.6496	.5138	.3823	.2660
	4	1.0000	.9999	.9984	.9901	.9672	.9219	.8497	.7515	.6331	.5044
	5	1.0000	1.0000	.9999	.9986	.9936	.9803	.9527	.9051	.8338	.7384
	6	1.0000	1.0000	1.0000	.9999	.9991	.9965	.9894	.9740	.9452	.8980
	7	1.0000	1.0000	1.0000	1.0000	.9999	.9996	.9984	.9952	.9877	.9726
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9995	.9983	.9955
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9997
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	**0**	.8953	.5688	.3138	.1673	.0859	.0422	.0198	.0088	.0036	.0014
	1	.9948	.8981	.6974	.4922	.3221	.1971	.1130	.0606	.0302	.0139
	2	.9998	.9848	.9104	.7788	.6174	.4552	.3127	.2001	.1189	.0652
	3	1.0000	.9984	.9815	.9306	.8389	.7133	.5696	.4256	.2963	.1911
	4	1.0000	.9999	.9972	.9841	.9496	.8854	.7897	.6683	.5328	.3971
	5	1.0000	1.0000	.9997	.9973	.9883	.9657	.9218	.8513	.7535	.6331
	6	1.0000	1.0000	1.0000	.9997	.9980	.9924	.9784	.9499	.9006	.8262
	7	1.0000	1.0000	1.0000	1.0000	.9998	.9988	.9957	.9878	.9707	.9390
	8	1.0000	1.0000	1.0000	1.0000	1.0000	.9999	.9994	.9980	.9941	.9852
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998	.9993	.9978
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9998
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Tabelle A.5: Binomialverteilung. Fortsetzung.

		p									
n	**x**	**0.50**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**
8	**0**	.0039	.0017	.0007	.0002	.0001	.0000	.0000	.0000	.0000	.0000
	1	.0352	.0181	.0085	.0036	.0013	.0004	.0001	.0000	.0000	.0000
	2	.1445	.0885	.0498	.0253	.0113	.0042	.0012	.0002	.0000	.0000
	3	.3633	.2604	.1737	.1061	.0580	.0273	.0104	.0029	.0004	.0000
	4	.6367	.5230	.4059	.2936	.1941	.1138	.0563	.0214	.0050	.0004
	5	.8555	.7799	.6846	.5722	.4482	.3215	.2031	.1052	.0381	.0058
	6	.9648	.9368	.8936	.8309	.7447	.6329	.4967	.3428	.1869	.0572
	7	.9961	.9916	.9832	.9681	.9424	.8999	.8322	.7275	.5695	.3366
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	**0**	.0020	.0008	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0195	.0091	.0038	.0014	.0004	.0001	.0000	.0000	.0000	.0000
	2	.0898	.0498	.0250	.0112	.0043	.0013	.0003	.0000	.0000	.0000
	3	.2539	.1658	.0994	.0536	.0253	.0100	.0031	.0006	.0001	.0000
	4	.5000	.3786	.2666	.1717	.0988	.0489	.0196	.0056	.0009	.0000
	5	.7461	.6386	.5174	.3911	.2703	.1657	.0856	.0339	.0083	.0006
	6	.9102	.8505	.7682	.6627	.5372	.3993	.2618	.1409	.0530	.0084
	7	.9805	.9615	.9295	.8789	.8040	.6997	.5638	.4005	.2252	.0712
	8	.9980	.9954	.9899	.9793	.9596	.9249	.8658	.7684	.6126	.3698
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	**0**	.0010	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0107	.0045	.0017	.0005	.0001	.0000	.0000	.0000	.0000	.0000
	2	.0547	.0274	.0123	.0048	.0016	.0004	.0001	.0000	.0000	.0000
	3	.1719	.1020	.0548	.0260	.0106	.0035	.0009	.0001	.0000	.0000
	4	.3770	.2616	.1662	.0949	.0473	.0197	.0064	.0014	.0001	.0000
	5	.6230	.4956	.3669	.2485	.1503	.0781	.0328	.0099	.0016	.0001
	6	.8281	.7340	.6177	.4862	.3504	.2241	.1209	.0500	.0128	.0010
	7	.9453	.9004	.8327	.7384	.6172	.4744	.3222	.1798	.0702	.0115
	8	.9893	.9767	.9536	.9140	.8507	.7560	.6242	.4557	.2639	.0861
	9	.9990	.9975	.9940	.9865	.9718	.9437	.8926	.8031	.6513	.4013
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	**0**	.0005	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.0059	.0022	.0007	.0002	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0327	.0148	.0059	.0020	.0006	.0001	.0000	.0000	.0000	.0000
	3	.1133	.0610	.0293	.0122	.0043	.0012	.0002	.0000	.0000	.0000
	4	.2744	.1738	.0994	.0501	.0216	.0076	.0020	.0003	.0000	.0000
	5	.5000	.3669	.2465	.1487	.0782	.0343	.0117	.0027	.0003	.0000
	6	.7256	.6029	.4672	.3317	.2103	.1146	.0504	.0159	.0028	.0001
	7	.8867	.8089	.7037	.5744	.4304	.2867	.1611	.0694	.0185	.0016
	8	.9673	.9348	.8811	.7999	.6873	.5448	.3826	.2212	.0896	.0152
	9	.9941	.9861	.9698	.9394	.8870	.8029	.6779	.5078	.3026	.1019
	10	.9995	.9986	.9964	.9912	.9802	.9578	.9141	.8327	.6862	.4312
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Tabelle A.6: Kolmogorov-Smirnov-Verteilung.

