TECHNISCHE UNIVERSITÄT WIEN
Vienna | Austria

ILSB
Biomechanics

INTRODUCTION TO BIOMECHANICS 317.043, VU

Tutorial 3: Movement Biomechanics & Deformable Bodies

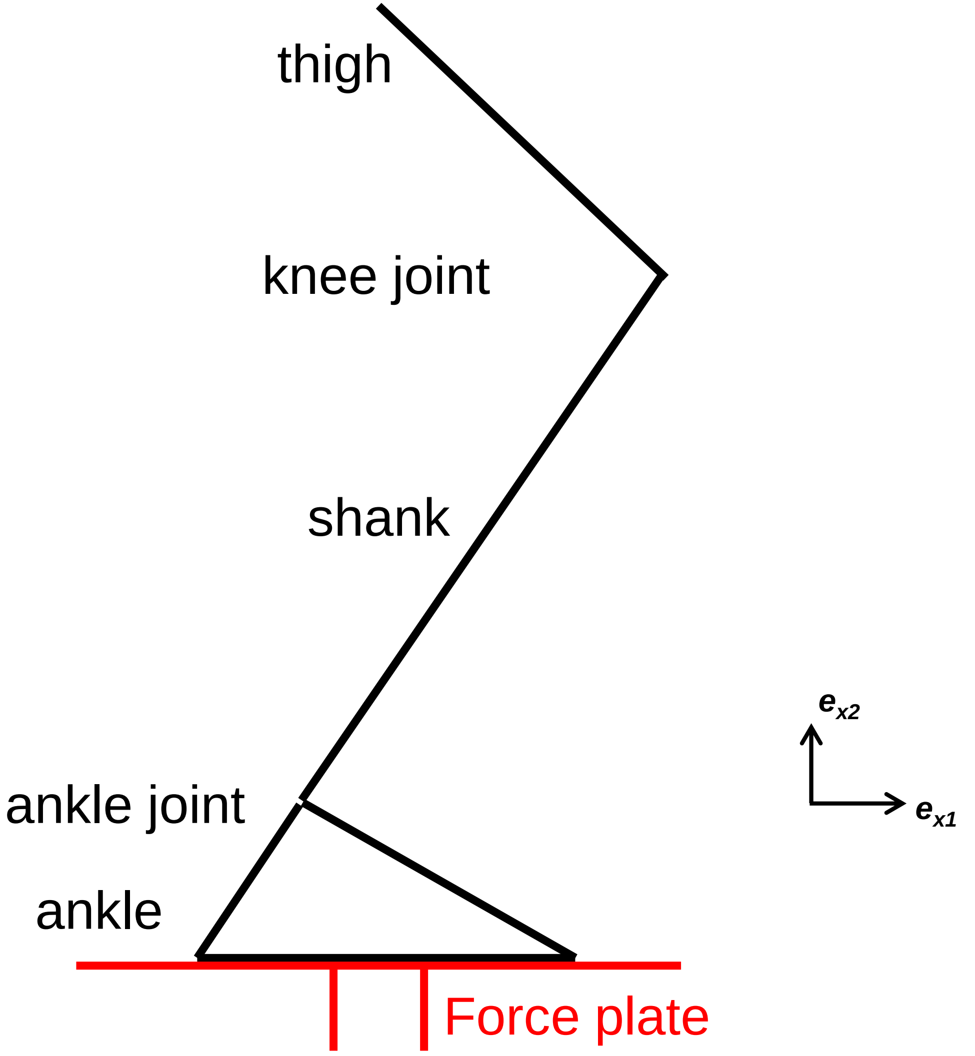
Katja Haslinger, Magdalena Fuchs

haslinger@ilsb.tuwien.ac.at
mfuchs@ilsb.tuwien.ac.at

Institute of Lightweight Design and Structural Biomechanics
TU Wien, Vienna, Austria

