This is the second homework assignment. Students should tick in TUWEL problems they have solved and upload their detailed solutions by **20:00** on Wednesday **October 20, 2021**.

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(1) **Bonferroni's inequality**
Prove that for any two events A and B Bonferroni's inequality holds

$$P(A \cap B) \geq P(A) + P(B) - 1.$$

(2) **Independence**
Let A and B be two independent events.

(a) Prove that A^c and B^c are also independent.

(b) If we additionally know that $P(A|B) = 0.6$ and $P(B|A) = 0.3$, compute the probabilities of the following events

(i) at most one of A or B

(ii) either A or B but not both.

(3) **Urn problem**
There are 8 red and 2 black balls in an urn. John takes n balls at once. What is the minimal number of balls that he should take so the probability that there is at least one black ball among those that he took, is greater than $\frac{2}{3}$?

(4) **Computer reliability**
A campus bookstore sells two types of computers: laptops and desctops. In the last semester it sold 56% laptops and 44% desktops. Reliability rates for the two types of machines are quite different. In the first year, 5% of desktops require service, while 15% of laptops have problems requiring service.

(a) Sketch a probability tree for this situation.

(b) What percentage of computers sold by the bookstore last semester required service?

(c) Given that a computer required service, what is the probability that it was a laptop?

(5) **Boxes**
Two boxes are given. There are 15 white and 12 black balls in the first box, and 14 white and 18 black balls in the second box. Anna provides the following experiment. Anna takes at once two balls from the first box and places them in the second box. Then, she takes one ball wihout looking from the second box.

(a) What is the probability that she took a white ball from the second box?

(b) Knowing that she took a black ball from the second box, what is the probability that she transferred two balls of different colors from the first box to the second box?

(6) **Widget factory**

The local widget factory is having a blowout widget sale. Everything must go, old and new. The factory has 450 old widgets, and 1550 new widgets in stock. The problem is that 12% of the old widgets are defective, and 4% of the new ones are defective as well. It is assumed that widgets are selected at random when an order comes in. You are the first customer since the sale was announced.

(a) You flip a fair coin once to decide whether to buy old or new widgets. You order two widgets of the same type, chosen based on the outcome of the coin toss. What is the probability that both of them will be defective?

(b) Given that both widgets turn out to be defective, what is the probability that they were old widgets?