Task 1 (General questions) [33 points]

(a) **Discrete Cosine Transformation (DCT)**
Name one advantage of using the DCT as a data compression technique for images, compared to the Discrete Fourier Transform (DFT).

[/ 3]

(b) **Time-frequency analysis**
Name one method which allows to observe the change in the frequency content of a signal over time and roughly explain how the method works in your own words (no formula required).

[/ 6]

(c) **Discrete Wavelet Transform (DWT)**
With one equation, show how the wavelets are generated from a single basis wavelet (called mother wavelet). For each parameter in the equation, explain its effect.

[/ 6]

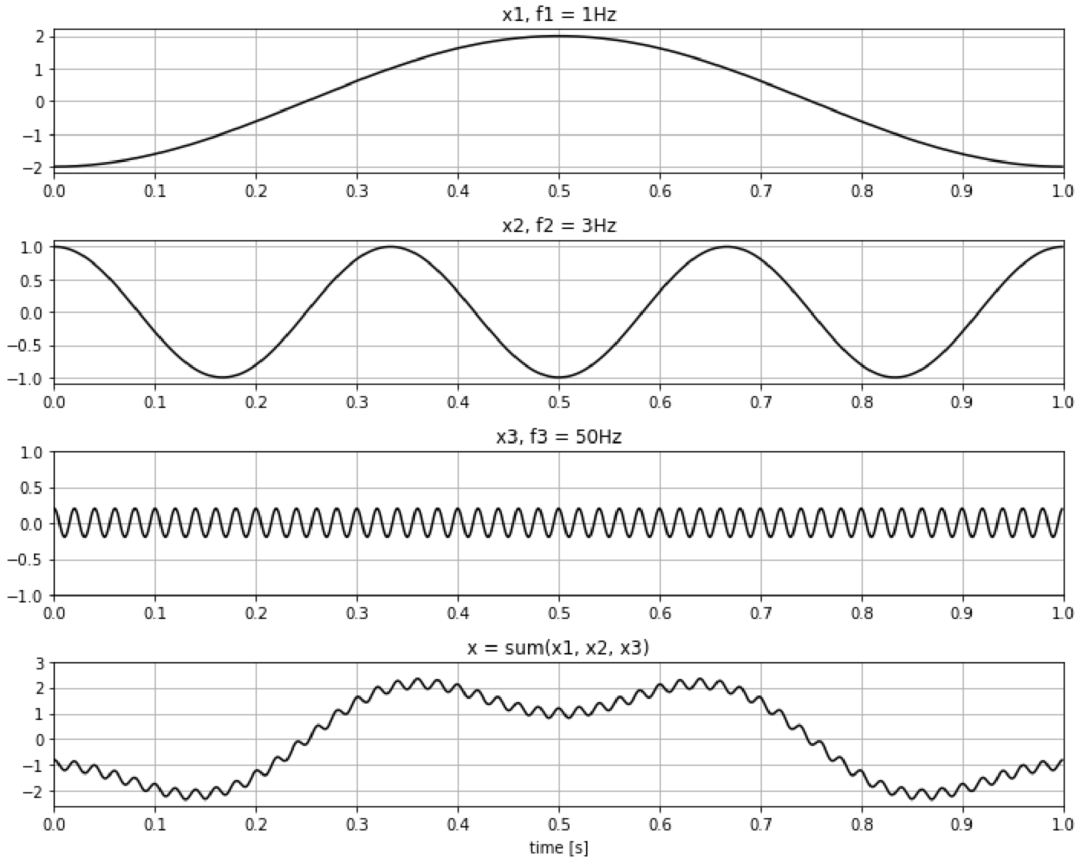
(d) **Stochastic processes**
Let $x(t)$ be a white Gaussian noise signal with zero mean and variance σ^2. Compute and draw its auto-covariance sequence $s(\tau)$ and its power spectral density (PSD) function $S(f)$. Carefully label all axes!

$s(\tau)$

$S(f)$

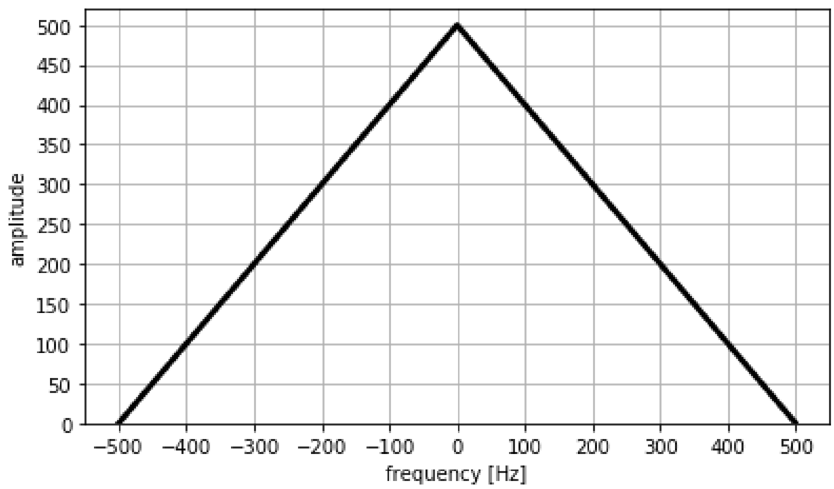
[/ 8]

(e) **High-pass filter in time domain**
The plot below shows $1s$ long sections of three infinite signals $x_1(t), x_2(t)$ and $x_3(t)$, as well as their sum $x(t) = \sum_{i=1}^{3} x_i(t)$. Sketch the signal which would result from applying an ideal 2Hz high-pass filter with zero-phase to $x(t)$. Carefully label all axes!