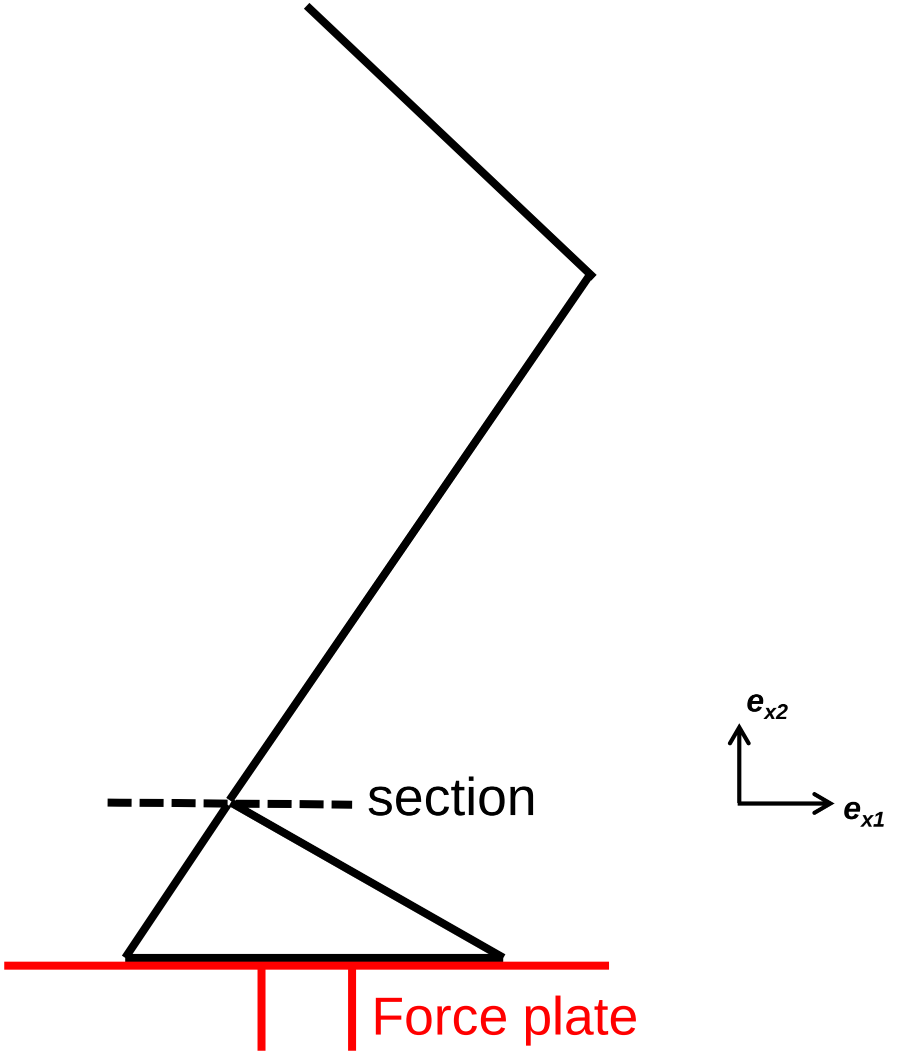
Inverse dynamics

- By knowing external forces, a body's inertial properties and its motion, internal forces are calculated



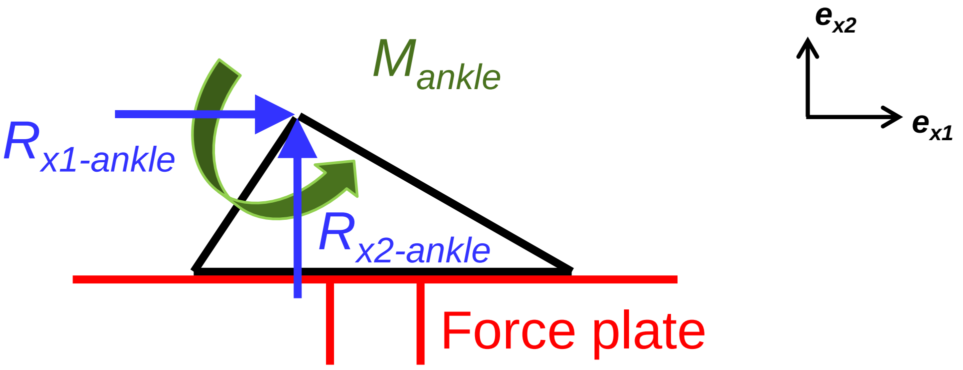
Inverse dynamics

- From force plate measurement the ground reaction force is known
- In a first step the ankle reaction force and moment are calculated



