TECHNISCHE UNIVERSITÄT WIEN
Vienna | Austria

ILSB
Biomechanics

INTRODUCTION TO BIOMECHANICS 317.043, VU

Tutorial 4: Hemodynamics and the Circulatory system

Katja Haslinger, Magdalena Fuchs

haslinger@ilsb.tuwien.ac.at
mfuchs@ilsb.tuwien.ac.at

Institute of Lightweight Design and Structural Biomechanics
TU Wien, Vienna, Austria

Exam

- **January 27th 17h00 – 20h00 - online**
- Same procedure as first exam (place your camera correctly, make sure your printer/scanner works etc.)
- You are allowed 1 A4 page (handwritten!) of “formula collection”
- Upload your formula collection in TUWEL until Tuesday 25th
- Don’t forget to register

Homework Nr. 5

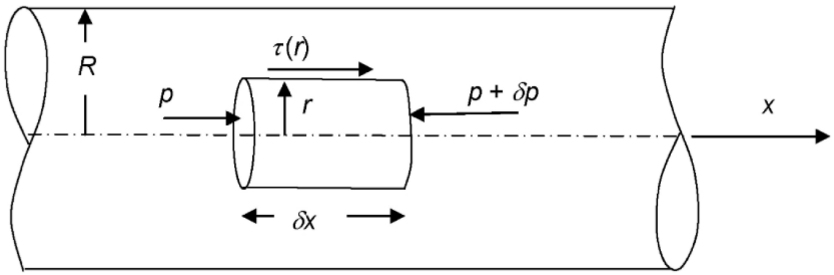
- **Is already online**
- **Caution:** due by Friday, 21st 12h00

  (So you can ask questions about the correction before the exam)

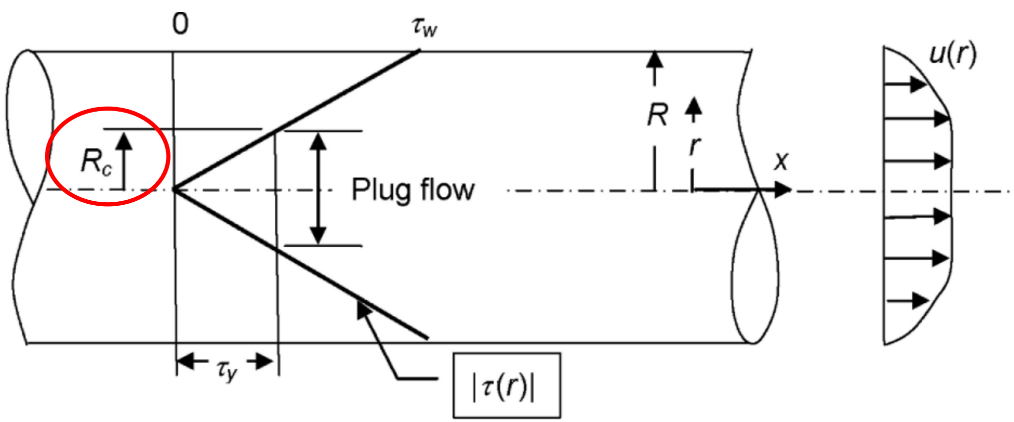
1.1 Casson Fluid – Flow Rate

Blood flows in a tube of radius 1 cm because of a pressure gradient of 0.4 dynes/cm^3. Treating the blood as a Casson fluid with yield stress 0.06 dynes/cm^2, what percentage of the total volume flowrate is from blood traveling in the central non-flowing “core” of the flow?

