1	2	3	4	5	Σ	Grade

Examination for "Logic and Reasoning in Computer Science"
October 11, 2024 **2nd Exam for SS 2024**

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This exam sheet consists of five problems, yielding a total of 100 points. Good luck!

Problem 1. (25 points) Consider the formula :

$$(\neg(p \land q) \rightarrow r) \land \neg((r \leftrightarrow p) \land r)$$

(a) Which atoms are pure in the above formula?

(b) Compute a clausal normal form C of the above formula by applying the CNF transformation algorithm with naming and optimization based on polarities of subformulas;

(c) Decide the satisfiability of the computed CNF formula C by applying the DPLL method to C. If C is satisfiable, give an interpretation which satisfies it.

Problem 2. (15 points) Formalize the following argument and verify whether it is correct:

1. If I stay late at work today then I finish my assignments.
2. Either I stay late at work or I walk my dog, and I cannot do both.
3. Therefore, I either walk my dog or finish my assignments.

Note that verifying whether an argument is correct means to prove a statement of the form: from the hypothesis P_1 and .. and P_n, the conclusion Q follows (that is, you need to either formally prove $P_1, \ldots, P_n \models Q$, or exhibit a counterexample for the statement).

Problem 3. (20 points) Let C be a set of clauses such that each clause in C contains at most one negative literal. Give an algorithm that decides the satisfiability of C in polynomial time. Justify the correctness of your algorithm and explain the polynomial time decidability of your solution.

Problem 4. (15 points) Provide a proof or a counterexample for the following statement:

$$\forall y \forall z (P(y) \land P(z)) \models \forall x (P(x) \land Q(x))$$

If you provide a counterexample, you must show that it is in fact a counterexample.

Problem 5. (20 points) Consider the formula:

$$a + 2 = b - 1 \land f(b+1) = c - 2 \land (f(a+3) \neq c \lor read(write(A, a+1, 3), b-2) = 2)$$

where b, c are constants, f is a unary function symbols, A is an array constant, $read, write$ are interpreted in the array theory, and $+, -, 1, 2, 3, ...$ are interpreted in the standard way over the integers.

Use the Nelson-Oppen decision procedure for reasoning in the combination of the theories of arrays, uninterpreted functions, and linear integer arithmetic. Use the decision procedures for the theory of arrays and the theory of uninterpreted functions and use simple mathematical reasoning to derive new equalities among the constants in the theory of linear integer arithmetic. If the formula is satisfiable, give an interpretation that satisfies the formula.