Introduction to Biomechanics VU 317.043

Tutorial 3

30.11.2021

1 Leg – joint reaction forces and moments

Given figure 1 below calculate the joint reaction forces at the knee and the hip at the 50% stance phase of the gait cycle.

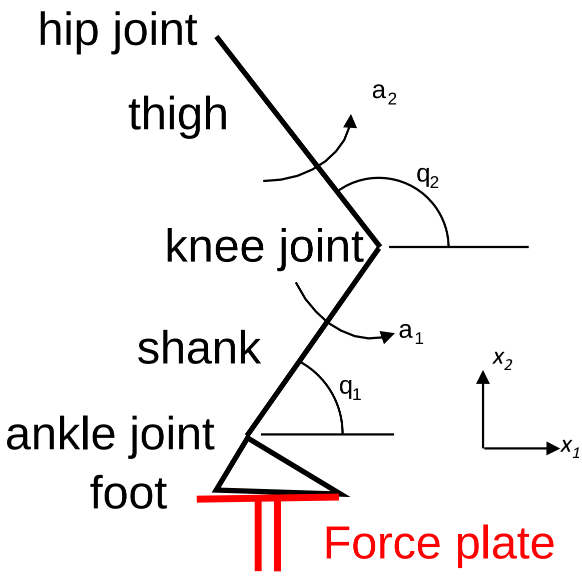


Figure 1

Use the following data for your calculations:

Anthropometric data

Limb segment	Segment length	Segment mass*	Centre of mass**	Radius of gyration r_g #
Thigh	32 cm	0.1 M	0.433	0.323
Shank	43 cm	0.0465 M	0.433	0.302

* Expressed as a fraction of the total body mass, M, where M = 70 kg
** Expressed as a fraction of the total segment length, measured from the proximal end of the segment
\# Expressed as a fraction of the segment length and for rotation around the center of mass

Thigh and shank kinematic data

% of stance phase of gait cycle	Shank angle Θ_1[°]	Thigh angle Θ_2[°]	Shank acceleration α_1 [rad·s^{-2}]	Thigh acceleration α_2 [rad·s^{-2}]
0	108.4	109.4	-32.87	0.13
10	100.3	108.4	-2.78	3.58
20	91.9	106.9	18.47	-28.36
30	86.5	101.2	7.87	-4.84
40	82.5	94.3	-0.22	7.81
50	78.7	88.7	-0.22	0.01
60	74.9	83.1	-2.99	-1.71
70	70.2	77.5	-13.88	11.95
80	63.2	73.8	-13.69	16.49
90	53.6	73.4	-8.16	27.13
100	42.9	78.1	25.25	30.36

Ankle reaction forces and muscle moments for the foot segment

% of stance phase of gait cycle	Reaction force, x_1 direction [N]	Reaction force, x_2 direction [N]	Muscle moment, [Nm]
0	-60.1	-101.7	-1.7
10	109.1	-572.1	6.9
20	135.1	-728.3	-0.5
30	58.3	-628.0	-6.6
40	27.7	-456.0	-18.7
50	21.6	-453.5	-30.3
60	-2.6	-534.2	-45.4
70	-49.9	-670.2	-71.2
80	-107.9	-744.2	-89.7
90	-130.5	-555.8	-68.6
100	-32.9	-125.7	-12.3