Die Tabelle gibt Quantile der Statistiken K_n^+, K_n^- für den einseitigen Test an.

	$\boldsymbol{\alpha}$				
n	**0.9**	**0.95**	**0.975**	**0.99**	**0.995**
1	.9000	.9500	.9750	.9900	.9950
2	.6838	.7764	.8419	.9000	.9293
3	.5648	.6360	.7076	.7846	.8290
4	.4927	.5652	.6239	.6889	.7342
5	.4470	.5094	.5633	.6272	.6685
6	.4104	.4680	.5193	.5774	.6166
7	.3815	.4361	.4834	.5384	.5758
8	.3583	.4096	.4543	.5065	.5418
9	.3391	.3875	.4300	.4796	.5133
10	.3226	.3687	.4092	.4566	.4889
11	.3083	.3524	.3912	.4367	.4677
12	.2958	.3382	.3754	.4192	.4490
13	.2847	.3255	.3614	.4036	.4325
14	.2748	.3142	.3489	.3897	.4176
15	.2659	.3040	.3376	.3771	.4042
16	.2578	.2947	.3273	.3657	.3920
17	.2504	.2863	.3180	.3553	.3809
18	.2436	.2785	.3094	.3457	.3706
19	.2373	.2714	.3014	.3369	.3612
20	.2316	.2647	.2941	.3287	.3524
21	.2262	.2586	.2872	.3210	.3443
22	.2212	.2528	.2809	.3139	.3367
23	.2165	.2475	.2749	.3073	.3295
24	.2120	.2424	.2693	.3010	.3229
25	.2079	.2377	.2640	.2952	.3166
26	.2040	.2332	.2591	.2896	.3106
27	.2003	.2290	.2544	.2844	.3050
28	.1968	.2250	.2499	.2794	.2997
29	.1935	.2212	.2457	.2747	.2947
30	.1903	.2176	.2417	.2702	.2899
31	.1873	.2141	.2379	.2660	.2853
32	.1844	.2108	.2342	.2619	.2809
33	.1817	.2077	.2308	.2580	.2768
34	.1791	.2047	.2274	.2543	.2728
35	.1766	.2018	.2242	.2507	.2690
36	.1742	.1991	.2212	.2473	.2653
37	.1719	.1965	.2183	.2440	.2618
38	.1697	.1939	.2154	.2409	.2584
39	.1675	.1915	.2127	.2379	.2552
40	.1655	.1891	.2101	.2349	.2521
Approximation für $n > 40$	$\frac{1.07}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.52}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

Anhang B

Zusammenfassung einiger statistischen Tests

Tests auf μ ($\overline{X}$–Test)

Gegeben n unabhängige Stichprobenvariablen $X_1, ..., X_n \sim N(\mu, \sigma^2)$.

(1) σ bekannt

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$\mu = \mu_0$	$\mu > \mu_0$	$\overline{X} > c_1$	$c_1 = \mu_0 + \frac{\sigma}{\sqrt{n}} z_{1-\alpha}$
$\mu = \mu_0$	$\mu < \mu_0$	$\overline{X} < c_2$	$c_2 = \mu_0 - \frac{\sigma}{\sqrt{n}} z_{1-\alpha}$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\overline{X} < c_3$ oder $\overline{X} > c_4$	$c_3 = \mu_0 - \frac{\sigma}{\sqrt{n}} z_{1-\alpha/2}$ $c_4 = \mu_0 + \frac{\sigma}{\sqrt{n}} z_{1-\alpha/2}$

Dabei ist z_p definiert durch $\Phi(z_p) = p$.

