TECHNISCHE UNIVERSITÄT WIEN
Vienna | Austria

ILSB
Biomechanics

INTRODUCTION TO BIOMECHANICS
317.043, VU

Tutorial 1: Statics & Dynamics

Katja Haslinger, Magdalena Fuchs

haslinger@ilsb.tuwien.ac.at
mfuchs@ilsb.tuwien.ac.at

Institute of Lightweight Design and Structural Biomechanics
TU Wien, Vienna, Austria

Office hours

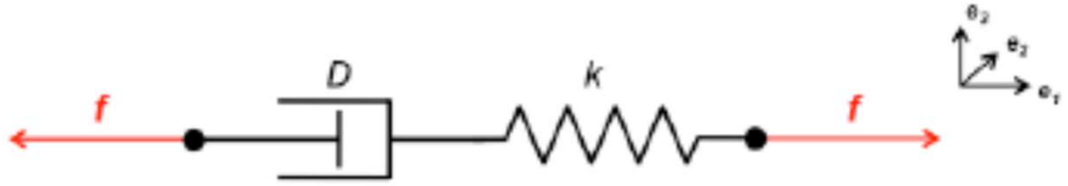
- If you have questions regarding calculation exercises, you can come to the office hour.
- Weekly
  - Office hours M. Fuchs & K. Haslinger:

    Tuesday 08:00 - 09:00
  - Tutorial-Zoom & office hours link: https://tuwien.zoom.us/j/95352431725?pwd=MW40ODhqdXphSVFSY1ZLL0FqQ2x0UT09
  - Password: Tut_21-22

Maxwell body

1 Mechanical elements - Maxwell body

Consider the following lumped parameter model of the Maxwell body:



Hint: The extension in the dashpot and spring is different, whereas their force is the same.

Assuming the extension to jump to instantaneously rise to a magnitude x_0 (relaxation experiment):

- derive an expression for the force $f_1(t)$ of the body
- sketch the progression of $f_1(t)$ qualitatively

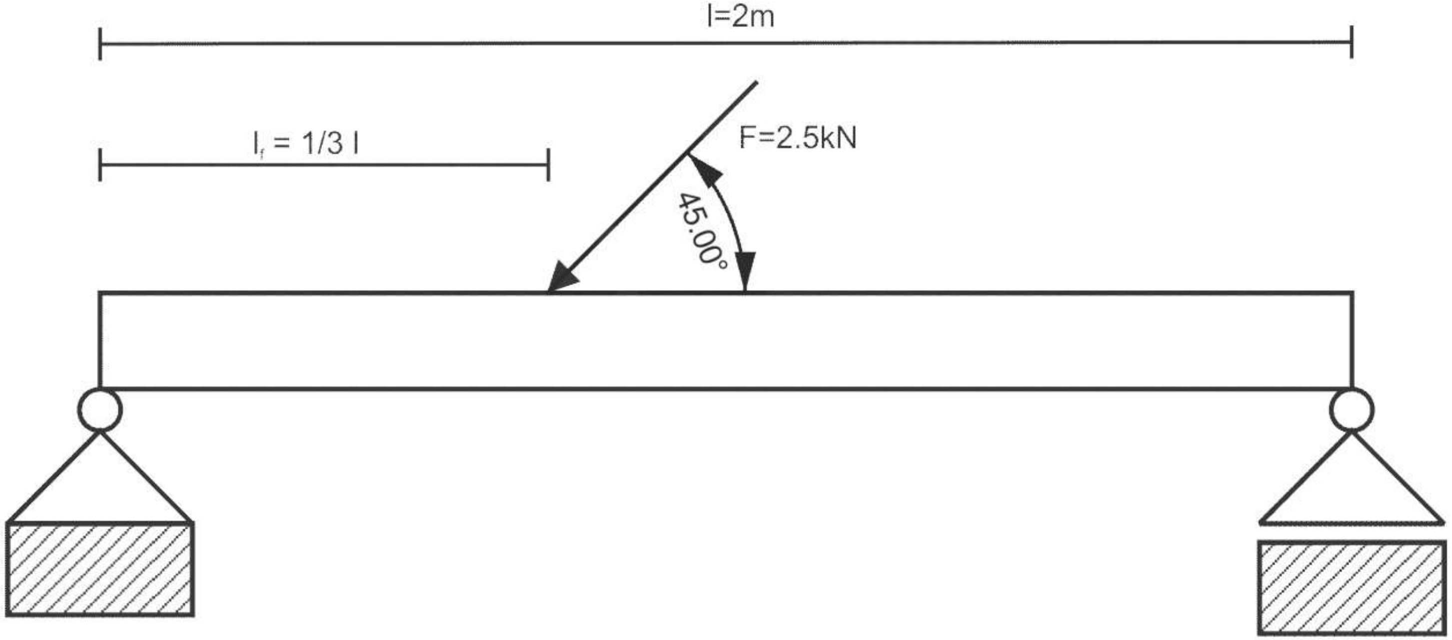
Now assume the force to jump to instantaneously rise to a magnitude f_0 (creep experiment):