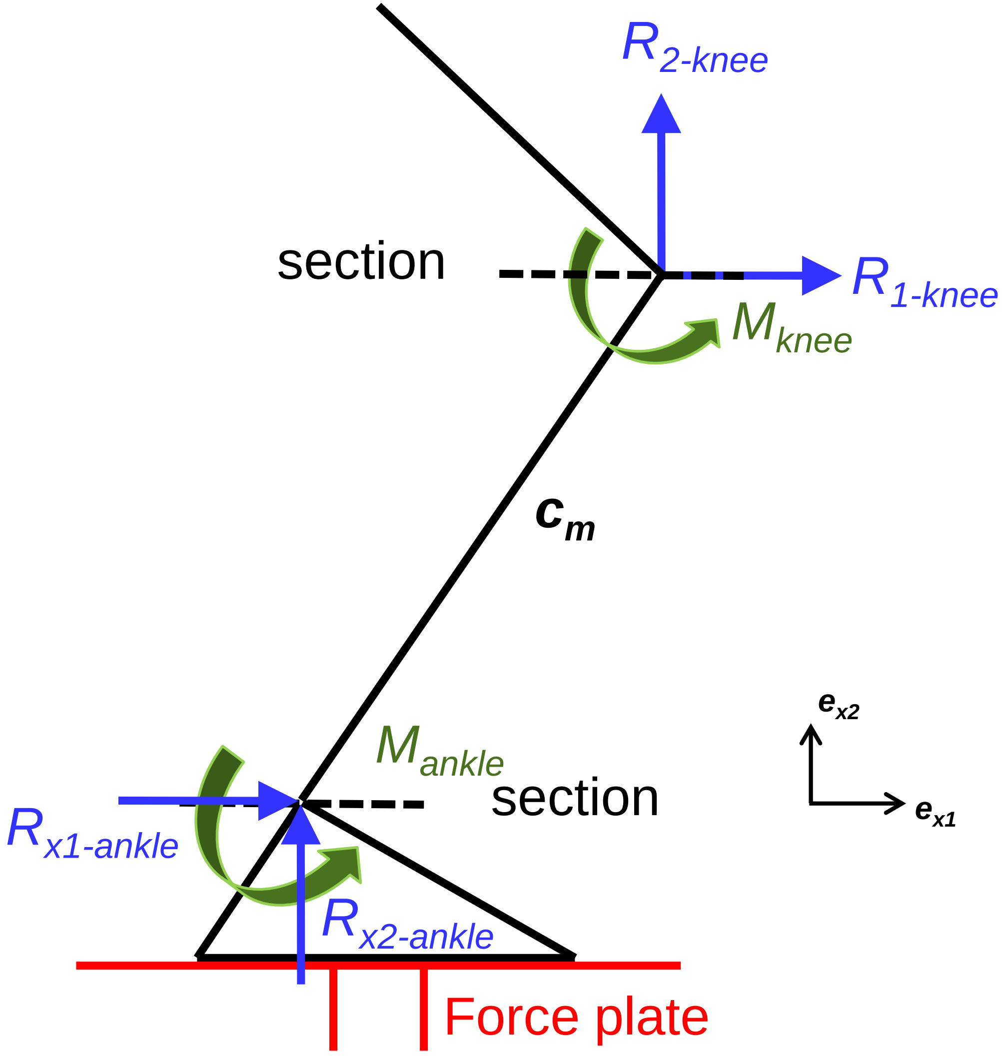
Inverse dynamics

- In a first step the ankle reaction force and moment has been calculated
- Now how do we get to the knee reaction force and moment?



