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Statistik und Wahrscheinlichkeitstheorie VO (Levajkovic)

In the situation of a right-sided one-sample t-test we find $\bar{x} = -12, s = 21$ and $n = 49$. For a given significance level we find the rejection region $R = [2.2, \infty)$. Then for the null hypothesis $H_0 : \mu = -3$ it holds

- a. we do not reject H_0, but we would reject if only the significance level was chosen small enough
- b. we reject H_0, and we would also reject for any smaller significance level
- c. we do not reject H_0, and we would also not reject for any smaller choice of the significance level
- d. we reject H_0, and we would also reject for any larger significance level

For a statistical test of significance level α it holds

- a. the rejection area does not depend on the distribution of the test statistic under the null hypothesis
- b. rejection at level α implies rejection at level $\alpha/2$
- c. the rejection area shrinks when α is increased
- d. the rejection area depends on $1 - \alpha$

For the P-value of a statistical test of significance level α it always holds true

- a. $P \leq \alpha$, if the null hypothesis was not rejected
- b. $P \leq \alpha/2$, if the null hypothesis was rejected
- c. $P > 2\alpha$, if the null hypothesis was rejected
- d. $P \geq 0$, if the null hypothesis was rejected

In the situation of a right-sided one-sample t-test we find $\bar{x} = -12, s = 21$ and $n = 49$. For a given significance level we find the rejection region $R = [2.2, \infty)$. Then for the null hypothesis $H_0 : \mu = -3$ it holds

- a. we reject H_0, and we would also reject for any smaller significance level
- b. we do not reject H_0, but we would reject if only the significance level was chosen small enough
- c. (selected) we do not reject H_0, and we would also not reject for any smaller choice of the significance level
- d. we reject H_0, and we would also reject for any larger significance level

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Let Z be a standard normal random variable and let $X = 4Z - 0.5$. Calculate $P(|X| \leq 2.1)$. Use the values given in the table below.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051

Table 1: Cumulative distribution function of the standard normal distribution

- a. 0.2422
- b. 0.0868
- c. 0.3976
- d. 0.1554

Let X be a random variable with probability density function of the form

$$f(x) = \begin{cases} -2x, & -1 \leq x \leq 0 \\ 0, & \text{else} \end{cases}.$$

Compute $P(-\frac{3}{4} < X < -\frac{1}{2})$.

- a. 19/64
- b. 5/16
- c. 7/8
- d. 1/2

In the context of the goodness of fit χ^2-test for three categories let the observed frequencies be $10, 20$ und 30. Let the null hypothesis be that no category is preferred. Further let the rejection region be $R = [7, \infty)$. Then,

- a. we can not say of whether we reject the null hypothesis
- b. due to the data type we should have performed an ANOVA
- c. we do not reject the null hypothesis
- d. we reject the null hypothesis (selected)

Suppose box A contains 4 red and 5 blue coins and box B contains 6 red and 3 blue coins. A coin is chosen at random from box A and placed in box B. Finally, a coin is chosen at random from among those that are now in box B. What is the probability a red coin was transferred from box A to box B given that the coin chosen from box B is blue?

- a. 5/8
- b. 2/9
- c. 16/45
- d. 3/8

The selected answers are not necessarily correct!

Let X be a random variable with the probability density function

$$f_X(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{else} \end{cases}.$$

If F_Y is the cumulative distribution function of $Y = -\ln X$, then $F_Y(\ln 3)$ equals

- a. 2/3
- b. 5/6
- c. 1/3
- d. 1/6

For (real-valued) data $x_1, \ldots, x_n$ (with $n \ge 2$) it always holds that

- a. both the empirical standard deviation and the empirical variance are not negative
- b. the emprical median is not equal to the emprical mean if the data is sampled from an asymmetric distribution
- c. their empirical median is unique if the sample size is even
- d. both their empirical variance and their interquartile range are positive

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Let $\mathfrak{X} \sim mult(4, p)$ with $p = (1/2, 1/2, 0)$. Which statement is **not** correct?

- a. $P(\mathfrak{X} = (2, 1, 1)) = 0$
- b. $P(\mathfrak{X} = (4, 0, 0)) \ne 1/16$
- c. $P(\mathfrak{X} = (1, 1, 2)) = 0$
- d. $P(\mathfrak{X} = (2, 2, 0)) = 3/8$

In a linear regression model ('y_i modeled as a linear function of x_i plus error') the parameters are estimated via least squares. For the mean and the empirical standard deviation of the x and y values we obtain $\bar{x} = 3, s_x = 4, \bar{y} = 7$ and $s_y = 3$. It holds that

- a. the slope of the regression line is smaller than $-3/4$
- b. the slope of the regression line is larger or equals $-3/4$
- c. the regression line goes through $(3, 4)$
- d. the regression line goes through $(7, 3)$

Let $X_1, X_2, \ldots X_{81}$ be an i.i.d. sample from a population with population mean $\mu = 5$ and population variance $\sigma^2 = 4$ and let $S = X_1 + X_2 + \ldots X_{81}$. Approximate the probability $P(369 \leq S \leq 441)$ using the Central limit theorem.

