Signal theory:

Convolution: weighted sum of two waves
3D Object -> sampling -> 3D reconstruction -> 2D image -> 2D reconstruction -> 2D picture
Fourier transformation: transform analogue signal (real) to spectrum (sinusoid function, virtual)

1. **From signal-theoretic point of view explain the Nyquist criterion**

If the Nyquist criterion is fulfilled the signal can be perfectly represented by a limited number of samples (the number of this discrete samples is taken two times greater than the highest frequency component in the continuous waveform)
Minimum sampling frequency to reliably reconstruct the original signal, the signal doesn't get undersampled.
f_s the amount of sampling per second
$\omega_s > 2\omega_m$ (= $f_{abtast} > 2^* f_{signal}$), for not overlapping sampling signals
sampling frequency > 2* highest frequency

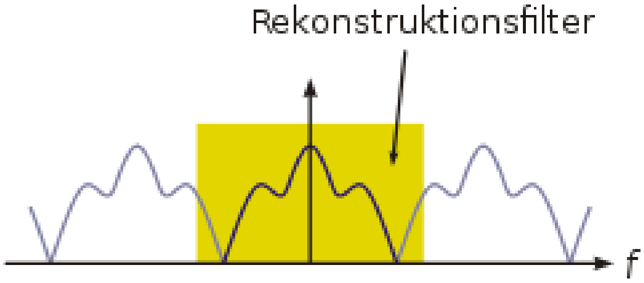
2. **From the signal-theoretic point of view explain the origin of aliasing**

Distortions and artifacts.
If the image data is not properly processed during sampling or reconstruction, the reconstructed image will differ from the original image, and an alias (error) is seen. it can occur when undersampled (below the niquist criteria) or by using a bad reconstruction filter and artifacts overlap the original signal.

3. **Explain the difference between prealiasing and postaliasing**

Prealiasing: Wrong frequencies appear if the Nyquist criterion is not fulfilled,
e.g undersampling because of insufficient sampling rate (below the niquist criterion).
High original frequency is actually a low frequency when sampling.

Postaliasing: Wrong frequencies appear also if the reconstruction filter support is too broad.
Wenn der Rekonstruktionsfilter nicht nur das Nutzsignal isoliert, sondern auch Teile von dessen Kopien mit einbezieht (siehe Schema rechts), kommt es zu Postaliasing.
Reconstruction filter is imperfect.



4. **Can post-aliasing be avoided and how? Explain from the signal theoretic point of view**

Post-Aliasing can theoretically be avoided by using an ideal filter, but an ideal filter doesn’t exist. Ideal filter would avoid overlappings of the signal. Post-Aliasing can be reduced by using a better filter for the image.

5. **Can pre-aliasing be avoided and how? Explain from the signal theoretic point of view**

It may be possible to sample above the Nyquist limit, but if this is not possible then pre-aliasing can also be avoided by low-pass filtering the volume to reduce its frequency content. Low-pass filtering removes “störsignale” (=artifacts) in highfrequency space for correct sampling.

Zur Vermeidung solcher Aliasing-Effekte wird das Eingangssignal durch einen Tiefpass gefiltert (Anti-Aliasing-Filter). Die Filterwirkung dieses Abschneidens der hohen Frequenzen kann auch durch die Begriffe Höhensperre, Höhenfilter, High Cut und Treble Cut beschrieben werden. Diese Filterung muss vor der Digitalisierung geschehen – eine nachträgliche Korrektur von Alias-Effekten ist nicht mehr möglich.

6. **From the signal theoretic point of view explain reconstruction of sampled data**

Direct reconstruction of a continuous signal from discrete samples:
- The spectrum replicas have to be suppressed by a reconstruction filter
- The ideal reconstruction filter is a box function:

$$R(\omega_x, \omega_y) = \begin{cases} K \text{ if } |\omega_x| \leq \omega_{xc} \wedge |\omega_y| \leq \omega_{yc} \\ 0 \text{ else} \end{cases}$$

Converts a discrete signal into a continuuing signal with recounstruction filter. interpolates sampled data

box function:

$$\text{box}(\omega) \leftrightarrow \text{sinc}(s) = \frac{\sin(s)}{s}$$

7. Qualitatively explain the differences between reconstruction using linear and cubic reconstruction filters

Interpolation in all dimensions over separable filters.

