Examination for "Logic and Computability"

July 9, 2014 — 4th Exam for WS13/14

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Task 1:
Prove or provide a counter example for the following statment:

- $Q(x) \to \forall x P(x)$ is logically equivalent to $\forall x(Q(x) \to P(x))$

Task 2:
Provide a formal proof of the following statement in the classical sequent calculus LK:

- $\{\forall x(A(x) \to B(x)), \forall x A(x)\} \models \forall x B(x)$

Is your proof also a valid LJ-proof?

Task 3:
If I is a recursive set and J is recursively enumerable, what can be said about
$I \backslash J = \{x \mid x \in I \text{ and } x \notin J\}$ in the following cases:
(a) $J = \emptyset$,
(b) J is recursive,
(c) I is finite,
(d) J is infinite and not recursive.

Task 4:
Compute all Robinson-Resolvents of the two clauses $\neg p(x, g(y))$ and $p(y, z) \vee p(g(x), y)$.

Task 5:
Prove or refute: $(A \wedge \neg\Box B) \vee (\Diamond\neg B \to \neg A)$ is valid in every Kripke frame.

Task 6:
Suppose that the system Σ over some standard artithmetic language is sound and that for all Turingmachines M the statement 'M terminates on every input' can be expressed in Σ. Can one conclude that Σ is incomplete? Why and how?