2021W Geometry for Computer Science

Exercise sheet A

Exercise 1. Given n points $P_1, \dots, P_n$ in affine space, prove that any affine combination of $P_1, \dots, P_n$ is well-defined, i.e., it does not depend on the choice of origin.

Exercise 2. Consider the following conic sections:

$$\mathcal{Q}_1 \colon x^2 + y^2 + 2(x - y) = 0,$$
$$\mathcal{Q}_2 \colon x^2 + y^2 - 2xy - \sqrt{2}(x + y) = 0.$$

Are $\mathcal{Q}_1$ and $\mathcal{Q}_1$ affine equivalent? Motivate your answer.

Exercise 3. Consider the matrix

$$\mathbb{A} = \begin{pmatrix} 1 & 2 & 5/2 \\ 2 & 4 & -1/2 \\ 5/2 & -1/2 & 4 \end{pmatrix}.$$

1. Determine the affine type of the associated conic section $\mathcal{Q}$.
2. What are the hyperplane coordinates of the tangent plane of $\mathcal{Q}$ at $p \in \mathcal{Q}$?

Exercise 4. Consider the line in three-dimensional space that passes from x and is parallel to the unit vector v. Show that the distance between such line and a point $p \in \mathbb{R}^3$ can be computed by

$$d = \|(p - x) \times v\|.$$

Exercise 5. Show that the affine map $F \colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} 8/5 & -6/5 \\ 3/10 & 2/5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

has no fixed point. (A *fixed point of* F is a point $x \in \mathbb{R}^2$ such that $F(x) = x$.)