- derive an expression for the extension $x_1(t)$ of the body
- sketch the progression of $x_1(t)$ qualitatively

Statics – Beams

2 Statics- Beams

Determine required reaction forces and moments of the weightless beam. Sketch the internal forces n*(x) (normal force) and v(x) (shear force)*, and the moment curve *M(x)*.



Statics – Beams: Definition of system

- **Idealisations:**
  - Geometry: point mass vs. rigid body
  - Solid materials: deformable vs. rigid
    - Forces acting on a rigid body -> dependent on line of action, independent of point of application

Statics – Beams: Description

- **Free-body diagrams:**

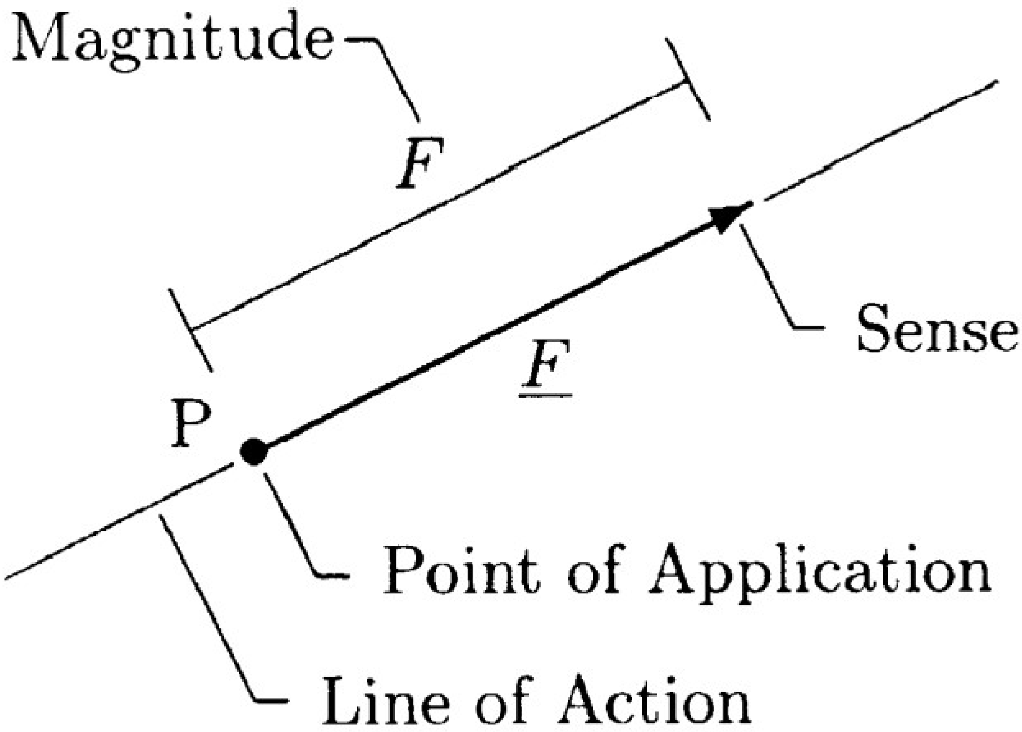
  surrounding parts of the structure are replaced by equivalent forces:

  - <u>external forces</u>
  - <u>inner forces</u>

- **Types of forces:**

  - <u>active forces</u>
    - Sense of the force vector is either given (e.g.: externally applied loads) or dependent on relevant physical law (e.g.: weight force)
  - reactive forces:
    - Sense of the force vector depends on equilibrium laws