- a. 5%
- b. 32%
- c. 95%
- d. 68%

In the context of a statistical test the null hypothesis was not rejected. Which interpretation is reasonable?

- a. the null hypothesis was not significant
- b. the null hypothesis is not compatible with an alternative hypothesis
- c. the data are hardly compatible with the null hypothesis
- d. the data barely give us a reason to doubt the null hypothesis

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For a statistical test of a significance level α it always holds true

- a. the probability to commit the β-error is larger than α
- b. the probability to commit the α-error is smaller than $\alpha/2$
- c. the power is not larger than the probability of any sure event
- d. the probability to commit the α-error is larger than 2α

Five groups are compared with an ANOVA. The size of the jth group is 5 if j is even, and 25 if j is odd, for $j = 1, 2, \ldots, 5$. Let f denote the Fisher-statistic calculated on the data. The following table shows the 99%-quantiles of the $\mathcal{F}(df_1, df_2)$-distribution.

		df_1				
		4	5	6	7	8
	55	3.68	3.37	3.15	2.98	2.85
	60	3.65	3.34	3.12	2.95	2.82
df_2	65	3.62	3.31	3.09	2.93	2.80
	70	3.60	3.29	3.07	2.91	2.78
	75	3.58	3.27	3.05	2.89	2.76
	80	3.56	3.26	3.04	2.87	2.74

From the information given, we conclude that

- a. For $f = 3.58$ we do reject the null hypothesis on the 5%-level, and we know of whether we reject it on the 1%-level
- b. For $f = 3.68$ we do not reject the null hypothesis on the 1%, but we do not know of whether we reject it on the 5%-level
- c. For $f = 3.68$ we do reject the null hypothesis on the 1%-level, but we do not know of whether we reject it on the 5%-level
- d. For $f = 3.58$ we do reject the null hypothesis on the 1%-level, but we do not know of whether we reject it on the 5%-level

The P-values of six tests are $1/25, 0.01, 1/(8\pi), 2/7^2, 0.08\%$ and $(10^{-4}/10^{-3})/3$. Consider a significance level $\alpha = 5\%$ for each test (prior Bonferroni). How often do we reject the associated null hypothesis after Bonferroni correction?

- ○ a. thrice
- ○ b. once
- ○ c. twice
- ○ d. four times

A fair die is rolled. Find the probability of getting an even number or a number bigger than 2.

Wählen Sie eine Antwort:

- ○ a. 1/3
- ○ b. 7/12
- ○ c. 5/6
- ○ d. 2/3

You perform a χ^2-test for independence in R using chisq.test(). A sufficient input is

- ○ a. the vector of all observations
- ◉ b. the matrix of absolute cell-frequencies
- ○ c. the matrix of relative cell-frequencies
- ○ d. the total number of observations

A plumbing contractor obtains 60% of her boiler circulators from a company whose defect rate is 0.005, and the rest from a company whose defect rate is 0.01. What proportion of the circulators can be expected to be defective? If a circulator is defective, what is the probability that it came from the first company?

- ○ a. 0.034 and 0.882
- ○ b. 0.034 and 0.118
- ○ c. 0.007 and 0.571
- ○ d. 0.007 and 0.429

In the one sample situation for $n > 1$ binary data let h denote the relative frequencies of 'successes'. Which statement is in general **correct**?

- ○ a. $1/h \in (0, 1]$, if $h > 0$
- ○ b. $h^2 \geq h(1-h)$
- ○ c. $h \geq h(1-h)$
- ○ d. $1/h^2 \in (0, 1]$, if $h > 0$

For a statistical test of a significance level α it always holds true

- a. the power is smaller than the probability of an impossible event
- b. the probability to commit the β-error is smaller than α
- c. the probability to commit the α-error is smaller or equals 2α
- d. the probability to commit the β-error is larger than α

In the situation of a right-sided one-sample t-test we find $\bar{x} = -12, s = 21$ and $n = 49$. For a given significance level we find the rejection region $R = [2.2, \infty)$. Then for the null hypothesis $H_0 : \mu = -3$ it holds

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Regarding the data $11, 21, 22, 9, 3, 5$ it holds

- a. the first quartile is not unique
- b. 8 is median
- c. the set of 50%-quantiles is [11,21]
- d. 22 is a 5/6-quantile

If X is a random variable with probability density function

$$f(x) = \begin{cases} 2x - a, & 1 \leq x \leq 3 \\ 0, & \text{else} \end{cases}.$$

then the constant a equals

- a. 2
- b. 3/2
- c. 3.5
- d. 4