Linear: Touching pixels average their values.
Cubic: Touching pixels average their values so central pixels maintain the most value.
reconstruction using a linear filter

Qualitative differences:
Linear uses less pixels than cubic; therefore cubic is slower, but produces less artifacts than linear eg.: bilinear: 4 pixels (2×2) and bicubic: 16 pixels (4×4).
(source: https://en.wikipedia.org/wiki/Bicubic_interpolation)

8. The DICOM Standard, basic characteristics of the format, role of DICOM images in a hospital

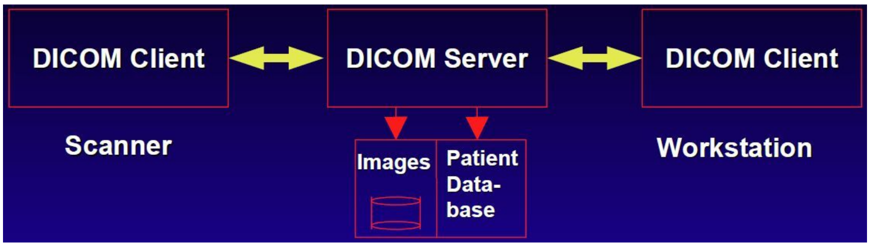
DICOM stands for Digital Imaging and Communications in Medicine:
It is a standard for handling, storing, printing and transmitting information in medical imaging. For that it includes a file format definition as well as a network communication protocol. It acts like a container, grouping various information (image data, patient identification and demographics as well as technical information) into data sets. So the patient name for example is also saved within the file. Almost every manufacturer of imaging systems (X-ray, MR, CT,...) implements this standard to provide interoperability between different medical institutions like hospitals but also smaller entities.
DICOM beginns with meta information, which are tagged.
A sequence of data elements orderered by Group and Data elements numbers.
Even numbers predefined, odd numbers vendor specific.

The format is used in hospitals for image data today. It defines:

- A set of protocols to be followed by devices claiming DICOM conformance
- The syntax and semantics of Commands and associated information which can be exchanged using these protocols
- Information that must be supplied with an implementation for which the DICOM conformance is claimed

DICOM Concepts:

- Scanners store their data on the server
- Workstations query server for list of image data managed by the server
- Server talks to database and provides results of query to the workstation
- Workstation retrieves desired data and server gets data from disk or other media



9. Classification of 3D grids and their representation in a computer

  - Scattered Points
  - Unstructured Grid
  - Structured Grid
  - Rectilinear Grid the cells are axisaligned, but grid spacings along the axes are arbitrary
  - Regular Grid
  - Cartesian Grid

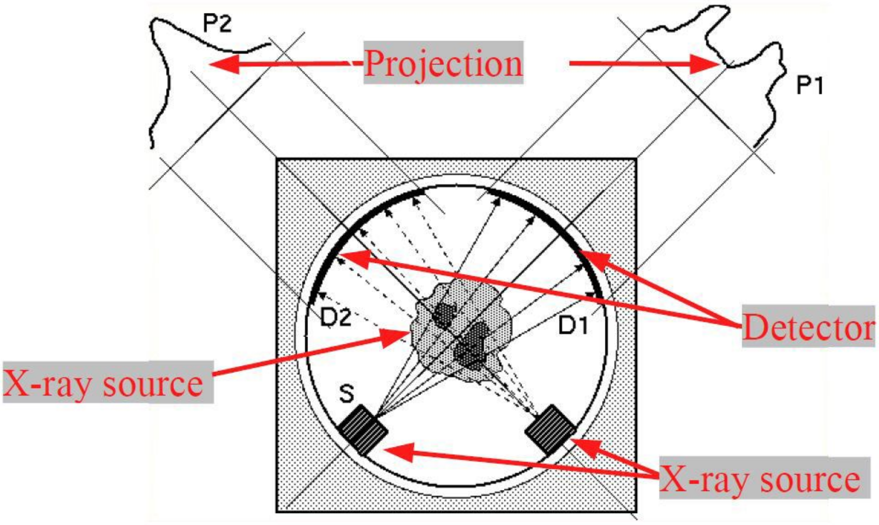
10. CT tomography, basic description, physical background and CT data characteristics

- Measurement of X-ray attenuation along a viewing ray (projections)
- Production of images: reconstruction from projections
- CT is a medical imaging procedure which utilizes computer-processed X-rays to produce tomographic images or 'slices' of specific areas of the body. These cross-sectional images (tomograms) are used for diagnostic and therapeutic purposes in various medical disciplines. Digital geometry processing is used to generate a three-dimensional image of the inside of an object from a large series of two-dimensional X-ray images taken around a single axis of rotation.