Statics – Beams: force vector



Statics – Beams: support types

Pinned

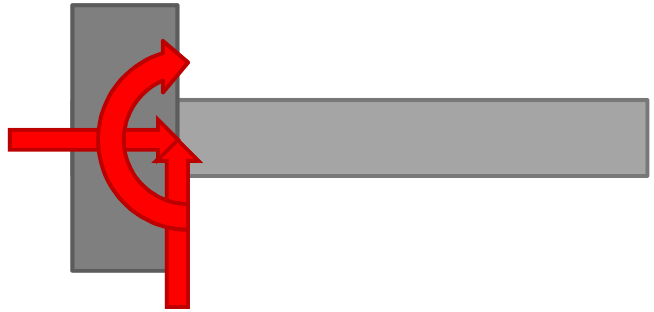
Fixed

Roller

Statics – Beams: reactive forces

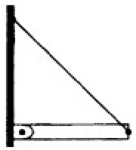
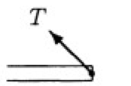
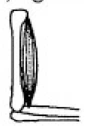
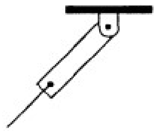
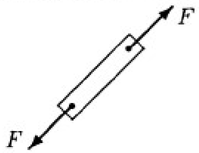
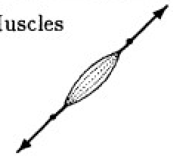
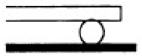
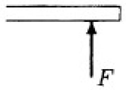
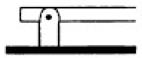
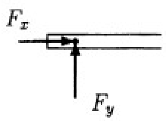
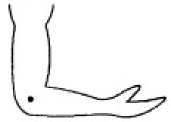
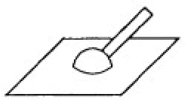
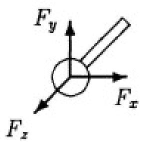
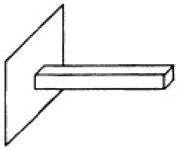
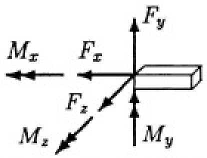
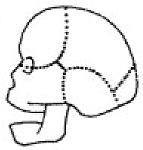
Pinned

Fixed



Roller

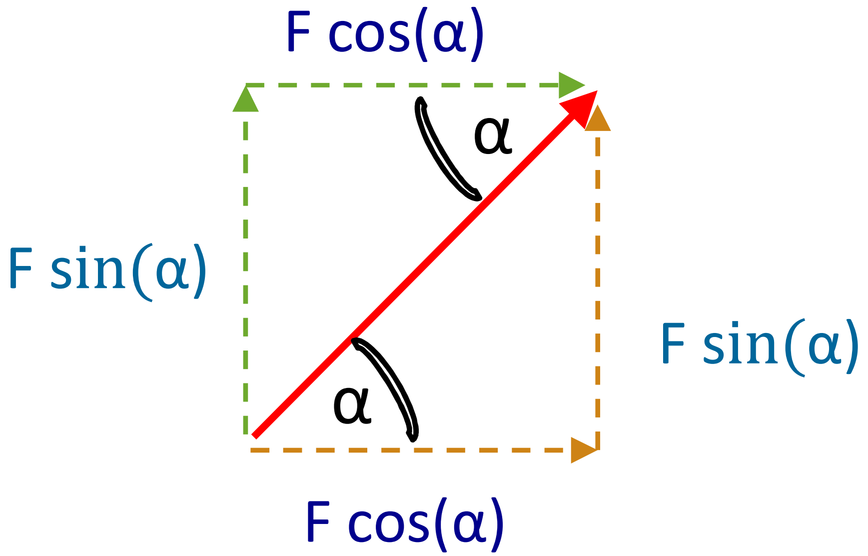
Statics – Body Supporting Structure:

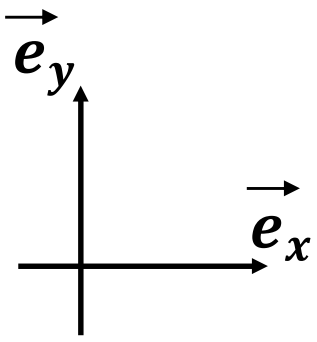
TYPE OF SUPPORT OR JOINT	SAMPLE GRAPHICAL REPRESENTATIONS	REPRESENTATIONS FOR USE IN FREE-BODY DIAG.	BIOMECHANICS EXAMPLES	UNKNOWNS
Flexible members (cables, ropes)			Muscles, ligaments 	Magnitude of the tension in the cable or muscle
Two-force members			Muscles 	Force magnitude
Rollers and other simple supports (no friction)			Bone-to-bone contact 	Force magnitude
Hinge or pin connection			Elbow 	Magnitude and direction of force (force components)
Ball-and-socket			Hip 	Magnitude and direction of force
Fixed, welded, or built-in			Skull 	Magnitude and direction of force and moment

Nihat Özkaya: Fundamentals of biomechanics : equilibrium, motion, and deformation New York, NY, Springer 2012

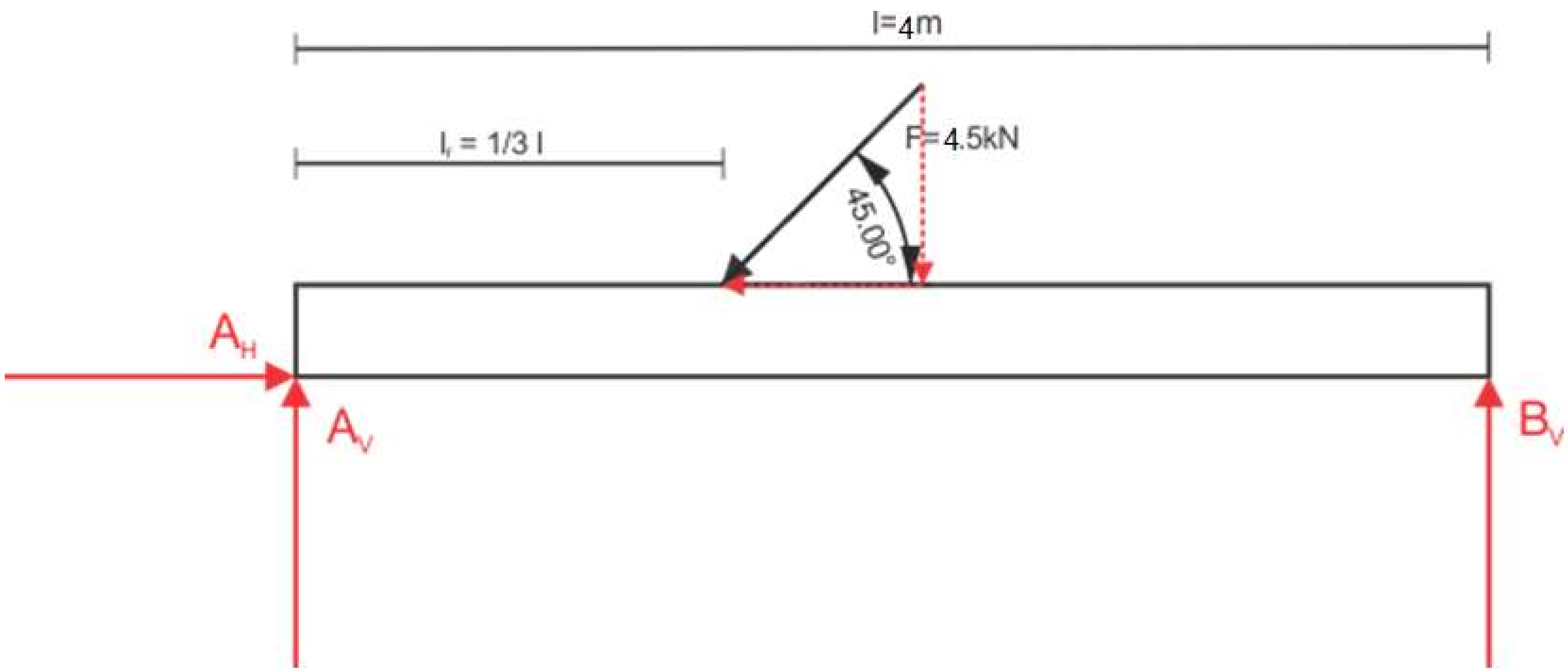
Force Components:

Decompose force vectors into directions of main axes





Statics – Beams: free body diagram



Statics – Beams: Equations of Motion

- Newton's first law states that a body that is originally at rest will remain at rest, or a body in motion will move in a straight line with constant velocity, if the net force acting upon the body is zero.

$$\boxed{\sum_{i=1}^{k} \mathbf{f}_i^{(e)} = 0}$$

$$\boxed{\sum_i \mathbf{M}_i = \sum_i (\mathbf{r}_i \times \mathbf{f}_i) = 0}$$

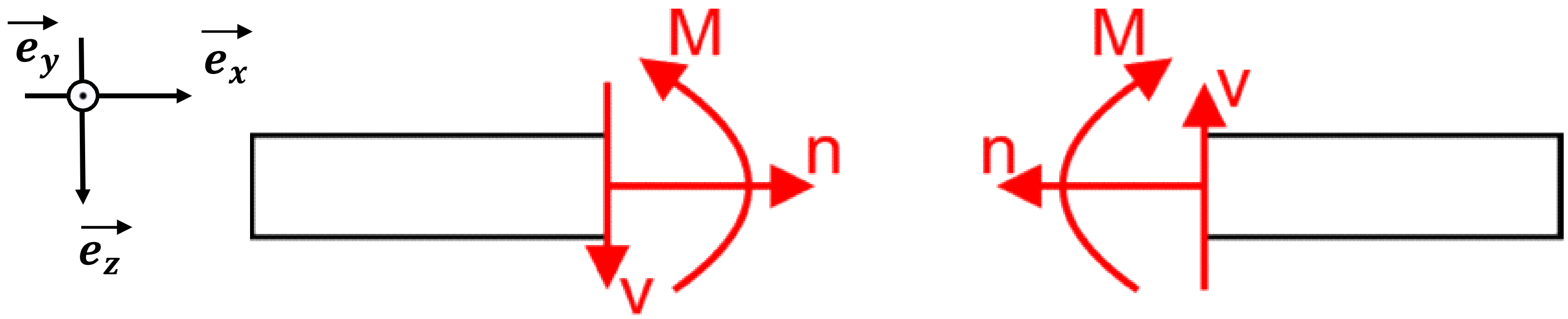
- Newton's second law states that a body with a net force acting on it will accelerate in the direction of that force, and that the magnitude of the acceleration will be directly proportional to the magnitude of the net force and inversely proportional to the mass of the body.

$$\boxed{\sum_{i=1}^{k} m_i \mathbf{a}_i = \sum_{i=1}^{k} m_i \ddot{\mathbf{r}}_i = m \ddot{\mathbf{c}}_m = \sum_{i=1}^{k} \mathbf{f}_i^{(e)}}$$

- Newton's third law states that to every action there is always an equal reaction, and that the forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and have the same line of action.