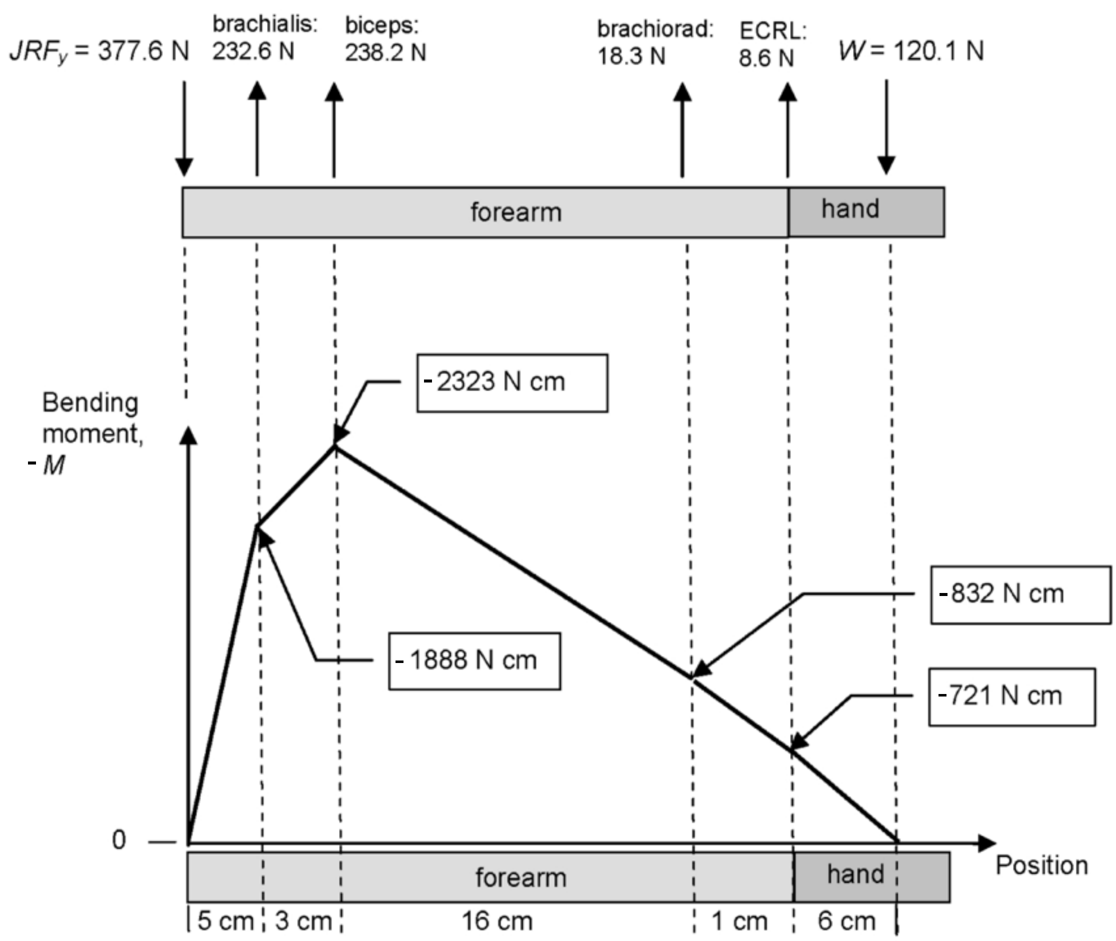
Shank and thigh acceleration data

% of stance phase of gait cycle	Shank acceleration x_1 direction [ms^{-2}]	Shank acceleration x_2 direction [ms^{-2}]	Thigh acceleration x_1 direction [ms^{-2}]	Thigh acceleration x_2 direction [ms^{-2}]
0	-5.87	2.26	-0.31	4.29
10	-4.01	2.75	-4.05	2.63
20	-6.62	-1.74	-4.85	-2.32
30	-3.89	0.40	-4.13	-1.01
40	0.82	-0.53	-0.65	-2.12
50	1.21	-0.95	1.15	-1.46
60	0.45	1.62	1.16	0.67
70	3.58	1.43	3.58	0.20
80	9.13	0.09	7.98	-1.12
90	14.31	0.82	9.09	-0.68
100	8.47	1.66	-2.10	3.27

Hint Reaction forces and muscle moments measured via a force plate or calculated for a certain segment need to be inverted to be valid for the next segment.

2 Elbow joint – Maximum internal stresses

For the given free body diagram of the forearm (below) with known joint reaction force, muscle forces and weight applied to the hand, determine:



i) The bending moment diagram along the forearm

ii) Assuming equal distribution of moments between the two bones of the forearm, i.e. the radius and the ulna, at each position, determine the maximum experienced stress.