Data properties:

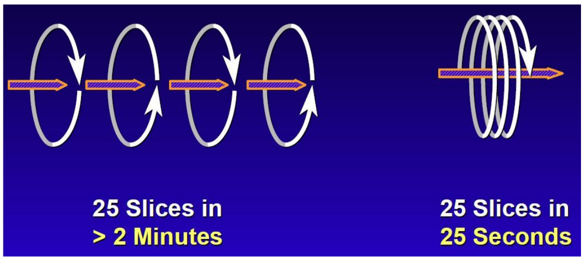
- Axial slices 0.5-10 mm thick
- Images 256 x 256 to 512 x 512 pixels of 0.5-2 mm side

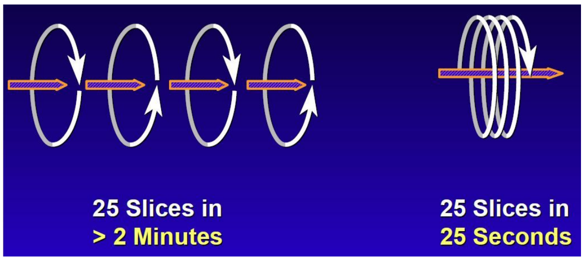
- Up to 2000 images per study
- High spatial resolution
- Consistent values (HU scale; Air: -1000 HU, Water: 0 HU, Bone: >1000 HU)
- (HU = Hounsfield Unit)
- X-ray irradiation



State of the art:

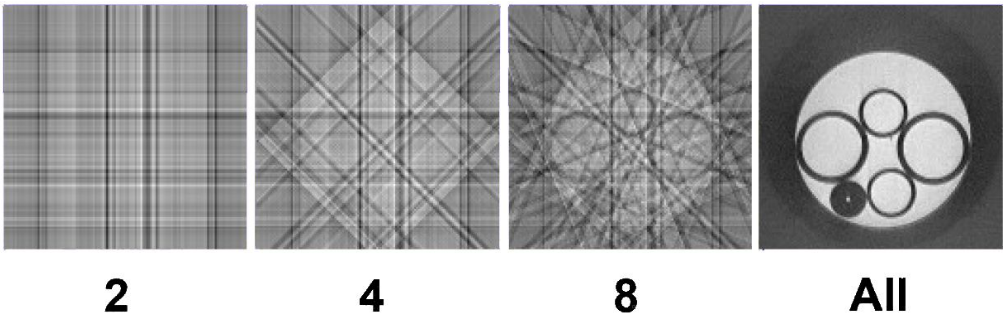
- Up to 256 detectors
- Spacing < 1mm between detectors
- Scanning speed 1m/30s





Reconstruction methods:

- Algebraic methods
- Fourier methods
- Filtered backprojection
  - Projection filtration (high pass filter, derivative)
  - Distribution of the filtered projection to the image in the direction of the projection



11. MRI, basic description, physical background and MRI data characteristics

Magnetic Resonance Imaging, ein bildgebendes Verfahren zur Darstellung der Gewebestrukturen im Körperinneren.

basic description, physical background:

- MRI makes use of the property of nuclear magnetic resonance (NMR) to image nuclei of atoms inside the body
- MRI detects the interaction of atom nuclei with an external magnetic field
- Equilibrium - Absorption of Radiofrequency energy - Relaxation
    - The principle behind the use of MRI machines is that they make use of the fact that body tissue contains lots of water (and hence protons) which gets aligned in a large magnetic field. Each water molecule has two hydrogen nuclei or protons. When a person is inside the powerful magnetic field of the scanner, the average magnetic moment of many protons becomes aligned with the direction of the field. A radio frequency current is briefly turned on, producing a varying electromagnetic field. This electromagnetic field has just the right frequency, known as the resonance frequency, to be absorbed and flip the spin of the protons in the magnetic field. After the electromagnetic field is turned off, the spins of the protons return to thermodynamic equilibrium and the bulk magnetization becomes re-aligned with the static magnetic field. During this relaxation, a radio frequency signal (electromagnetic radiation in the RF range) is generated, which can be measured with receiver coils.

Properties of MR data:

- Measurement in arbitrary planes
- Typically 256x256 (512x512)pixels
- No absolute scale for measured values
- Significant level of noise
- Good soft tissue contrast

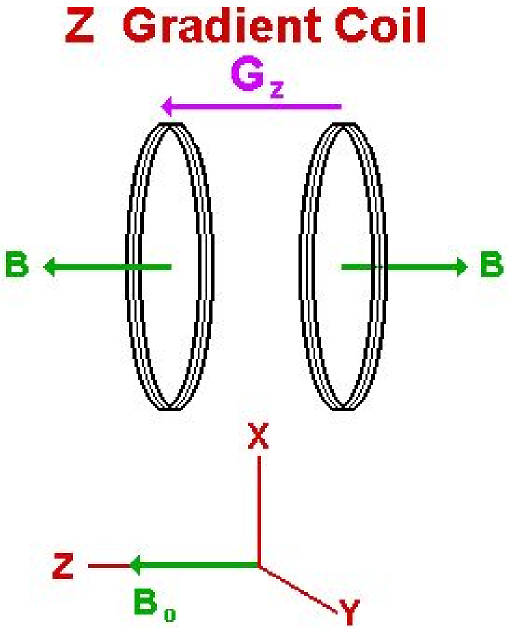
- Spatial inhomogeneities - bias
- No irradiation

