

Discrete Math Exam

February 28, 2025

1 Graphs

- a) Prove that for a connected planar graph, the average degree of vertices must be less than 10.
- b) Prove for a matroid (E, S) , if A and B are bases of the matroid, then $|A| = |B|$.

2 Generating Functions

Determine explicitly the coefficient a_n of the power series

$$f(x) = \frac{1}{(1-x)(1+3x)} + \frac{3}{\sqrt{1+2x}}$$

3 Abstract Algebra

For $\mathbb{Z}_3$, show which is irreducible

$$f(x) = x^3 + x - 1$$

$$g(x) = x^4 - x + 1$$

4 Pigeonhole Principle

Given a set A , which is a subset of $\{1, \dots, 41\}$, and $|A| = 21$, prove that there must exist $x, y \in A$ such that $x+y = 42$.