Discrete Math Exam

March 1, 2025

1 Graphs

a) Prove that for a connected planar graph with at least 10 vertices, the average degree of vertices must be less than 10.
b) Prove for a matroid (E, S), if A and B are bases of the matroid, then $|A| = |B|$.

2 Generating Functions

Determine explicitly the coefficient a_n of the power series
$f(x) = \frac{1}{(1-x)(1+3x)} + \frac{3}{\sqrt{1+2x}}$

3 Abstract Algebra

For $\mathbb{Z}_3$, show which is irreducibile
$f(x) = x^3 + x - 1$
$g(x) = x^4 - x + 1$

4 Pigeonhole Principle

Given a set A, which is a subset of $\{1, ..., 41\}$, and $|A| = 21$, prove that there must exist $x, y \in A$ such that x+y= 42.