1.1 Casson Fluid – Flow Rate

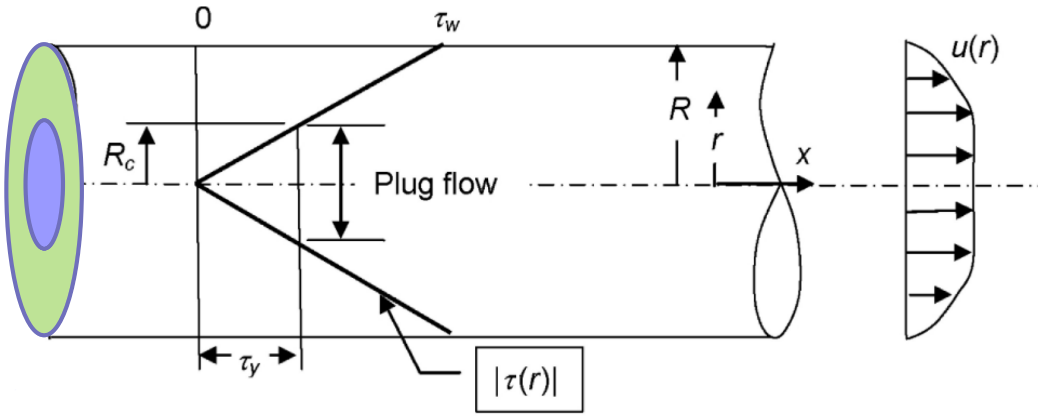


$$\rightarrow \tau(r) = \frac{r}{2}\frac{dp}{dx}$$



$$\rightarrow \tau(R_c) = \tau_y \;\rightarrow R_c$$

1.1 Casson Fluid – Flow Rate



$$Q_{plug} = \int_0^{R_c} u(R_c) * 2\pi r\, dr$$

$$Q_{flow} = \int_{R_c}^{R} u(r) * 2\pi r\, dr$$

$$Q_{total} = = -\frac{\pi R^4}{8\mu}\frac{dp}{dx}F(\xi)$$

→ Solution: ca. 13%

1.2 Casson Fluid – Pressure Drop

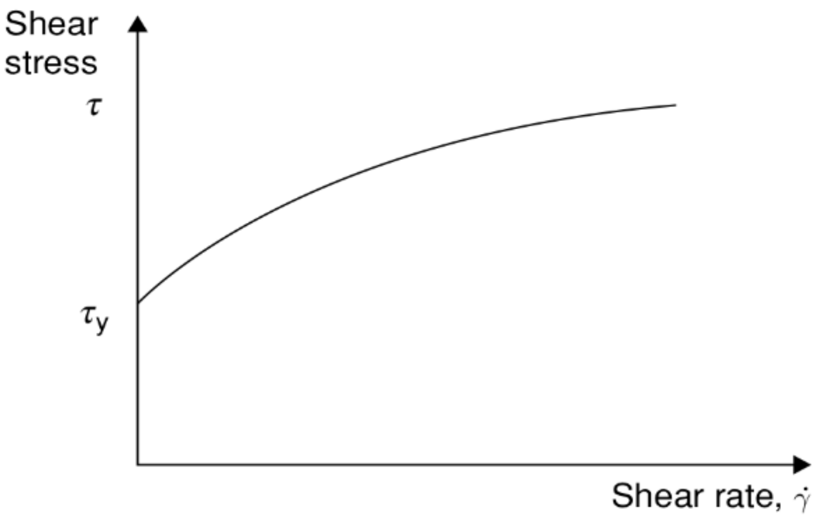
A 35 cm long tube (internal diameter, 1 mm) is filled with blood, which has been citrated to prevent it from clotting. (Assume that the citration process does not alter the rheological properties of the blood.) Use property values for blood: μ = 3.5 cP, τ_y = 0.05 dynes/cm^2, ρ = 1.06 g/cm3.

(a) At what pressure difference between the tube ends (Δp) does the blood begin to flow?

(b) What is the flow rate (Q) when Δp = 10 Pa?

1.2 Casson Fluid – Pressure Drop

- When $\tau < \tau_y$, no flow
- $\tau(r)$ increases with r



- for flow to happen:

