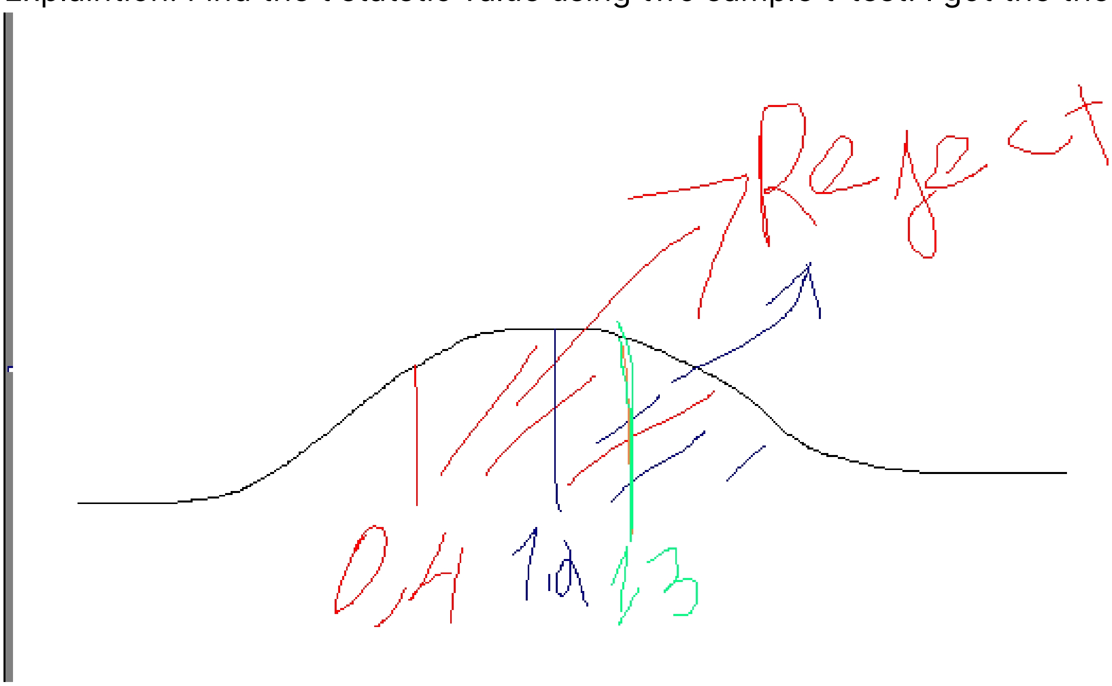
Exam 30.01.2023 (Solutions)

My attempt at solving the exam. The eaxm can be found here
No idea if everything is right. Or explaination is right.

1. (b) the regression line goes through (3, 7).
   Explaination: According to the slides, the regression line goes through $(\bar{x}, \bar{y})$.
   Furthermore, to find the slope of the line $\frac{s_y}{s_x}$

2. (a) The larger the value of the sample size n, the closer the standard deviation of the sampling distribution of $\bar{x}$ is to the standard deviation of the population.

3. (d) we do neither reject for q = 0.4 nor for q = 1.2.
   Explaintion: Find the t statstic value using two sample t-test. I got the the $1.3\dot{3}$.



   As you can see that the area to the left of 0.4 and 1.2 must be rejected. And the value $1.3\dot{3}$ is to the left of both them.

4. (c) $\sqrt{6}$
   Explaination: Remember

$$f(x) = \frac{\lambda^x}{x!} \times e^{-\lambda}$$

We know $f(1) = f(3)$, then we can

$$\frac{\lambda^1}{1!} \times e^{-\lambda} = \frac{\lambda^3}{3!} \times e^{-\lambda}$$

$$\lambda = \frac{\lambda^3}{6}$$

$$\lambda = \sqrt{6}$$

5. (d) Multiplies the interval size by $\sqrt{2}$.
   Explaination:

$$sem = \frac{s}{\sqrt{n}} = \frac{s}{\sqrt{\frac{n}{2}}} = \frac{\sqrt{2} \times s}{\sqrt{n}}$$

6. (b) The students do not have sufficient evidence to reject the fast food chain's claim..
   Population standard deviation is unknown $\rightarrow$ t-test.

$$H_0 : \mu = 150$$

$$H_1 : \mu \neq 150$$

The alpha is $0.1 \rightarrow 0.05$, because its two sided test and degree of freedom $df = n - 1 = 16 - 1 = 15$. Read the value from the t-table: -1.753 and 1.753.
t-test stastic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$t = \frac{144 - 150}{\frac{15}{\sqrt{16}}}$$