12. Explain the basics of spatial localization in MR tomography and of measurement in arbitrary planes

Spatial Localization in MRI
Position encoding by means of gradient fields:

- Z-gradient: slice selection by changing the Larmor frequency fL = (B0 + Gz.z)

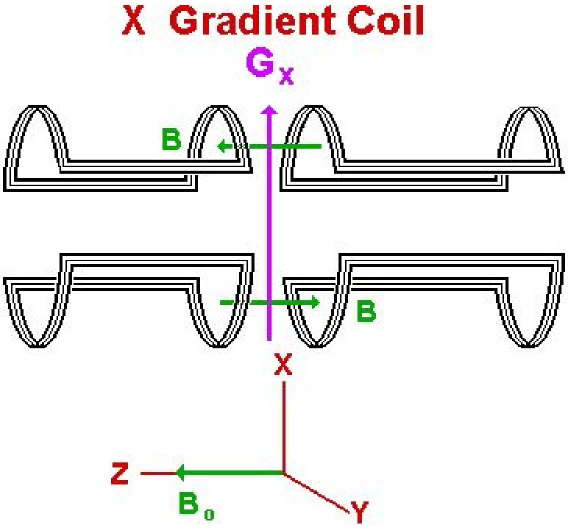
- Z-gradient is applied during the RF pulse



- X, Y encoding

  Similar tricks to encode the x and y coordinates

  Only one row measured in one excitation



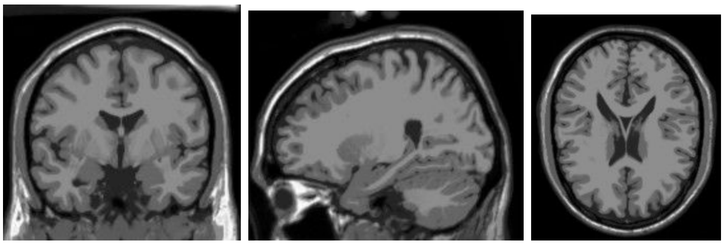
- Imaging in Other Planes

  - B_0 always remains in the same direction

  - Choice of imaging plane depends on the order of gradients' application

  - Oblique planes: simultaneous application of 2 gradients

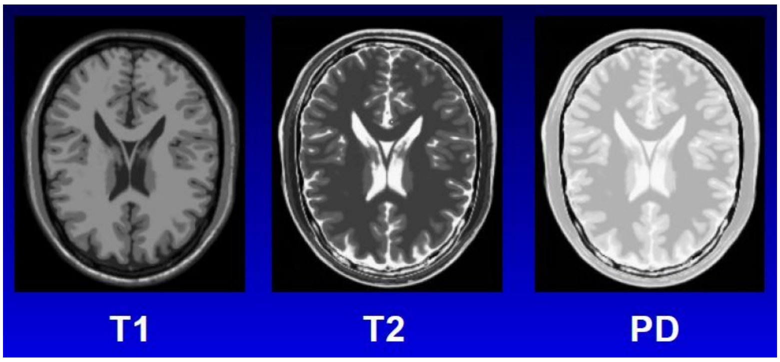
  Sagittal Coronal Transversal



13. MRI, scanning protocols, T1, T2 and proton density images

- Scanning Protocols
  - ○ Protocol is a sequence of pulses, gradients and signal measurements
  - ○ Protocols influence image properties and examination time
  - ○ Patented and sold by scanner vendors
  - ○ Examination Time:
    - ■ T = TR * R * N [ms]
      - TR: repetition Time
      - R: number of image rows
      - number of accumulations (noise)
      - Example T = 2000 * 256 * 2 = 17 min

- T1,T2, PD images



  - Different combinations of TR (Repetition Time) and TE (Echo time) yield different tissue contrast

    - Longitudinal relaxation (rate T1), vector return back to direction
        - Short TR (repetition Time)
        - Short TE (excitation Time)
    - Transversal relaxation (rate T2), loose synchronization
        - Long TR
        - Long TE
    - Proton Density (PD)
        - Long TR
        - Short TE