$$\boxed{\mathbf{f}_{ij} = -\mathbf{f}_{ji}}$$

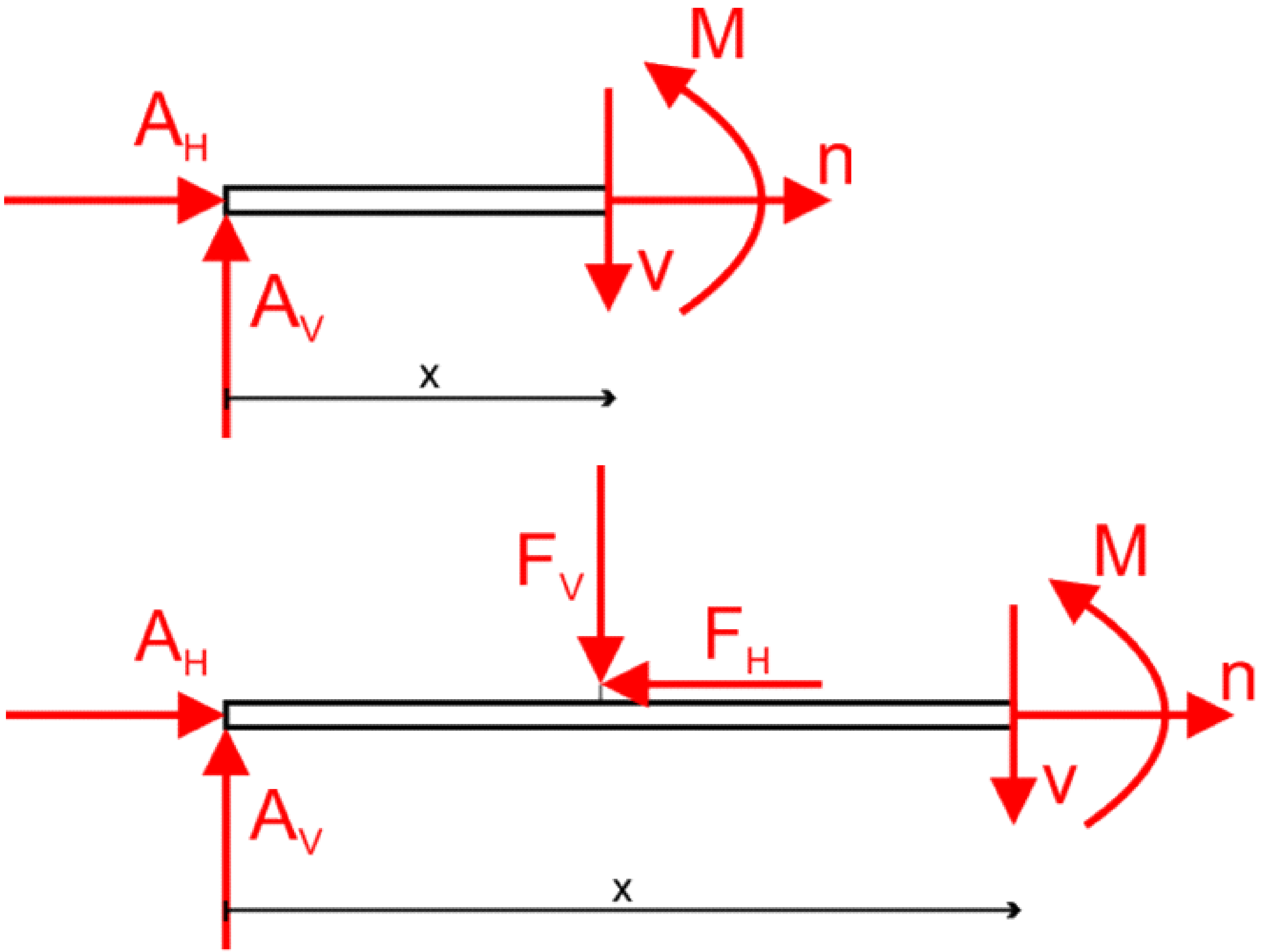
Statics – Beams: internal forces



Recall internal forces: shear force *v*
normal force *n*
internal moment *M*

Their magnitudes depend on the position where the beam is cut

Statics – Beams: internal forces (2)



Summary: Solution of a biomechanical problem

Description

1. Definition of system (dimensions, symmetry, constraints)
2. Choose reference frame
3. Draw forces and moments
4. Define state variables

Resolution

1. Find applicable laws
2. Express conservation laws
3. Write equations of motion
4. Study the solution

Interpretation

1. Discussion of results

Dynamics - jump

3.1 Jump

A person with a mass of 80kg is performing a jump from rest in a crouched position. The duration of the take of phase is τ = 180 ms and the vertical ground reaction force f_3 [N] can be described by:

$$f_3(t) = 2400\sin\left(\frac{\pi t}{\tau}\right) + 800\left(1 - \frac{t}{\tau}\right)$$

Calculate the peak height *h*, that the center of mass (COM) raises above its initial position.