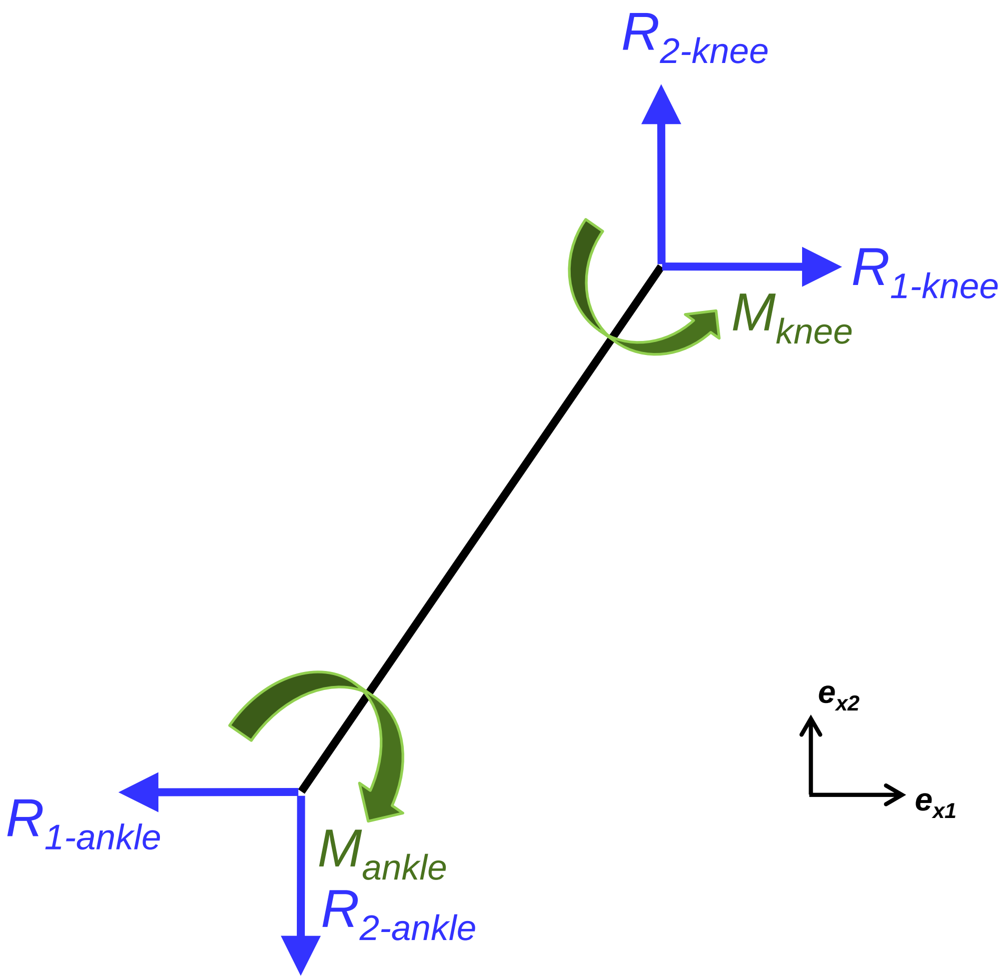
Inverse dynamics

- Make 2 sections
- To go from ankle to knee segment invert the reaction force and moment (equilibrium)



Inverse dynamics

- Then add kinetic, kinematic as well as anthropometric data



Inverse dynamics

- Then, the three equations to solve for the unknown reaction force can simply be written down e.g.