Explanation:

- certain atoms interact with magnetic fields (resonance)
- absorb energy and later produce energy to get to equilibrium state

Resonance -> energy travels from one to another medium with same frequency, two systems with same characteristic frequency
Elements have own magnetic moment (characteristic frequency)

Speed of T1, T2 depends on material (tissue)
T1: “move” tissue
T2: “move” water
PD: water densitiy

radiofrequency coil: invokes radiowaves, object absorbs energy
gradient field coil: induce magnetic field to retain informations

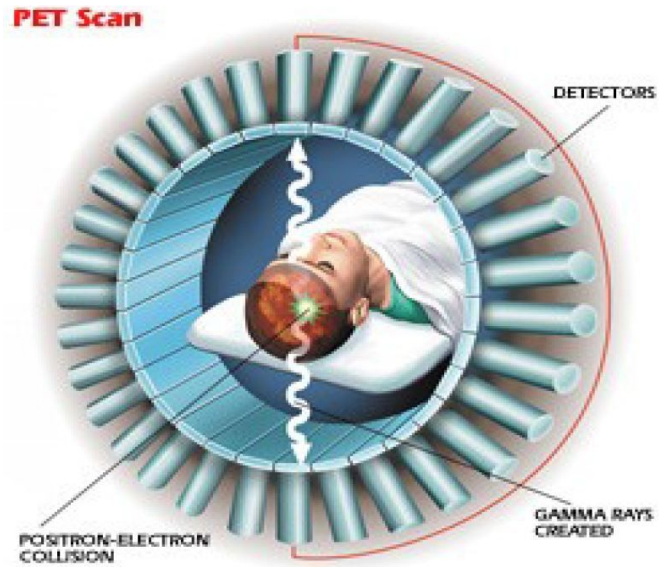
radiofrequency absorbed -> start to rotate synchonizly (=Excitation) -> after switching the magnetic field off you get the signal
Procedure: Equilibrium -> Excitation -> Relaxation -> Equlibrium
TE: time from excitation (when to measure), measure in the right time after excitation

14. Anatomic and functional acquisition techniques, PET & SPECT

<u>PET:</u>

- Positron Emission Tomography
- nuclear medicine imaging technique
- 3D-images of functional processes in the body
- short-lived radioactive isotope injected in patient for beta+ decay (positrons)
- Annihilation - a pair of photons with energy 511keV in opposite directions (gamma rays emitted indirectly)
- Detection without collimators: registration of concurrent events in detector pairs

(better results without collimators because they ignore 99,99% of radiation)
- 3D measurement, statistical reconstruction (MRF)
- often in combination with CT scan



SPECT:

- Single Photon Emission Computed Tomography
- similar to PET but not the same
- radioactive isotopes injected in patient for emitting gamma-photons (Tc-99, I-125, I-131)
- Uniform distribution of photons in all directions
- Scanner - a set of detectors with collimators - gamma camera
- lower resolution than PET
- significantly less expensive than PET (able to use longer-lived more easily-obtained radioisotopes)

15. Volume viewing techniques

Volume Viewing techniques:

- Slice-by-slice viewing
- Multiplanar reconstruction (Definition of new cutplanes)
- Curved planar reconstruction (Volume cutting along a line)
- Reprojection (Add all values along a viewing ray; Simulation of X-ray projection)
- Maximum intensity projection (Register the brightest value along a viewing ray; Suitable for thin structures)

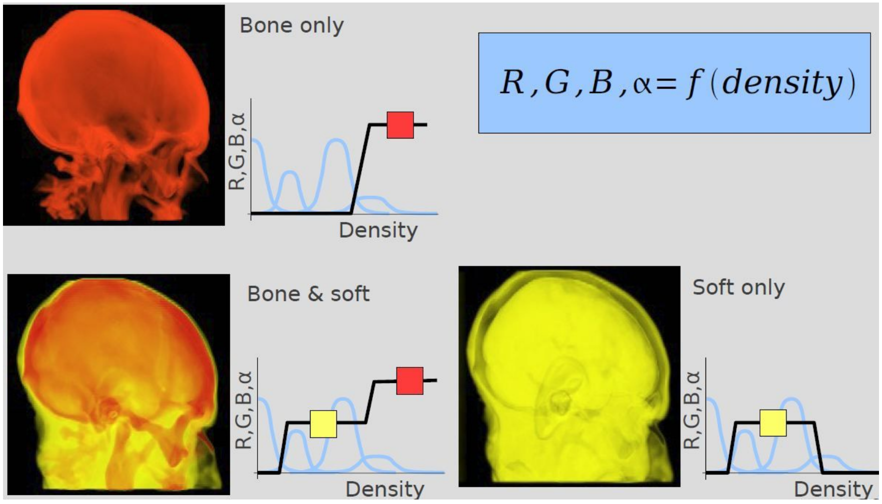
16. Basics of visualization with mapping

Mapping: Assignment of visual attributes to data (transparency, color, reflectance, etc.)