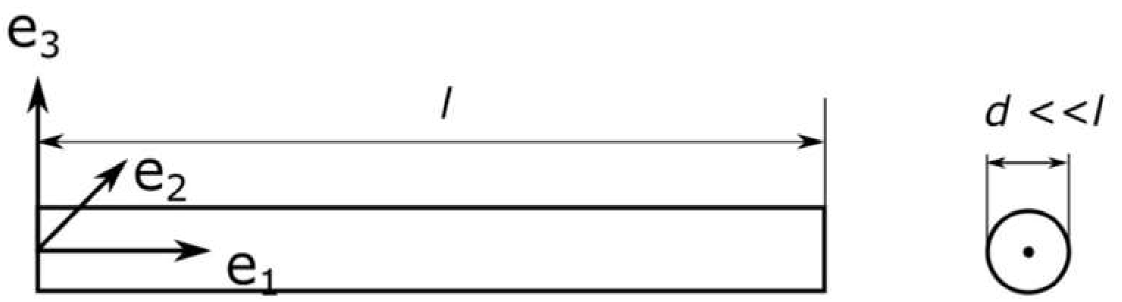
Dynamics – moment of inertia

3.2 Inertia

A slender, circular rod with cross-section $A = \frac{d^2\pi}{4}$ length *l* and density ρ is located with one end at the origin and oriented parallel to axis e_1. For a rotation around axis e_1, with the rotation centre located at the centre of mass calculate the moment of inertia.

Calculate the moment of inertia around axis e_3 with the rotation centre located at the centre of mass and at the origin. Further, calculate the radius of gyration, with the assumption that *d* is much smaller than *l*.

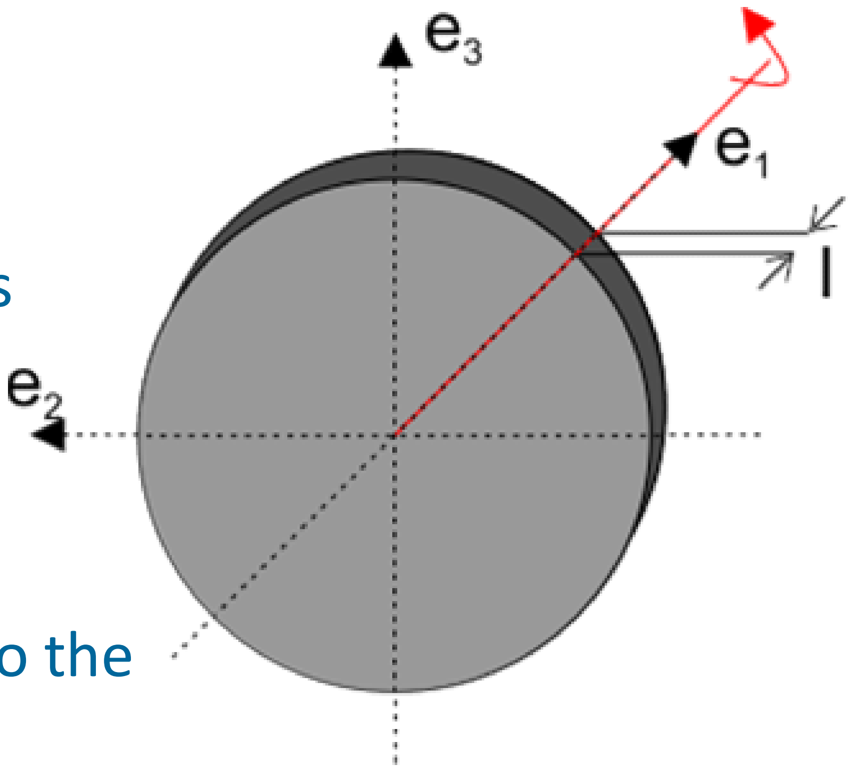
Hint: Use the parallel axis theorem to calculate the moment of inertia with respect to a new axis. $I = I_{cm} + md^2$, where I_{cm} is the moment of inertia at the centre of mass, *m* is the mass and *d* is the perpendicular distance to the new axis.



Dynamics – moment of inertia (2)

The moment of inertia depends on the axis of rotation and is the rotational analog of mass for linear motion. For a point mass it is described by