[/ 6]

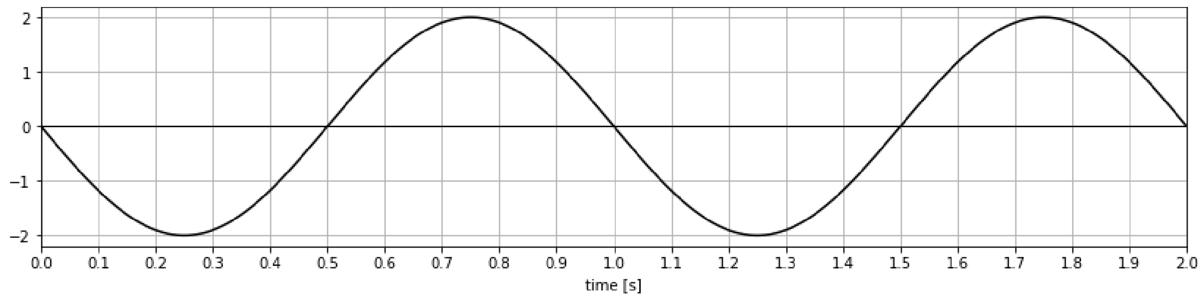
(f) **Band-pass filter in frequency domain**
The plot below shows the amplitude spectrum of a signal. Sketch the amplitude spectrum which would result from applying an ideal $100 - 300$Hz band-pass filter to it. Carefully label all axes!



[/ 4]

Task 2 (Analytic signal) [23 points]

The plot below shows a pure oscillation signal $x(t)$ for $t \in [0, 2]$.



(a) What is the frequency and phase of $x(t)$?

[/ 4]

(b) Sketch the amplitude and phase spectrum of $x(t)$. Carefully label all axes!

Amplitude spectrum

Phase spectrum

[/ 6]

(c) Let z be the analytic signal of x. Draw the imaginary part, instantaneous amplitude, instantaneous phase and instantaneous frequency of z. Carefully label all axes!

Imaginary part

Instantaneous amplitude

Instantaneous phase

Instantaneous frequency

[/ 8]

(d) Draw z in the complex plane. Indicate which point corresponds to $t_1 = 0$ and $t_2 = \frac{1}{4}$. Carefully label all axes!

[/ 5]

Task 3 (Information theory) [26 points]

To transmit messages between computers, we usually need to encode them into binary or hexadecimal code. To increase the efficiency of data transmission, many data compression techniques have been proposed, and one of the most famous algorithms is Huffman coding. Assume that we are going to transmit the following sentence:

`She sells seashells by the seashore.`

(a) With the standard ASCII, each character is encoded into 8 digits of binary code, e.g., 'A' = 0x41 = 0100 0001. Compute how many digits of binary code are needed to encode the whole sentence (including space key ' ' and '.').

[/ 4]

(b) The total number of each character in the sentence is listed below. Compute the probability of each character.

('s', 8)	('e', 7)	(' ', 5)	('h', 4)	('l', 4)	('a', 2)
('b', 1)	('y', 1)	('t', 1)	('o', 1)	('r', 1)	('.', 1)

[/ 4]

(c) Use Huffman's method to encode all presented characters (including space key ' ' and '.') and determine how many binary digits are needed to encode the given sentence.

[/ 10]

(d) Name two necessary conditions under which Huffman encoding is optimal, and determine whether they are fulfilled in this case.

[/ 8]

Task 4 (Application: Brain-Computer Interfaces (BCIs)) [18 points]

This task is designed for you to show that you can apply your signal processing knowledge to a real world problem. The task uses the example of a BCI experiment, but no previous knowledge of BCIs is required to solve it.

Suppose you are tasked with recording and evaluating brain signals for a BCI experiment with the following hypothesis: When the study participant is moving their right hand, the dominant frequency in the brain signal increases by about 1Hz compared to the dominant frequency during a relaxed state.

(a) You are interested in recording brain signals with frequencies up to 100Hz. What is the minimum sampling frequency you need for your recordings?

[/ 2]

(b) You decide to record with a sampling frequency of 1000Hz. During the experiment you record several trials (repetitions) of the participant relaxing, and several trials of the participant moving their right hand. Each trial lasts 20 seconds. In this case, what is the frequency resolution for each of the trials? Is it enough to evaluate the hypothesis above? Justify your answer.

[/ 4]

(c) After you finished recording the data, you notice that it takes up too much space on your hard disk, so you want to down-sample the data to a sampling frequency of 500Hz. Using an infinite impulse response (IIR) filter, name the steps in doing so.

[/ 6]

(d) Explain one way how to calculate the mean difference in the dominant frequency between the two cases (relaxing vs. moving the right hand) after having recorded all data.

[/ 6]