Mapping by Density Classification Assumptions:

- Areas of interest can be identified solely by density value
- Neighbors in histogram are neighbors in space

Density Classification by Transfer Functions:



Mapping Summary:

- Transfer function based
  - Color & transparency assigned to voxels
  - Semitransparent volumes
  - Display of volumes
- Segmentation-based
  - Unambiguous object definition
  - Color & transparency assigned to objects
  - Display of surfaces

17. Compare direct volume rendering and isosurfacing

https://en.wikipedia.org/wiki/Volume_rendering#Direct_volume_rendering

DVR vs. Isosurfacing:

- Rendering algorithms are similar
- Isosurfacing is a limit case of DVRs with special TF(=Transfer functions) and parameter setting

18. Basics of direct volume rendering DVR techniques Object order and image order rendering

Basics & Techniques
A direct volume renderer requires every sample value to be mapped to opacity and a color. This is done with a "transfer function" which can be a simple ramp, a piecewise linear function or an arbitrary table. Once converted to an RGBA (for red, green, blue, alpha) value, the composed RGBA result is projected on corresponding pixel of the frame buffer. The way this is done depends on the rendering technique.

- Simplified light interaction with semitransparent material
- Light attenuation and emission along a ray
- No shadows, no reflections
- Numerical evaluation
    - Per-segment compositing by Porter&Duff's operators (Front-to-back or Back-to-front order)

Techniques:

- Rendering by compositing
- Splatting
- Texture mapping
- Volume ray casting

Object-order algorithms (splatting):

- Compositing in image plane
- Projection of samples from volume to image

Image-order algorithms:

Ray casting based
Sequence of samples along the ray Compositing along the ray

19. Problems which may occur in perspective visualization of volume data and their possible solutions

Questions:

- Which is the correct sampling density along a ray
- Which is the influence of gradient on the correct sampling density
- Which is the correct sampling density in perspective rendering
    - Incorrect spatial variable sampling
    - Oversampling (long time)
    - Undersampling (low image quality)

Solution:

- VR is a resampling process
- Antialiasing by low-pass filtering

Practical Solution:

- 3D-mip map (multum in parvo, much in a small space)
- Adaptive Sampling
- Exponential Regions Perspective

20. Methods for specification of density-based transfer functions in volume rendering.

- Transfer functions of any shape and complexity but they assign the same value to a particular density regardless of its position and environment.
- Densitybased transfer functions assign opacities and chromacities to each density, i.e.:

$$\rho(p)=f(d(p))$$
$$k(p)=g(d(p))$$

f and g ... transfer functions
p ... a position in the volume
d(p) ... density at p

21. The special case of transfer functions to be used for CT data.

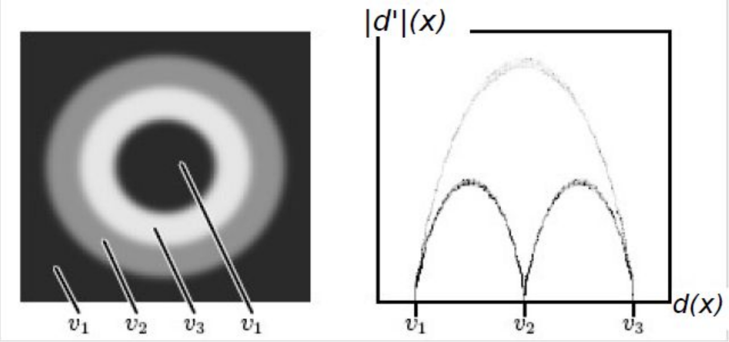
R,G,B, alpha = f (density) , f = transfer function
Use of a transfer function to map density values to a RGBA. (A = Opacity)
When displaying density in a historgram the neighbors in the histogram are neighbors in space.
Density classification is done by transfer functions. Different transfer functions are used for classifying differces types of the body, e.g Bone, Soft, Bone & Soft

Density based transfer function is one-dimensional, in some cases more dimensions better.

