Statistik und Wahrscheinlichkeitstheorie - Exam group D

Mo, 30.01.2023

1. In a linear regression model ('y_i modeled as a linear function of x_i plus error') the parameters are estimated via least squares. For the mean and the emprirical standard deviation of the x and y values we obtain $\bar{x} = 3$, $s_x = 4$, $\bar{y} = 7$ and $s_y = 3$. It holds that

   (a) the slope of the regression line is smaller than $-3/4$

   (b) the regression line goes through $(3, 7)$

   (c) the regression line goes through $(7, 3)$

   (d) the slope of the regression line is larger than $3/4$

2. Which one of the following is an **incorrect** statement?

   (a) The larger the value of the sample size n, the closer the standard deviation of the sampling distribution of $\bar{x}$ is to the standard deviation of the population.

   (b) The sampling distribution of $\bar{x}$ has mean equal to the population mean μ even if the population is not normally distributed.

   (c) When n is large, the sampling distribution of $\bar{x}$ is approximately normal even if the population is not normally distributed.

   (d) The sampling distribution of $\bar{x}$ has standard deviation $\sigma/\sqrt{n}$ even if the population is not normally distributed.

3. Two features of a novel operating system are compared using a two-sample t-test. The statistics for the first feature are $\bar{x} = 21$, $s_x = 10$ and $n_x = 4$ and those for the second features are $\bar{y} = 29$, $s_y^2 = 55$ and $n_y = 5$. The rejection region is given through $R = (-\infty, -q] \cup [q, \infty)$. Then it holds

   (a) we reject for $q = 0.4$ but not for $q = 1.2$

   (b) we do not reject for $q = 0.4$ but for $q = 1.2$

   (c) we reject for both $q = 0.4$ and $q = 1.2$

   (d) we do neither reject for $q = 0.4$ nor for $q = 1.2$

4. Let X be a random variable with a Poisson distribution. If it holds

$$P(X = 1) = P(X = 3),$$

   then the expectation $\mathbb{E}X$ equals

   (a) 6

   (b) 3

   (c) $\sqrt{6}$

   (d) $\sqrt{3}$

5. In general, how does halving the sample size change the confidence interval size?

   (a) Doubles the interval size

   (b) Halves the interval size

   (c) Divides the interval size by $\sqrt{2}$

   (d) Multiplies the interval size by $\sqrt{2}$

6. A fast food chain advertises that their large bag of french fries has a weight of 150 grams. Some high school students, who enjoy french fries at every lunch, suspect that they are getting less than the advertised amount. With a scale borrowed from their physics teacher, they weigh a random sample of 16 bags. Assuming the level of significance $\alpha = 10\%$, what would be the conclusion if the sample mean is 144 g and standard deviation is 15 g? Assume that all conditions for inference are met.

   (a) There is sufficient evidence to prove the fast food chain advertisement is true.

   (b) The students do not have sufficient evidence to reject the fast food chain's claim.

   (c) The students have sufficient evidence to reject the fast food chain's claim.

   (d) There is sufficient evidence to prove the fast food chain advertisement is false.

7. We toss two fair coins simultaneously and independently. If the outcomes of the two coin tosses are the same, we win, otherwise, we lose. Let A be the event that the first coin comes up heads, B be the event that the second coin comes up heads and C be the event that we win. Which one of the following statements is **true**?

   (a) The probability of winning is 3/4.

   (b) Events A and C are independent.

   (c) Events A and B are not independent.

   (d) Events B and C are not independent.

8. Which of the following statements about t-istribution are **true**?

   I Like the normal, t-distributions are always symmetric.

   II The smaller the number of degrees of freedom, the closer the curve is to the normal curve.

   III Twenty degrees of freedom gives the normal curve.

   (a) I and II

   (b) III only

   (c) I only

   (d) I and III

9. The income per household in a certain country is assumed to be normally distributed with the mean 9500 Euro and standard deviation of 1750 Euro. The middle 95% of incomes (in Euro) are between what two values?

   (a) 5422 and 13578

   (b) 6621 and 12379

   (c) 6070 and 12930

   (d) 8049 and 10951

10. Let $X \sim Poi(2)$ and $Y \sim Bin(8, 0.5)$ be two random variables with the correlation $\mathbb{C}orr(X, Y) = 0.4$. Compute $\mathbb{C}ov(X, Y + 3)$.

    (a) 1.6

    (b) 0.8

    (c) -2.2

    (d) -1.4

11. Let $X_1, \ldots, X_{64}$ be a random sample from a distribution with the expectation -1.2 and variance 4. Let

$$\bar{X} = \frac{1}{64} \sum_{i=1}^{64} X_1$$

    be the sample mean. Determine the approximate value of $P(\bar{X} > -0.9)$ using the Central limit theorem and express it in terms of a suitable R-function.