$$I_{axis} = m\, r^2$$



For a body, the moment of inertia is the sum of all point masses

$$I_{axis} = \int r^2\ dm$$

r is the distance of the point mass to the axis of rotation

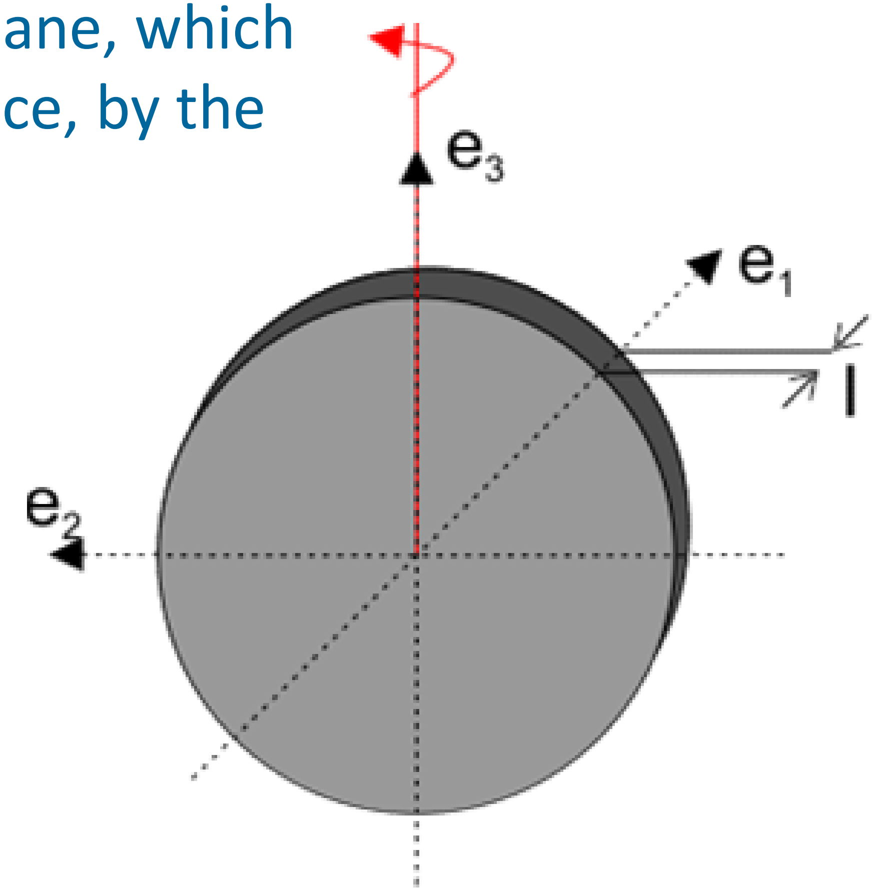
Perpendicular axis theorem

Axes 2 and 3 lie in the same plane, which is perpendicular to axis 1. Hence, by the perpendicular axis theorem:

$$dI_1 = dI_2 + dI_3$$

By symmetry we know:

$$dI_2 = dI_3$$



Parallel axis theorem

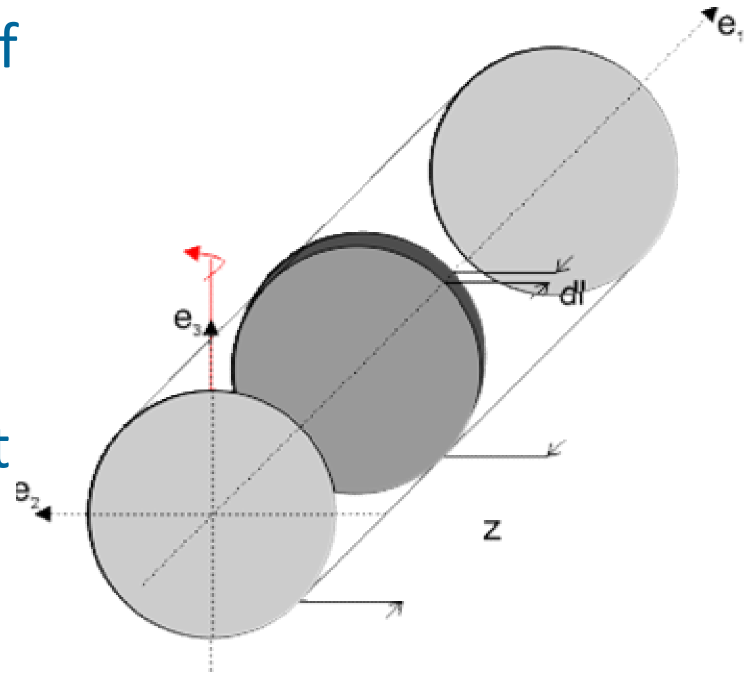
The moment of inertia about any axis parallel to that axis through the center of mass is given by

$$I = I_{COM} + m\,z^2$$

z is the distance between these axes

The moment of inertia of any thin disk at distance *z* can be expressed by

$$dI_3 = \frac{1}{4}R^2 dm + z^2 dm$$



22

Thanks for your attention!