This is the fifth homework assignment. Students should tick in TUWEL problems they have solved and upload their detailed solutions by **20:00** on Monday **November 21, 2022**.

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(1) **Human resource testing**
Some human resource deprtments administer standard IQ tests to all employees. The Stanford-Binet test scores are well modeled by a Normal model with expectation 100 and standard deviation 16.

(a) If the applicant pool is well modeled by this distribution, what is the probability that a randomly selected applicant would have the score ibetween 84 and 116?

(b) For the IQ test administered by human resources, what cutoff value would separate the middle 95%?

(2) **Coin throws**
An unfair coin is thrown 600 times. The probability of geting a tail in each throw is $\frac{1}{4}$.

(a) Use a Binomial distribution to compute the probability that the number of heads obtained does not differ more than 10 from 440.

(b) Use a Normal approximation without a continuity correction to calculate the probability in (a). How does the result change if the approimation is provided with a continuity correction?

(3) **Cars arrivals**
Suppose cars arrive at a parking lot at a rate of 50 per hour. Assume that the process is modeled by a Poisson random variable with $\lambda = 50$.

(a) Compute the probability that in the next hour the number of cars that arrive at this parking lot will be between and including 54 and 62.

(b) Compare the value obtained in (a) with the probability calculated by using a Normal approximation.

(4) **Sum and average**
Let X be a random variable with $\mathcal{N}(5, 4)$. Let $X_1, X_2, \ldots, X_{50}$ be independent identically distributed copies of X. Let S be their sum and $\bar{X}$ their average, i.e.

$$S = X_1 + \cdots + X_{50} \qquad \text{and} \qquad \bar{X} = \frac{1}{50}(X_1 + \cdots + X_{50}).$$

(a) Plot the density of X.

(b) What are the expectation and the standard deviation of S and of $\bar{X}$?

(c) Use `R` to generate a sample of 50 numbers from $\mathcal{N}(5, 4)$. Plot the histogram for this sample. Do the same for a sample of 500 numbers from $\mathcal{N}(5, 4)$.

(5) **Histograms of averages of** exp(1)

(a) Generate a frequency histogram of 1000 samples from an exp(1) random variables.

(b) Generate a density histogram for the average of 2 independent exp(1) random variable.

(c) Using `rexp`(), `matrix`() and `colMeans`() generate a density histogram for the average of 50 independent exp(1) random variables. Make 10000 sample averages and use a binwidth of 0.1 for this. Look at the spread of the histogram.

(d) Add a graph of the pdf of $\mathcal{N}(1, 1/50)$ on your plot in problem (c).