(2) σ unbekannt ($t - Test$)

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$\mu = \mu_0$	$\mu > \mu_0$	$\overline{X} > c_1$	$c_1 = \mu_0 + \frac{S}{\sqrt{n}} t_{n-1;1-\alpha}$
$\mu = \mu_0$	$\mu < \mu_0$	$\overline{X} < c_2$	$c_2 = \mu_0 - \frac{S}{\sqrt{n}} t_{n-1;1-\alpha}$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\overline{X} < c_3$ oder $\overline{X} > c_4$	$c_3 = \mu_0 - \frac{S}{\sqrt{n}} t_{n-1;1-\alpha/2}$ $c_4 = \mu_0 + \frac{S}{\sqrt{n}} t_{n-1;1-\alpha/2}$

$t_{n-1;p}$ ist das p-Quantil der t-Verteilung mit n-1 Freiheitsgraden und $S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2}$.

Test auf σ^2 (χ^2-Test)

Gegeben: n unabhängige Stichprobenvariablen $X_1, ..., X_n \sim N(\mu, \sigma^2)$.

(1) μ bekannt

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$\sigma^2 = \sigma_0^2$	$\sigma^2 > \sigma_0^2$	$T > c_1$	$c_1 = \sigma_0^2 \chi^2_{n;1-\alpha}$
$\sigma^2 = \sigma_0^2$	$\sigma^2 < \sigma_0^2$	$T < c_2$	$c_2 = \sigma_0^2 \chi^2_{n;\alpha}$
$\sigma^2 = \sigma_0^2$	$\sigma^2 \neq \sigma_0^2$	$T < c_3$ oder $T > c_4$	$c_3 = \sigma_0^2 \chi^2_{n;\alpha/2}$ $c_4 = \sigma_0^2 \chi^2_{n;1-\alpha/2}$

$T = \sum_{i=1}^{n} (X_i - \mu)^2$. $\chi^2_{n;p}$ ist das p-Quantil der χ^2-Verteilung mit n Freiheitsgraden.

(2) μ unbekannt

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$\sigma^2 = \sigma_0^2$	$\sigma^2 > \sigma_0^2$	$T > c_1$	$c_1 = \sigma_0^2 \chi^2_{n-1;1-\alpha}$
$\sigma^2 = \sigma_0^2$	$\sigma^2 < \sigma_0^2$	$T < c_2$	$c_2 = \sigma_0^2 \chi^2_{n-1;\alpha}$
$\sigma^2 = \sigma_0^2$	$\sigma^2 \neq \sigma_0^2$	$T < c_3$ oder $T > c_4$	$c_3 = \sigma_0^2 \chi^2_{n-1;\alpha/2}$ $c_4 = \sigma_0^2 \chi^2_{n-1;1-\alpha/2}$

$T = \sum_{i=1}^{n}(X_i - \overline{X})^2$. $\chi^2_{n-1;p}$ ist das p-Quantil der χ^2-Verteilung mit $n-1$ Freiheitsgraden.

Test auf Gleichheit zweier Erwartungswerte (t-Test)

Gegeben: n_1 unabhängige Stichprobenvariablen $X_1, ..., X_{n_1} \sim N(\mu_X, \sigma_X^2)$ und n_2 unabhängige Stichprobenvariablen $Y_1, ..., Y_{n_2} \sim N(\mu_Y, \sigma_Y^2)$.

(1) unverbundene Stichproben

Die Variablen X_i und Y_j sind ebenfalls unabhängig, $\sigma_X = \sigma_Y$ (unbekannt).

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$\mu_X = \mu_Y$	$\mu_X > \mu_Y$	$T > c_1$	$c_1 = t_{n_1+n_2-2;1-\alpha}$
$\mu_X = \mu_Y$	$\mu_X < \mu_Y$	$T < c_2$	$c_2 = -t_{n_1+n_2-2;1-\alpha}$
$\mu_X = \mu_Y$	$\mu_X \neq \mu_Y$	$T < c_3$ oder $T > c_4$	$c_3 = -t_{n_1+n_2-2;1-\alpha/2}$ $c_4 = t_{n_1+n_2-2;1-\alpha/2}$

$$T = \frac{\overline{X} - \overline{Y}}{\sqrt{\frac{(n_1-1)S_X^2 + (n_2-1)S_Y^2}{n_1+n_2-2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}},$$

$t_{n_1+n_2-2;p}$ ist das p-Quantil der t-Verteilung mit $(n_1 + n_2 - 2)$ Freiheitsgraden.

(2) verbundene Stichproben

Die Variablen X_i und Y_i sind abhängig (paarweise an einem Merkmalsträger erhoben), $n_1 = n_2 = n$. Die Variablen $D_i = X_i - Y_i \;\; (i = 1, ..., n)$ sind unabhängig.