22. Multidimensional transfer functions

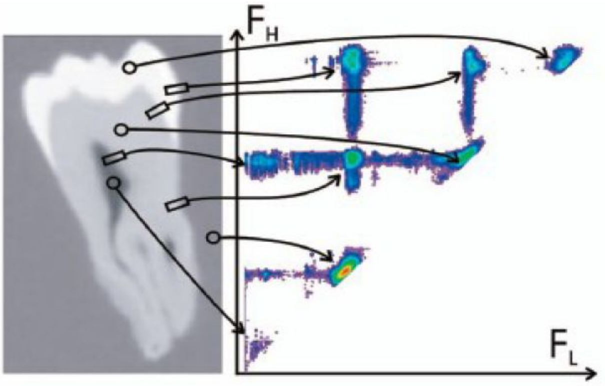
Multidimensional transfer functions are a very effective way to extract materials and their boundaries for both scalar and multivariate data.
Rather than classifying a sample based on a single scalar value, multi-dimensional transfer functions allow a sample to be classified based on a <u>combination of values</u>.
These values are the axes of a multi-dimensional transfer function.

- |d'| vs. d scatterplots
  TF design paradigm based on |d'| vs. d scatterplot analysis



Observation: special arc-shaped d/d' scatterplot of blurred data

- LH (Low-High) -histograms
  - Downhill and uphill stationary values
  - A boundary is represented by a single point in LH histogram



23. Surface rendering by the Marching Cubes approach and its problem Compare the Marching Cubes and the Marching Tetrahedra approaches. Advantages and disadvantages of surface rendering. Surface Shading

MC-Algorithmus:
1.Consider a cell

2.Classify each vertex as inside or outside (greater or lower the threshold)
3.Build an index (binary)
4.Get edge list from table[index] (256 cases)
5.Interpolate the edge location
6.Go to next cell

Marching Cubes Disadvantages:
- Several tessellation possibilities for certain vertex configuration
- Surface holes may appear Incorrect combination
- Correction needed

Marching Tetrahedra
- Solves ambiguity in Marching Cubes tessellation
- Disambiguation by cell subdivision in tetrahedra
- Doubles the number of triangles

Der Algorithmus für Marching Cubes ist sehr rudimentär. Er nutzt beispielsweise nicht aus, dass bereits berechnete Informationen wieder verwendet werden können: Teilen sich zwei benachbarte Kuben eine Kante, so müssen darauf liegende Knoten nur einmal interpoliert werden; der Nachbar kann die bereits gefundenen Knoten einfach übernehmen.

Da die Laufzeit des Algorithmus nur von der Anzahl der betrachteten Kuben abhängig ist, liegt in der Verminderung dieser Anzahl das größte Einsparpotenzial. Weitere Optimierungsansätze versuchen daher, vor dem Marching Cubes-Durchlauf diejenigen Würfel aus der Datenmenge herauszufiltern, die ohnehin nicht mit der Isooberfläche in Berührung kommen. Dies sind diejenigen Kuben, die vollständig innerhalb oder vollständig außerhalb des Objektes liegen.

Marching tetrahedra computes up to nineteen edge intersections per cube, where marching cubes only requires twelve. Only one of these intersections cannot be shared with an adjacent cube (the one on the main diagonal), but sharing on all faces of the cube complicates the algorithm and increases memory requirements considerably. On the other hand, the additional intersections provide for a slightly better sampling resolution.

The number of configurations, determining the size of the commonly used lookup tables, is much smaller, since only four rather than eight separate vertices are involved per tetrahedron. There are six tetrahedra to process instead of one single cube. The process is unambiguous, so no additional ambiguity handling is necessary.

The downside is that the tessellation of a cube with tetrahedra requires a choice regarding the orientation of the tetrahedra, which may produce artificial "bumps" in the isosurface because of interpolation along the face diagonals.[2]

Surface Extraction Advantages:
- Uses known rendering methods

- Can take advantages of hardware
- View/light changes require only re- rendering (no pre-processing)
- Compact storage and transmission

Surface Extraction Disadvantages:

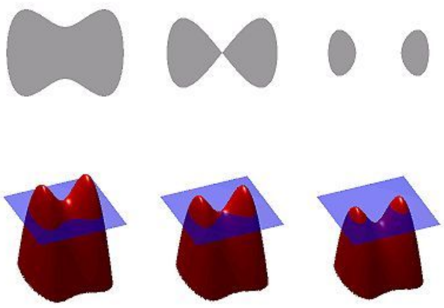
- Requires binary classification
- Throws away data
- Handles small features poorly
- Requires user intervention sometimes
- Cannot represent translucent data and weak surfaces

Surface Shading

- Depth Shading (distance to obersver stored for postprocessing)
- Lambert Shading (I = n x p, n surface normal vector, p light direction vector)
- Phong Shading

