142) Man diskutiere die Funktion $f(x) = e^{-x^2}$ (d. h. man bestimme Nullstellen, Extremwerte, Wendepunkte, Grenzwerte, Symmetrieeigenschaften, ...) und skizziere den Funktionsgraphen.

$$f(x) = e^{-x2}$$
$$f'(x) = -2x * e^{-x^2}$$
$$f''(x) = (-2)e^{-x^2} + (-2x) * (-2x)e^{-x^2} = e^{-x^2}(4x^2 - 2)$$
$$f'''(x) = 8x\, e^{-x^2} + (-2x)e^{-x^2}(4x^2 - 2) = e^{-x^2}(8x - 8x^3 + 4x) = e^{-x^2}(-8x^3 + 12x)$$

Nullstellen:

$$f(x) = 0 => e^{-x2} = 0, e^{-x2} \mathit{kann\ nie\ 0\ werden} => \mathit{keine\ Nullstelle!}$$

Extrema:

$$f'(x) = 0 => -2x * e^{-x^2} = 0, e^{-x2} \mathit{kann\ nie\ 0\ werden!}$$
$$-2x = 0 => x = \frac{0}{-2} => \mathit{Extremwert\ bei\ } x = 0!$$

$$\mathit{Minimum\ wenn\ } f''(x) > 0, \mathit{Maximum\ wenn\ } f''(x) < 0$$
$$f''(0) = e^{-0^2}(4 * 0^2 - 2) = 1(0 - 2) = -2 => \mathit{Maximum!}$$
$$f(0) = e^{-0^2} = 1 => \mathit{Maximum\ an\ der\ Stelle\ } (0,1)$$

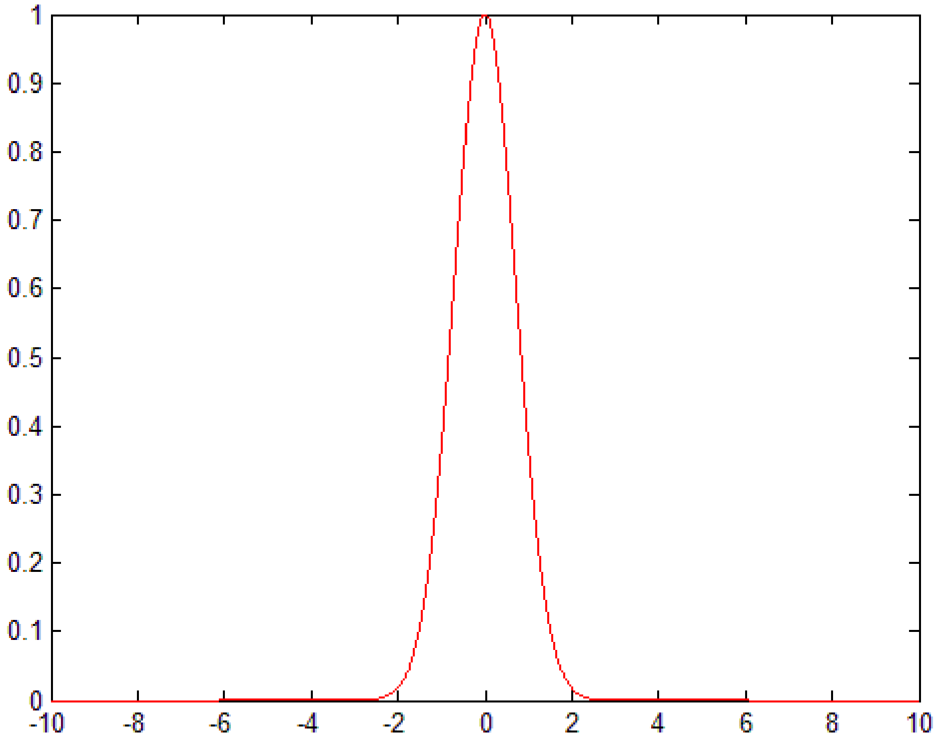
Wendepunkte:

$$f''(x) = 0 => e^{-x^2}(4x^2 - 2) = 0, e^{-x2} \mathit{kann\ nie\ 0\ werden!}$$
$$4x^2 - 2 = 0 => x^2 = \frac{2}{4} => x = \pm\sqrt{\frac{1}{2}}, \mathit{wenn\ Wendepunkt, dann\ muss\ } f'''(x) \neq 0 \mathit{\ sein!}$$

$$f'''(x) \neq 0 => e^{-x^2}(-8x^3 + 12x) \neq 0 => -8x^3 + 12x \neq 0 => -2x^3 + 3x \neq 0$$

$$f'''\left(\sqrt{\frac{1}{2}}\right) = -2 * \sqrt{\frac{1}{2}}^3 + 3 * \sqrt{\frac{1}{2}} = 1.41 => \mathit{Wendepkt.}$$
$$f'''\left(-\sqrt{\frac{1}{2}}\right) = -2 * (-\sqrt{\frac{1}{2}})^3 + 3 * -\sqrt{\frac{1}{2}} = -1.41 => \mathit{Wendepkt.}$$

$$\mathit{Wendepunkt}_1\left(\frac{1}{\sqrt{2}}, e^{-\frac{1}{2}}\right), \mathit{Wendepunkt}_2(-\frac{1}{\sqrt{2}}, e^{-\frac{1}{2}})$$



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10