    (a) `pnorm(-0.9, -1.2, 0.25)`

    (b) `pnorm(1.2)`

    (c) `pnorm(-1.2)`

    (d) `pnorm(-0.9, 1.2, 0.5)`

12. Suppose the null hypothesis $H_0 : p = 0.4$, and the power of the test for the alternative hypothesis $H_A : p = 0.35$ is 0.75. Which of the following is a valid conclusion?

    (a) If the null hypothesis is false, the probability of failing to reject it is 0.6

    (b) The probability of committing a Type I error is 0.05.

    (c) The probability of committing a Type II error is 0.65.

    (d) If the alternative hypothesis is true, the probability of failing to reject the null is hypothesis 0.25.

13. Two classes take the same exam. Suppose a certain score is at the 40th percentile for the first class and at the 80th percentile for the second class. Which of the following is the most reasonable conclusion?

    (a) One of the classes has twice the number of students as the other.

    (b) Students in the second class generally scored higher than students in the first class.

    (c) Students in the first class generally scored higher than students in the second class.

    (d) A score at the 50th percentile for the first class is at the 90th percentile for the second class.

14. Out of the students in a class, 60% are playing chess, 70% love ice skating, and 40% fall into both categories. Compute the probability that a randomly selected student is neither a chess player nor an ice skating lover.

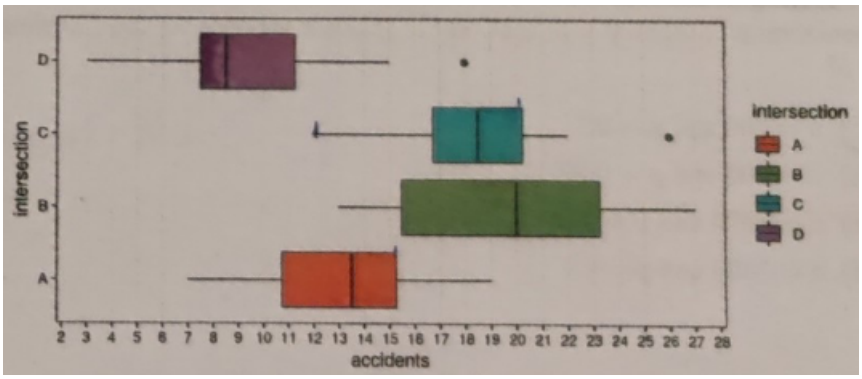
    (a) 0.4

    (b) 0.1

    (c) 0.6

    (d) 0.9

15. Data on the number of yearly accidents were collected from four intersections (A-D) over a 20 year period and are presented below. Which one of the following statements is **false**?



    (a) During at least 5 years, fewer than 10 accidents occurred at section A.

    (b) During at least 75% of years, intersection C had more accidents than the lowest 75% of years at intersection A.

    (c) The minimum accident total at intersection C was higher than the number of accidents observed at intersection D in 75% of years.

    (d) The minimum number accidents that occurred in a single intersection was 3.

16. For a project, a high school student randomly picks 100 fellow Statistics students to survey on whether each has either a PC or Apple at home (all students in the school have a home computer) and what score (1, 2, 3, 4, 5) each expects to receive on the Statistics exam. A chi square test of independence results in a test statistic of 8. How many degrees of freedom are there?

    (a) 4

    (b) 9

    (c) 7

    (d) 1

17. For a statistical test of significance level α it holds

    (a) the rejection area does not depend α

    (b) rejection at level α implies rejection at level $\alpha/2$

    (c) the rejection area shrinks when α is increased

    (d) the rejection area depends on the distribution of the test statistic under the null hypothesis

18. A study is to be performed to estimate the proportion of voters who believe the economy is "heading in the right direction." Which of the following pairs of sample size n and population proportion p will result in the smallest variance for the sampling distribution of $\hat{p}$?

    (a) $n = 100$ and $p = 0.1$

    (b) $n = 100$ and $p = 0.99$

    (c) $n = 1000$ and $p = 0.5$

    (d) $n = 1000$ and $p = 0.1$

19. Consider the two sets $X = \{10, 30, 45, 50, 55, 70, 90\}$ and $Y = \{10, 30, 35, 50, 65, 70, 90\}$. Which one of the following answers is **false**?

    (a) The sets have identical ranges.

    (b) The sets have identical medians.

    (c) None of the rest are false.

    (d) The sets have identical means.

20. Let $X \sim \mathcal{N}(-1, 4)$. Express the probability

$$P(X^2 + 2X \leq 0)$$

in terms of the cumulative distribution function Φ of the standard normal random variable

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\, dt, \quad x \in \mathbb{R}.$$

(a) $\Phi(2) - 0.5$

(b) $2 \cdot \Phi(0.25) - 1$

(c) $2 \cdot \Phi(0.5) - 1$

(d) $2 - 2 \cdot \Phi(0.5)$