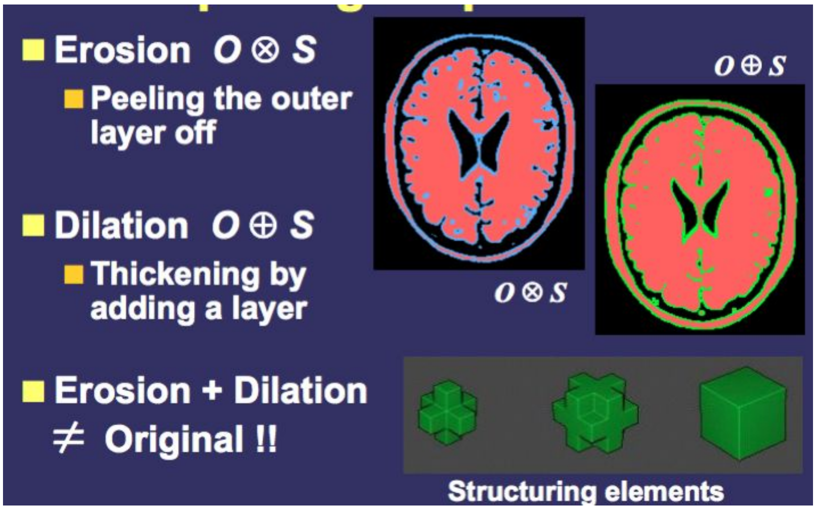
24. Segmentation of volumetric data.

LSF (Level Set Function)
very basic: there is a surface (LSF), it intersects a plane, that gives us a contour.
https://en.wikipedia.org/wiki/Level_set_method
Graph of a level set function ϕ determinies the shape. Used to calculate curves and the topology. With the LSF it is easier to calculate these curves and topologies because the function is well behaved. The underlying topology can easily change.
Front is the surface where $\phi = 0$.



ISEG (based on thresholding)
Interactive Segmentation: separate objects from areas that are not connected, some interactions can be broken.

Erosion: peel outer layer. take structured element and put it in object and paint covered area. blue areas partial out and partial in -> partially shrinks, little isolation objects detected
Dilatation: points touch input area -> add 1 layer of voxel to surface. little isolated objects added
Combination: Erosion -> Dilatation != Original

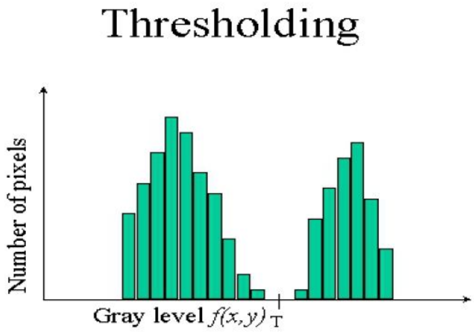


- **Erosion $O \otimes S$**
  - Peeling the outer layer off
- **Dilation $O \oplus S$**
  - Thickening by adding a layer
- **Erosion + Dilation $\neq$ Original !!**

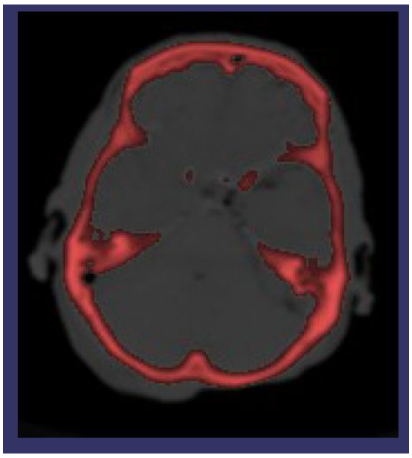
https://www.youtube.com/watch?v=8g9WI0y__jU

25. Segmentation by thresholding – comparison of CT and MRI data from the point of thresholding

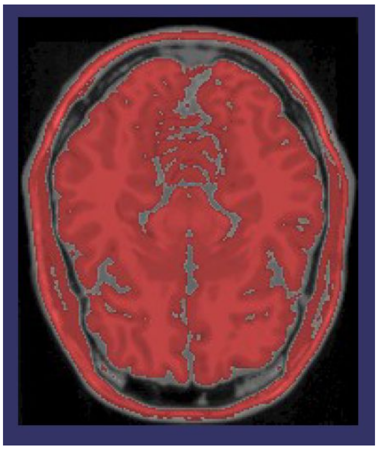
Labeling operation on a gray scale image that distinguishes pixels of a higher intensity from pixels with a lower intensity value.
Output: binary image
Works well when the image histogram is bi-modal



bimodal image



CT image



MRI image

https://en.wikipedia.org/wiki/Image_segmentation#Thresholding

-> CT: thresholds derived from radiograph instead of (reconstructed) image