Ana - UE 10 - 11902064

332) $f(x,y) = x^2(y-1) + xe^{y^2}$

$x_0 = (1,0)$

$f_x = 2x(y-1) + e^{y^2}$ $\qquad$ $f_x(1,0) = -1$

$f_y = x^2 + x(e^{y^2} \cdot 2y)$ $\qquad$ $f_y(1,0) = 1$

$f_{xx} = 2y - 2$

$f_{xy} = 2x + 2y\, e^{y^2} = f_{yx}$

$f_{yy} = x \cdot e^{y^2}(4y+2)$

$f_{xx}(1,0) = -2$

$f_{xy}(1,0) = 2$

$f_{yy}(1,0) = 1$

$\Rightarrow$ lin Approx:

$f(x,y) = (-1,1)\begin{pmatrix} x-1 \\ y \end{pmatrix} + R = \underline{-x+y+1+R}$

$\Rightarrow$ quadr Approx:

$f(x,y) = (-1,1)\begin{pmatrix} x-1 \\ y \end{pmatrix} + \frac{1}{2}(x-1,y)\begin{pmatrix} -2 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x-1 \\ y \end{pmatrix} + R =$

$= -x+y+1 \quad x^2+2xy+2x-2y+y^2 =$

$= \underline{-x^2+2xy+x-y+y^2+R}$

359) $f(x,y) = x^2 + xy + y^2 + x + y + 1$

$$f_x(x,y) = 2x + y + 1$$
$$f_y(x,y) = 2y + x + 1$$
$$f_{xx} = 2$$
$$f_{xy} = 1$$
$$f_{yy} = 2$$

$$\begin{pmatrix} 2x + y + 1 \\ 2y + x + 1 \end{pmatrix} = \vec{0}$$

$$x - y = 0$$
$$x = y$$

$$3x + 1 = 0$$
$$x = y = -\frac{1}{3}$$

Hesse-Matrix:

2

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

3 beide pos $\Rightarrow$ pos definit

$\Rightarrow \left(-\frac{1}{3}, -\frac{1}{3}\right)$ ist eine Tiefstelle

366) $f(x,y) = \cos(x+y) + \sin x + \sin y$

$f_x = -\sin(x+y) + \cos(x)$

$f_y = -\sin(x+y) + \cos(y)$

$f_{xx} = -\cos(x+y) - \sin(x)$

$f_{xy} = -\cos(x+y)$

$f_{yy} = -\cos(x+y) - \sin(y)$

$$\begin{pmatrix} -\sin(x+y) + \cos(x) \\ -\sin(x+y) + \cos(y) \end{pmatrix} = \vec{0}$$

$\cos(x) = \cos(y)$

Weil wir im Intervall $0; \frac{\pi}{2}$ sind folgt daraus, dass $x = y$ ist.

$-\sin(2x) + \cos(x) = 0$

$\cos(x) = \sin(2x)$

$\sin(x + \frac{\pi}{2}) = \sin(2x)$

$x_1 = \frac{\pi}{2}$

$x_2 = \frac{\pi}{6}$

Interessante Stellen: $(\frac{\pi}{2}; \frac{\pi}{2})$; $(\frac{\pi}{6}, \frac{\pi}{6})$

$(\frac{\pi}{6}; \frac{\pi}{6})$

-1

$$\begin{pmatrix} -\cos(\frac{\pi}{3}) - \sin(\frac{\pi}{6}) & -\cos(\frac{\pi}{3}) \\ -\cos(\frac{\pi}{3}) & -\cos(\frac{\pi}{3}) - \sin(\frac{\pi}{6}) \end{pmatrix} = \begin{pmatrix} -1 & -\frac{1}{2} \\ -\frac{1}{2} & -1 \end{pmatrix}$$

$\frac{3}{4}$

$\Rightarrow$ neg def $\Rightarrow$ Maximum

$f(\frac{\pi}{6}, \frac{\pi}{6}) = 1{,}5$

$\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$: am Rand des Def-Bereichs.

Kann es ein glob Extremum sein? nein!

$f\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ ist 1 und somit $< 1{,}5$

Ränder: $y = 0$ und $f_x = 0 \Rightarrow \sin x = \cos x$ $x = \frac{\pi}{4}$ die einzige Mögl. in $\left[0; \frac{\pi}{2}\right]$

$f\left(\frac{\pi}{2}, 0\right) = \sqrt{2} < 1{,}5$

$y = \frac{\pi}{2}$ und $f_x = 0$ $-\sin\left(x + \frac{\pi}{2}\right) + \cos(x = 0$

$\sin\left(x + \frac{\pi}{2}\right) = \sin\left(x + \frac{\pi}{2}\right)$

$\Rightarrow$ kein Anstieg, alle Werte für $y = \frac{\pi}{2}$ sind gleich

für x: äquivalent.