$$t = -1.6$$

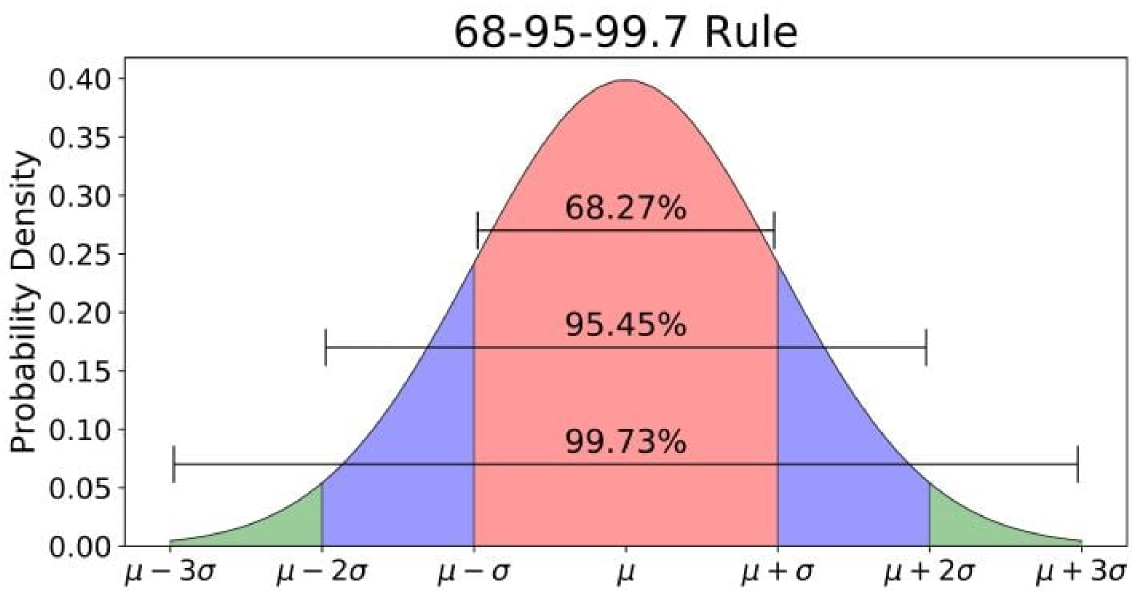
-1.6 is between -1.753 and 1.753.

7. (b) Events A and C are independent.
   Explaination: The probablity of winning is $\frac{1}{2}$. Therefore, a is wrong.
   We know you win the game, if the both coins land on same side. Knowing that first coin was heads (event A) doesn't determine that we win (event C), because we can also win when first coin is tails. Bascially, winning doesnt tell whether or not you flipped heads on first coin.

   Also the outcome of A doesn't influence B. Therefore, they both are independant events.

8. (c) I.
Explaination:
I: t-distributions are normal distribuations but with different peak and tails. Look up the properties for t-distribution. Like this question, they probably can ask something about it.
II: Second option is wrong because higher the degree of freedom, more closely it resembles the normal distribution.
III: It is close to normal. But not excatly normal. (Lel good luck thinking of this fact during the exam. Here's the explaination from crackap).

9. (c) 6070 and 12930
Explaination:



As you can see that about 95.45% of data is between two standard deviation. Not excatly 95%, but close enough. So to found lower bound: $9500 - 2 \times 1750 = 6000$ and upper bound: $9500 + 2 \times 1750 = 13000$.

10. (b) 0.8
Explaination: $Corr(x, y) = 0.4$ is given. Poision distribution mean is equals to its variance.
Furthermore, recall:

$$Corr(x, y) = \frac{Cov(x, y)}{\sqrt{var(x) \times var(y)}}$$

Use propertiy 5 to solve this problem.

And poision distribution mean is equals to its variance. Variance of binomial distribution $var = 8 * 0.5 * 0.5 = 2$
Plug in the values:

$$0.4 = \frac{Cov(x,y)}{\sqrt{2 \times 2}}$$

$$Cov(x,y) = 0.8$$

Some useful properites:

Lemma 5.3
The covariance has the following properties:

1. $\mathrm{Cov}(X, X) = \mathrm{Var}(X)$;
2. if X and Y are independent then $\mathrm{Cov}(X, Y) = 0$;
3. $\mathrm{Cov}(X, Y) = \mathrm{Cov}(Y, X)$;
4. $\mathrm{Cov}(aX, Y) = a\mathrm{Cov}(X, Y)$;
5. $\mathrm{Cov}(X + c, Y) = \mathrm{Cov}(X, Y)$;
6. $\mathrm{Cov}(X + Y, Z) = \mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z)$;
7. more generally,

$$\mathrm{Cov}\left(\sum_{i=1}^{m} a_i X_i, \sum_{j=1}^{n} b_j Y_j\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_i b_j \mathrm{Cov}(X_i, Y_j).$$

Note: Cov and corr of two independant variables is 0. But $cov(x,y)$ and $cor(x,y)$ for non independent can be 0.

11. Not sure. Probably, it is c. There is a similiar example on slides.

12. (d) If the alternative hypothesis is true, the probability of failing to reject the null is hypothesis 0.25.
Explaination: Recall: α is the probablity of making type 1 error, β is the probablity of making type 2 error. And the power of a test is: $P = 1 - \beta$. The power is given $P = 0.75$.
Therefore, $0.75 = 1 - \beta \rightarrow \beta = 0.25$.

13. (c) Students in the first class generally scored higher than students in the second class.
Explaination: In the first class, 40% of students score below the score x. Meanwhile, 80% of students of second class score below the score x.

14. (b) 0.1
    Explaination: $P(A \cup B) = 0.6 + 0.7 - 0.4 = 0.9$ gives us the probability of picking a student who either likes ice skating, chess or both. $\neg p = 1 - 0.9 = 0.1$ is the probabilty of liking nothing.

15. (a) During at least 5 years, fewer than 10 accidents occurred at section A. (Not sure)
    Explaination: $\frac{5}{20} = 0.25$. 25th quarantile doesn't have less than 10 accidents.

16. (a) 4
    Explaination: Recall, df of χ^2 test is $df = (rows - 1) * (columns - 1)$.
    $df = (4 - 1) * (2 - 1) = 4$

17. (d) the rejection area depends on the distribution of the test statistic under the null hypothesis.
    Explaination: (a) and (b) are completly wrong. (c) would have been true if the statement were to be: "The rejection area shrinks when α is **decreased**".
    Leaving us the option (d). Just see how different tests have different way of finding the rejection area.

18. (d) $n = 1000$ and $p = 0.1$
    Explaination: Recall, $var = \frac{p*(1-p)}{n}$. Larger the denominator, smaller the value.
    $(0.1) * (0.9) < (0.5)(0.5)$.

19. (c) None of the rest are false
    Explaination: Just do the calculations to find the values.

20. (b) IDK just a random guess.