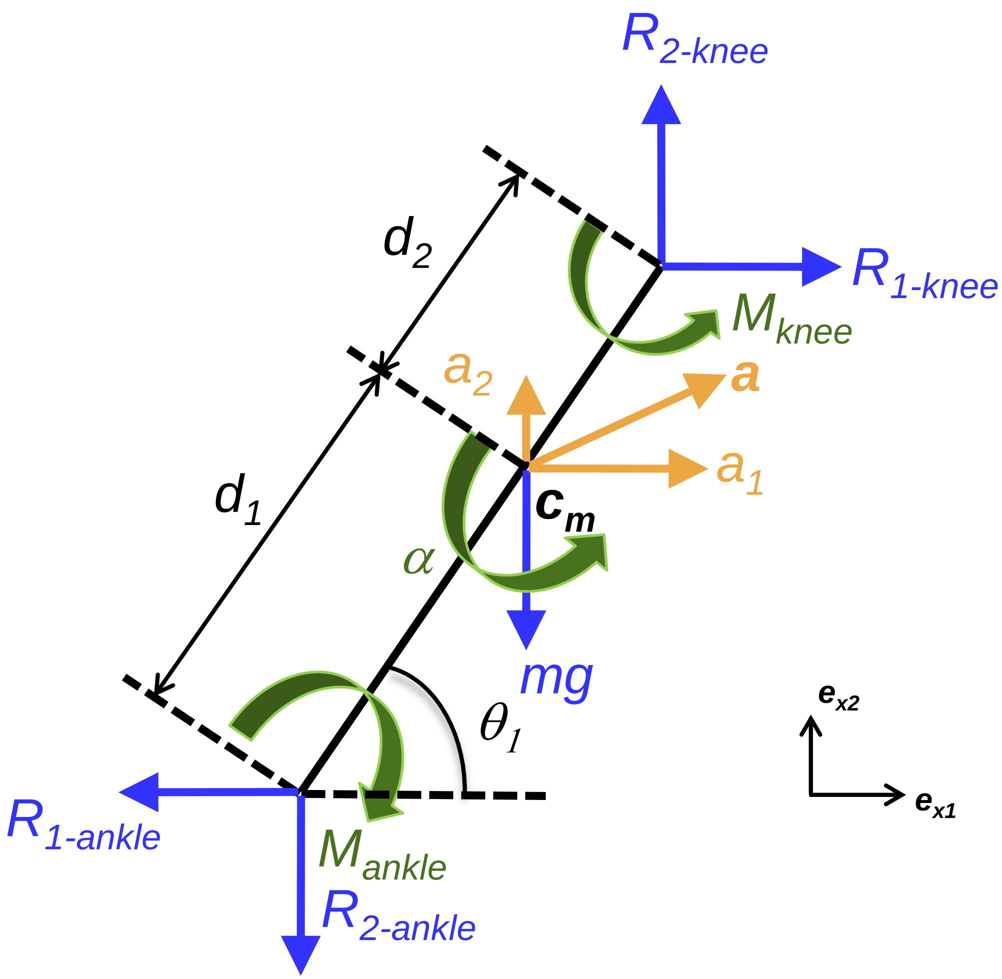
$$\sum_i f_{1i} = ma_{1i}$$

$$\sum_i f_{2i} = ma_{2i}$$

$$\sum_i M_i = I\alpha$$

For trigonometric functions (moment equation) care must be taken

when segment angles approach π, sign inversion might occur



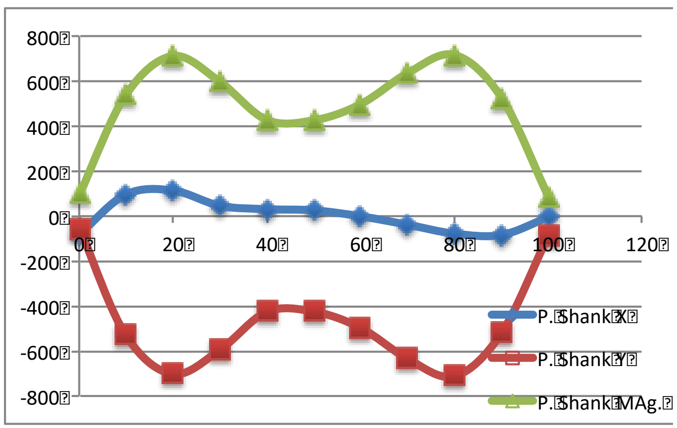
Inverse dynamics

- To obtain reaction forces of the hip – use same approach and invert forces and moments from the knee

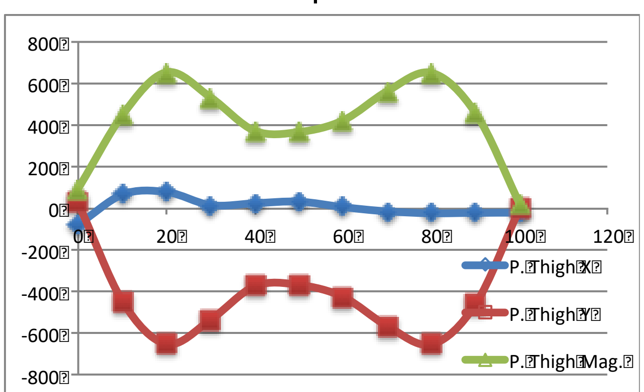
Inverse dynamics

- Use spreadsheet or other software to obtain forces and moments for full gait cycle