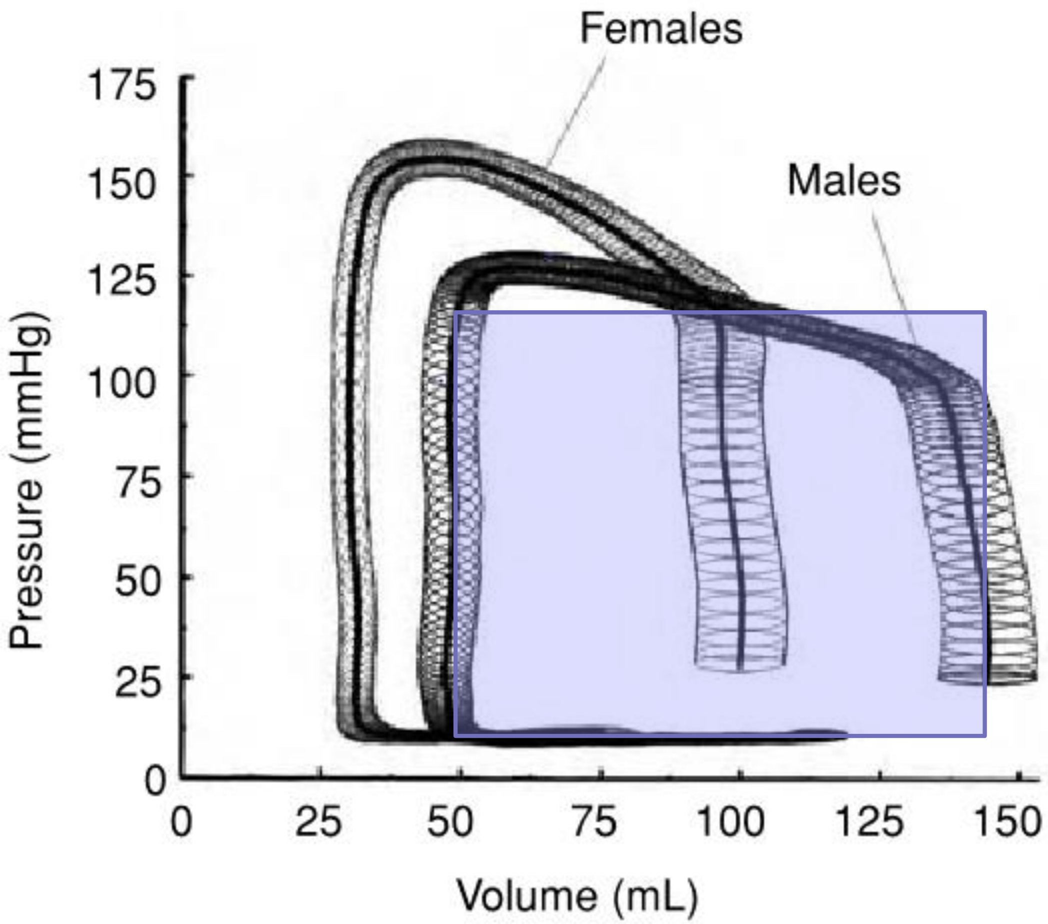
$$\tau(R) = \frac{R}{2}\frac{dp}{dx} > \tau_y$$

→ solve for pressure gradient

1.3 The Heart

Based on the averaged left-ventricular pressure-volume loop (Figure 1), calculate the pumping power of the female and the male heart (left ventricle only). Assume a resting heart rate of 70 beats per minute and approximate the area under the curve with a rectangle.

1.3 The Heart



$$P = \frac{\Delta W}{\Delta t}$$

1.4 Windkessel Model

In the Windkessel model, the product *RC* has units of time and can be thought of as a characteristic response time for the cardiovascular system. The purpose of this question is to estimate the magnitude of *RC*.

(a) Starting from the definition of compliance *C* given as $C = dV/dp$ show that $C = VD/Et$, where *V* is the volume of the compliant arteries, *D* is arterial diameter, *t* is arterial wall thickness, and *E* is the Young's modulus of the artery wall. To derive this formula, it is useful to first relate *C* to the arterial distensibility, *β*, as defined by $\beta = \frac{2}{D}\frac{\mathrm{d}D}{\mathrm{d}p} = \frac{D}{Et}$.

(b) Taking the volume of the compliant arteries as 700 ml and cardiac output as 5 l/min, estimate *RC*. The ratio between arterial wall thickness and diameter typically is $\frac{t}{D} = 0.07$ and the Young's modulus of the artery wall $E = 10 \cdot 10^5$ Pa. Use appropriate tablework to estimate the pressure drop across the systemic circulation.

Thanks for your attention!