Both the radius and ulna can assumed to be hollow circular shafts with inner diameter D_i = 0.7 cm and outer diameter D_o = 1.4 cm

iii) How do the maximum tensile and compressive stresses change if a compressive force of 50 N is applied at the hand (right-most position)?

iv) Assuming a Young's modulus of 17 GPa, what is the maximum strain encountered due to the experienced loading situations in ii) and iii)?

3 Strain and Stress Transformation

A rotation matrix $\mathbf{R}(\theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ produces a rotation in a counter-clockwise fashion (as shown in figure 4).

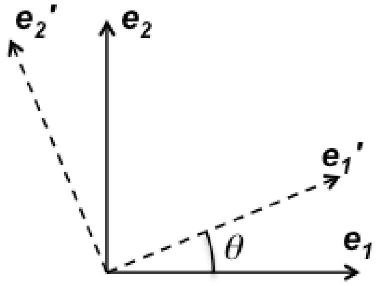


Figure 4

Using this rotation matrix within the transformation law for 2nd rank tensors:

$$\underline{\underline{\varepsilon}}' = \mathbf{R}^T \underline{\underline{\varepsilon}} \mathbf{R}$$

i) Calculate the strain tensor $\underline{\underline{\varepsilon}}'$ in the rotated reference frame.

ii) Further consider how a state of pure shear strain $\begin{pmatrix} 0 & \varepsilon_{12} \\ \varepsilon_{12} & 0 \end{pmatrix}$ transforms upon a rotation of Θ = 45°.

4 Generalised Hooke's law – shear component

Consider a pure shear test, i.e. a stress state: $\underline{\underline{\sigma}} = \begin{pmatrix} 0 & \sigma_{12} & 0 \\ \sigma_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{e_1,e_2}$

Upon a rotation of $q = 45\mathrm{deg}$ of the original configuration given above we get a stress state similar to the state in exercise 3 ii. Using Hooke's law in the rotated coordinate system (including the Poisson effect) calculate the corresponding strains and rotate these back to the original configuration.

Verify the shear components of the generalised Hooke's law for isotropic, homogeneous materials:

$$\varepsilon_{12} = \frac{1+\nu}{E}\sigma_{12}$$

5 Generalised Hooke's law – shear component

A strain gauge rosette measurement on a human femoral neck during a compression experiment (Figure 3) gives the following output:

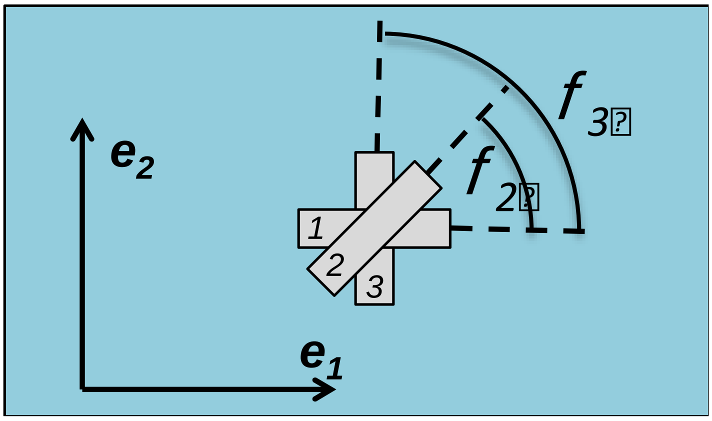


Figure 3

$\varepsilon_{n1} = 0.004$ $\varepsilon_{n2} = 0.002$ $\varepsilon_{n3} = -0.001$

Gauge 1 is aligned with the X1 direction whereas gauges 2 and 3 are at angles of Θ_2=45° and Θ_3 = 90°, respectively.

Using this information determine the 2D strain tensor at the measurement point.