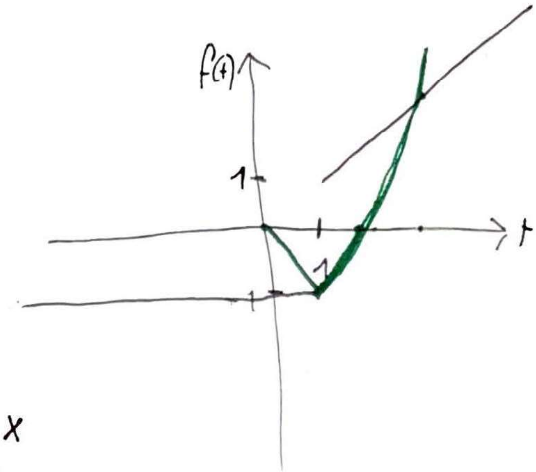
195) Für die Funktion $f(t) = \cancel{\begin{cases} -1 & (t \le 1) \\ 1 & (t > 1) \end{cases}}$ berechnen Sie $F(x) = \int_0^x f(t)\,dt$. Ist $F(x)$ stetig bzw. differenzierbar?

197) Wie 195) für $f(t) = \begin{cases} -1 & (t \le 1) \\ t & (t > 1) \end{cases}$

offensichtlich unstetig an $t=1$



Fallunterscheidung $\int_0^x f(t)$

$x \le 1$

$$F(x) = \int_0^x (-1)\,dt = -t\Big|_0^x = -x$$

$x > 1$

$$F(x) = \int_0^1 (-1)\,dt + \int_1^x t\,dt = -t\Big|_0^1 + \frac{t^2}{2}\Big|_1^x$$

$$= -1 + \left(\frac{x^2}{2} - \frac{1}{2}\right) = \frac{x^2}{2} - \frac{3}{2}$$

$$\lim_{x \to 1-} F(x) = -1 = \lim_{x \to 1+} F(x)$$

$$F\left(\lim_{x \to 1} x\right) = F(1) = -1 = \lim_{x \to 1} F(x)$$ stetig ✓

Diffcheck:

$$\lim_{x \to 1-} \frac{F(x) - F(1)}{x-1} = \lim_{x \to 1-} \frac{-x+1}{x-1} = \lim_{x \to 1-} (-1) \cdot \frac{\cancel{x-1}}{\cancel{x-1}} 1 = -1$$

$$\lim_{x \to 1+} \frac{F(x) - F(1)}{x-1} = \lim_{x \to 1+} \frac{\frac{x^2-3+2}{2}}{x-1} = \lim_{x \to 1+} \frac{x^2-1}{2x-2} = \lim_{x \to 1+} \frac{(x+1)\cancel{(x-1)}}{2\cancel{(x-1)}} = 1$$

nicht differenzierbar