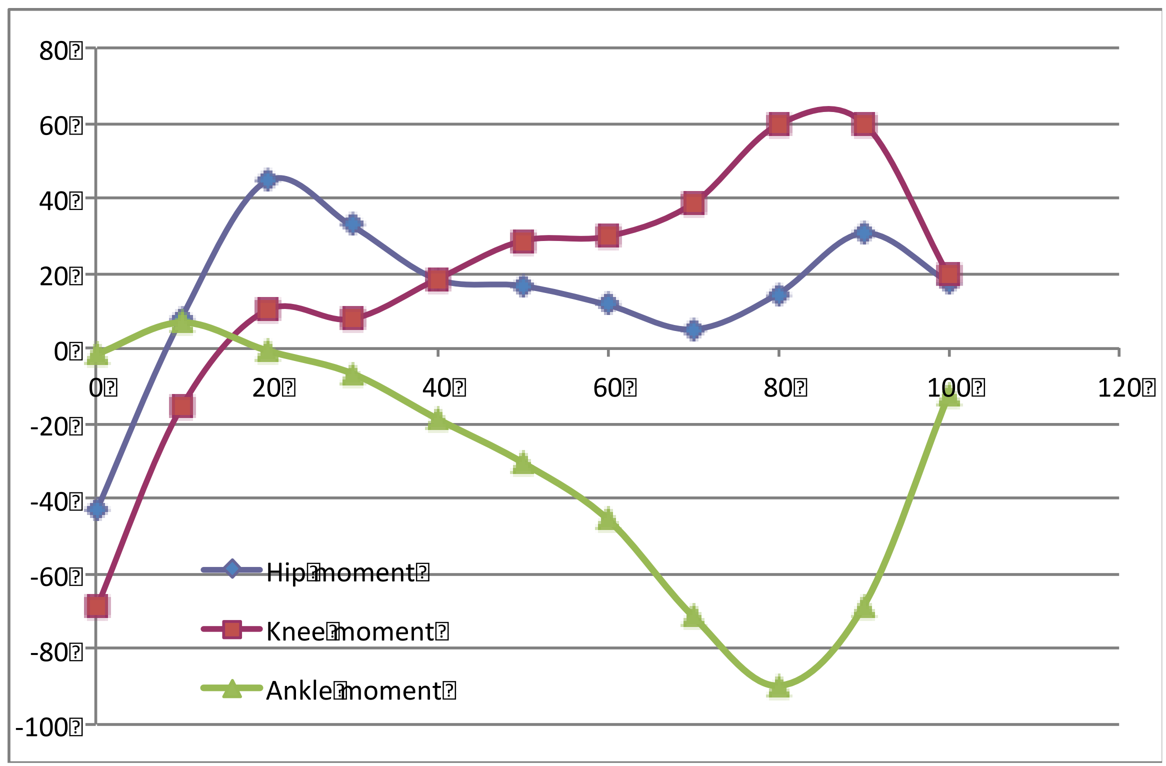
Knee



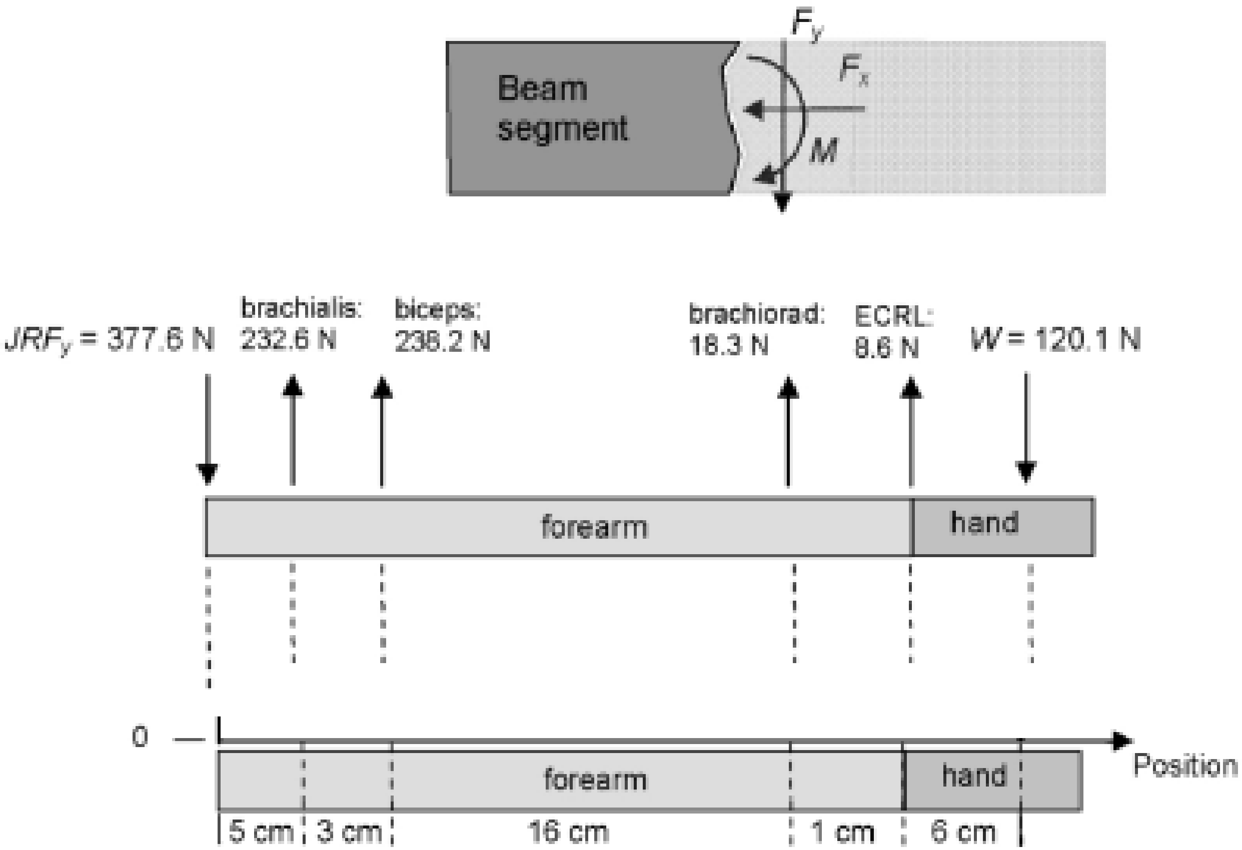
Hip



Joint moments

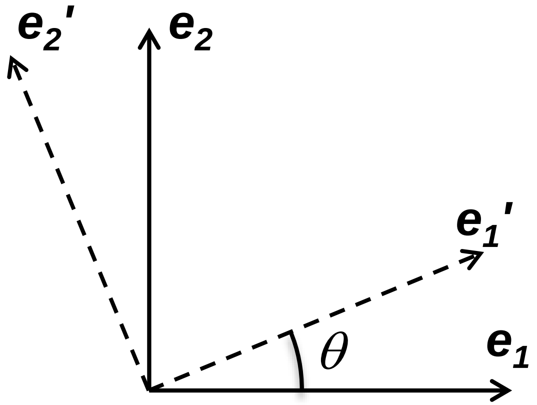


Elbow joint – Maximum internal stresses



Strain transformation

$$R(\theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$



$$\underline{\underline{\varepsilon}}' = \mathbf{R}^T \underline{\underline{\varepsilon}} \mathbf{R}$$

Strain transformation matrix

$$\varepsilon_1' = \varepsilon_1 \cos^2\theta + 2\varepsilon_{12}\sin\theta\cos\theta + \varepsilon_2 \sin^2\theta$$
$$\varepsilon_2' = \cos\theta\sin\theta\,(-\varepsilon_1 + \varepsilon_2) + \varepsilon_{12}(-\sin^2\theta + \cos^2\theta)$$
$$\varepsilon_{12}' = \varepsilon_1 \sin^2\theta - 2\varepsilon_{12}\sin\theta\cos\theta + \varepsilon_2 \cos^2\theta$$

$$\sin^2 + \cos^2 = 1$$
$$\sin 2\theta = 2\sin\cos$$
$$\cos 2\theta = \cos^2 - \sin^2$$