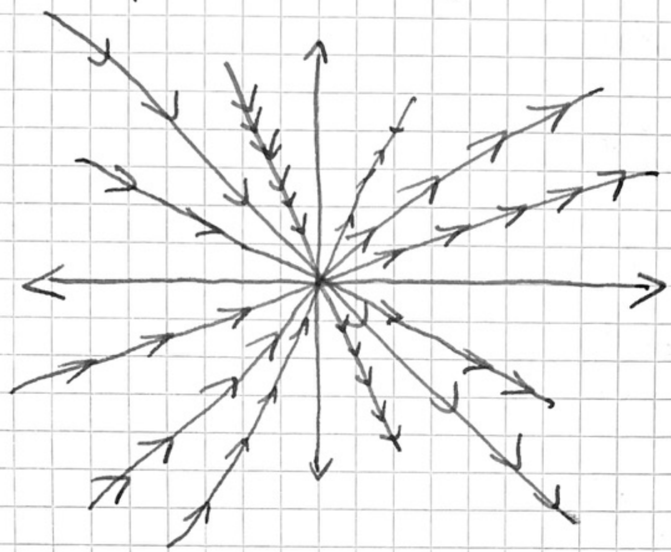
$x = y = 0$: $f(0,0) = 1$

3.73) $y' = \frac{y}{x}$

1. Mediane (ohne $x=0$) : 1
2. Mediane (ohne $x=0$) : -1

$y = 2x : 2$ $\quad y = -2x : -2$

$y = \pm \frac{1}{2} x : \pm \frac{1}{2}$



Es ist für jeden Punkt ~~mit~~ (x, y) mit $x \neq 0$ ist es eindeutig definiert, sonst ~~...~~ ist es undefiniert

382) $y' + \frac{1}{1-x} y = x^2 \qquad y(0) = 1$

homogene gl:

$y' + \frac{1}{1-x} \cdot y = 0$

$\frac{y'}{y} = \frac{1}{x-1}$

$\ln|y| = \int \frac{1}{x-1}\, dx$

$\ln|y| = \ln|x-1| + \tilde{C}$

$\cancel{y_h(x) = x-1 \cdot \tilde{C}}$

$y_h(x) = \tilde{C} \cdot (x-1)$

partikulärlösung:

$y_p(x) = C(x) \cdot (x-1)$

$y_p' = \frac{1}{1-x}\, y_p = x^2$

$C'(x) \cdot (x-1) + C(x) + \frac{1}{1-x} C(x) \cdot (x-1) = x^2$

$C'(x) \cdot (x-1) + C(x) - C(x) = x^2$

$C'(x) = \frac{x^2}{x-1}$

$u = x-1 \qquad \frac{du}{dx} = 1$

$C(x) = \int \frac{x^2}{x-1}\, dx$

NR: $\int \frac{x^2}{x-1}\, dx = \int \frac{(u+1)^2}{u}\, du =$

$= \int u\, du + \int 2\, du + \int \frac{1}{u}\, du =$

$= \frac{u^2}{2} + 2u + \ln|u| = \frac{x^2-2x+1}{2} + 2x - 1 + \ln|x-1| =$

$= \frac{x^2}{2} + x + \ln|x-1| + \tilde{C}$

$C(x) = \frac{x^2}{2} + x + \ln|x-1| + \tilde{C}$

$y_p(x) = \left(\frac{x^2}{2} + x + \ln|x-1| + \tilde{C}\right)(x-1)$

$(0 + 0 + \ln 1 + \tilde{C})(-1) = 1$

$\tilde{C} = -1$

$y(x) = \left(\frac{x^2}{2} + x + \ln|x-1| - 1\right) \cdot (x-1)$

384) $y' = \sin^2 x \cos^2 y$

$$\frac{y'}{\cos^2 y} = \sin^2 x$$

$$\int \frac{y'}{\cos^2 y}\,dx = \int \sin^2 x\,dx$$

$$\int \frac{\frac{dy}{dx}}{\cos^2 y}\,dx =$$

$$= \int \frac{dy}{\cos^2 y} = \tan y + C$$

$f(x) = \sin x \quad f'(x) = \cos x$

$g'(x) = \sin x \quad g(x) = -\cos x$

$$\int \sin^2 x\,dx = -\sin x \cos x - \int -\cos x \cos x\,dx =$$

$$= -\sin x \cos x + \int \cos^2 x\,dx =$$

$$= -\sin x \cos x + \int 1\,dx - \int \sin^2 x\,dx =$$

$$= -\sin x \cos x + x - \int \sin^2 x\,dx$$

$$\int \sin^2 x\,dx = -\sin x \cos x + x - \int \sin^2 x\,dx$$

$$2\int \sin^2 x\,dx = -\sin x \cos x + x$$

$$\int \sin^2 x\,dx = \frac{-\sin x \cos x + x}{2} + C$$

$$\tan y = \frac{-\sin x \cos x + x}{2} + C$$

$$y(x) = \arctan\left(\frac{-\sin x \cos x + x}{2} + C\right)$$