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$\mu_X = \mu_Y$	$\mu_X > \mu_Y$	$T > c_1$	$c_1 = t_{n-1;1-\alpha}$
$\mu_X = \mu_Y$	$\mu_X < \mu_Y$	$T < c_2$	$c_2 = -t_{n-1;1-\alpha}$
$\mu_X = \mu_Y$	$\mu_X \neq \mu_Y$	$T < c_3$ oder $T > c_4$	$c_3 = -t_{n-1;1-\alpha/2}$ $c_4 = t_{n-1;1-\alpha/2}$

$T = \frac{\overline{D}}{S_D}\sqrt{n}$. Weiters ist $t_{n-1;p}$ das p-Quantil der t-Verteilung mit $n-1$ Freiheitsgraden und

$$\overline{D} = \frac{1}{n}\sum_{i=1}^{n} D_i, \quad S_D = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(D_i - \overline{D})^2}.$$

Test auf Gleichheit zweier Varianzen (F-Test)

Gegeben: n_1 unabhängige Stichprobenvariablen $X_1, ..., X_{n_1} \sim N(\mu_X, \sigma_X^2)$ und n_2 unabhängige Stichprobenvariablen $Y_1, ..., Y_{n_2} \sim N(\mu_Y, \sigma_Y^2)$. Die Variablen X_i und Y_j sind ebenfalls unabhängig.

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$\sigma_X^2 = \sigma_Y^2$	$\sigma_X^2 > \sigma_Y^2$	$T > c_1$	$c_1 = F_{n_1-1,n_2-1;1-\alpha}$
$\sigma_X^2 = \sigma_Y^2$	$\sigma_X^2 < \sigma_Y$	$T < c_2$	$c_2 = F_{n_1-1,n_2-1;\alpha}$
$\sigma_X^2 = \sigma_Y^2$	$\sigma_X^2 \neq \sigma_Y^2$	$T < c_3$ oder $T > c_4$	$c_3 = F_{n_1-1,n_2-1;\alpha/2}$ $c_4 = F_{n_1-1,n_2-1;1-\alpha/2}$

$T = \frac{S_X^2}{S_Y^2}$. Weiters ist $F_{n_1-1,n_2-1;p}$ das p-Quantil der F-Verteilung mit $n_1 - 1$ und $n_2 - 1$ Freiheitsgraden.

Test auf Unkorreliertheit (Unabhängigkeit)

Gegeben: Paarige, unabhängige Stichprobenvariablen $(X_1, Y_1), ..., (X_n, Y_n) \quad \sim N_2(\mu_x, \mu_y, \sigma_X^2, \sigma_Y^2, \rho)$.

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$\rho = 0$	$\rho \neq 0$	$\lvert R\rvert\sqrt{\frac{n-2}{1-R^2}} > c$	$c = t_{n-2;1-\alpha/2}$

$R = \frac{\sum_{i=1}^n (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^n (X_i - \overline{X})^2 \sum_{i=1}^n (Y_i - \overline{Y})^2}}$.

$t_{n-2;p}$ ist das p-Quantil der t-Verteilung mit $n-2$ Freiheitsgraden.

Wichtige nichtparametrische Tests

Anpassungstests

(1) χ^2-Test

Gegeben seien n unabhängige Stichprobenvariablen $X_1, ..., X_n$ und eine vollständig spezifizierte Verteilungsfunktion F.

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$F(x) = F_0(x)\ \forall x$	$F(x) \neq F_0(x)$ für mindestens ein x	$T > c$	$c = \chi^2_{k-1;1-\alpha}$

Dabei ist $T = \sum_{i=1}^{k}(h_i - e_i)^2/e_i$ mit k Klassen der Daten, h_i den absoluten Häufigkeiten und e_i den theoretischen, absoluten Häufigkeiten aus F_0. Ist F_0 nicht vollständig bestimmt, d.h. nur bis auf r Parameter, die aus den Daten geschätzt werden, dann sind die kritischen Werte $c = \chi^2_{k-r-1;1-\alpha}$.

(2) Kolmogorov-Smirnov-Test

Gegeben seien n unabhängige Stichprobenvariablen $X_1, ..., X_n$ mit stetiger Verteilungsfunktion F.

H_0	H_1	Entscheidung gegen H_0, falls	kritische Werte
$F(x) = F_0(x)$ $\forall x$	$F(x) > F_0(x)$ mindestens ein x	$sup(F_n(x) - F_0(x)) > c_1$	$c_1 = k^-_{1-\alpha}$
$F(x) = F_0(x)$ $\forall x$	$F(x) < F_0(x)$ mindestens ein x	$sup(F_0(x) - F_n(x)) > c_2$	$c_2 = k^+_{1-\alpha}$
$F(x) = F_0(x)$ $\forall x$	$F(x) \neq F_0(x)$ mindestens ein x	$sup\lvert F_n(x) - F_0(x)\rvert > c_3$	$c_3 = k_{1-\alpha}$