$$\Rightarrow$$

$$\cos^2 = \frac{1+\cos 2\theta}{2}$$
$$\sin^2 = \frac{1-\cos 2\theta}{2}$$

Shear transformation matrix result

$$\varepsilon_1' = \frac{\varepsilon_1 + \varepsilon_2}{2} + \frac{\varepsilon_1 - \varepsilon_2}{2} \cos 2\theta + \varepsilon_{12} \sin 2\theta$$

$$\varepsilon_2' = \frac{\varepsilon_1 + \varepsilon_2}{2} + \frac{\varepsilon_2 - \varepsilon_1}{2} \cos 2\theta - \varepsilon_{12} \sin 2\theta$$

$$\varepsilon_{12}' = \varepsilon_{12} \cos 2\theta + \frac{\varepsilon_2 - \varepsilon_1}{2} \sin 2\theta$$

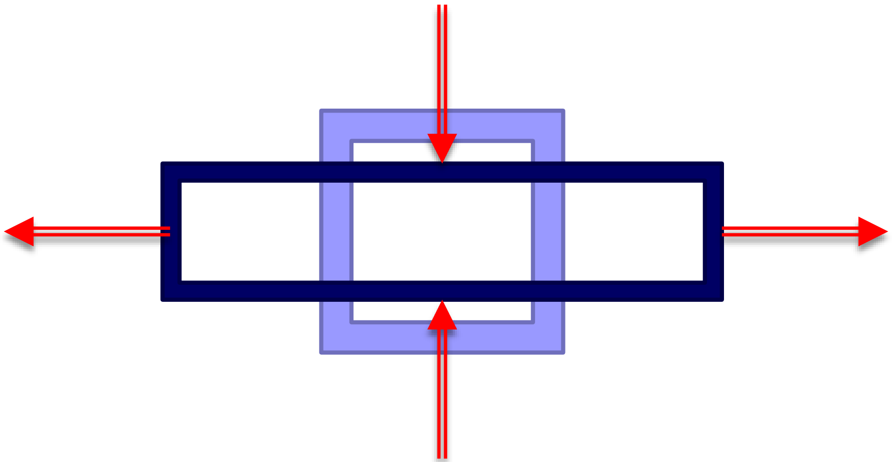
Generalised Hooke's law – shear component

Pure shear: $$\underline{\underline{\sigma}} = \begin{pmatrix} 0 & \sigma_{12} & 0 \\ \sigma_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{e_1,e_2}$$

Verify that: $$\varepsilon_{12} = \frac{1+\nu}{E}\sigma_{12}$$

Poisson's ratio

- ν is the signed ratio of transverse strain to axial strain
- Material property
- $\nu = -\frac{\varepsilon_2}{\epsilon_1}$



The strain gauge rosette

The strain tensor at the surface of hard tissues can be measured using strain gauge rosettes

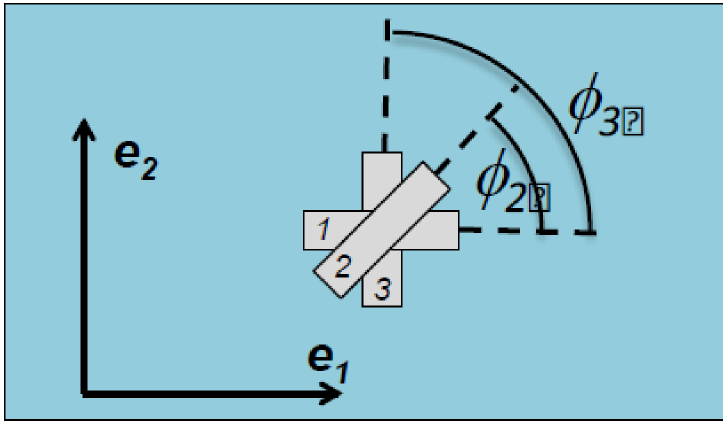


Figure 3

$$\varepsilon_n = \mathbf{n} \cdot \underline{\underline{\varepsilon}}\mathbf{n}$$

$$\mathbf{n} = \begin{bmatrix} \cos(\theta_n) \\ \sin(\theta_n) \end{bmatrix}$$

$\varepsilon_{n1} = 0.004$ $\varepsilon_{n2} = 0.002$ $\varepsilon_{n3} = -0.001$

Strain gauge rosette

Thanks for your attention!

18

Thanks for your attention!