192.067 VO Deductive Databases
March 18, 2022

	Matrikelnummer (student id)	Familienname (family name)	Vorname (first name)	

1.) Consider the following two databases:

$$D_1 = \{R(a,b), R(b,a)\}$$

$$D_2 = \{R(a,c), R(b,c), R(c,d)\}$$

Furthermore, consider the program P consisting of the following rules:

$$Q_1(Y,Y) \leftarrow R(X,Y)$$

$$Q_2(Y,X) \leftarrow R(X,Y)$$

$$Q_2(X,Z) \leftarrow Q_2(X,Y), R(Z,Y)$$

Compute the answer to the Datalog query (P, Q_1) over the database D_1.

Compute the answer to the Datalog query (P, Q_2) over the database D_2.

(12 points)

1)

Output of (P, Q_1) over D_1 is $\{(b,b), (a,a)\}$

2)

Output of (P, Q_2) over D_2 is $\{(c,a), (c,b), (d,c), (d,a), (d,b)\}$

2.) Consider a program P consisting of the following three rules:

$$b \leftarrow not\ b$$
$$b \leftarrow b$$
$$b \leftarrow not\ a$$

Present at least one stable model of P. Justify your answer (including the computation of the program reduct). (7 points)

Which one of the three rules should be deleted from P so that the resulting program P' has *no* stable models? Explain you answer. (5 points)

(12 points)

candidates

$M_1 = \emptyset$ $\quad M_3 = \{b\}$

$M_2 = \{a\}$ $\quad M_4 = \{c\}$

1)

P^{M_1}

$b \leftarrow$

$b \leftarrow b$

$b \leftarrow$

M_1 isn't model of P^{M_1}

P^{M_2}

$b \leftarrow$

$b \leftarrow b$

M_2 not a model

P^{M_3}

$b \leftarrow b$

$b \leftarrow$

M_3 model and minimal

-> stable

P^{M_4}

$b \leftarrow b$

M_4 model but not minimal

2) $b \leftarrow$ not a

P^{M_1}

$b \leftarrow$

$b \leftarrow b$

M_1 not a model

P^{M_2}

$b \leftarrow$

$b \leftarrow b$

M_2 not a model

P^{M_3}

$b \leftarrow b$

M_3 model but not minimal

P^{M_4}

$b \leftarrow b$

-"-

3.) Consider a program P consisting of the following rules:

$$c \leftarrow a$$
$$\cancel{a \leftarrow b, not\ d}$$
$$b \leftarrow c, not\ a$$
$$d \leftarrow not\ a$$

1) Present a nonempty set U_1 of atoms from P such that U_1 is unfounded w.r.t. $(P, \{\}, \{\})$.

2) Present a nonempty set U_2 of atoms from P such that U_2 is unfounded w.r.t. $(P, \overset{Pos}{\{a\}}, \overset{Neg}{\{\}})$. It should be the case that $U_1 \neq U_2$.

3) Justify your answer. **(12 points)**

1) use greatest unfounded set algorithm

1 $\{a, b, c, \cancel{d}\}$

2 $\{a, b, c\}$

2)

1. $\{b, \cancel{c}, d\}$
2. $\{b, d\}$

Check

$c \leftarrow a$ $\quad a \in U$ ✓

$a \leftarrow b, not\ d$ $\quad b \in U$ ✓

$b \leftarrow c, not\ a$ $\quad c \in U$ ✓

$d \leftarrow not\ a$ $\quad d \notin U$ ✓

Check

$c \leftarrow a$ $\quad c \notin U$ ✓

$a \leftarrow b, not\ d$ $\quad a \notin U$ ✓

$b \leftarrow c, not\ a$ $\quad a \in Pos$ ✓

$d \leftarrow not\ a$ $\quad a \in Pos$ ✓

4.) Consider an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ satisfying the following:

- $\Delta^{\mathcal{I}} = \{a, b, c\}$,
- $A^{\mathcal{I}} = \{a, b\}$ for the concept name A,
- $B^{\mathcal{I}} = \{a, c\}$ for the concept name B,
- $R^{\mathcal{I}} = \{(a, c), (b, a), (c, b)\}$ for the role name R, and
- $P^{\mathcal{I}} = \{(a, a), (b, b)\}$ for the role name P.

Compute the extension of $\cdot^{\mathcal{I}}$ for the following complex concepts (i.e. compute $C^{\mathcal{I}}$ for all complex concepts C listed below):

(1) $\neg A \sqcup \neg B$

(2) $A \sqcap \top$

(3) $\exists R.B$

(4) $\forall R.B$

(5) $\exists P.(\overbrace{A \sqcup \neg A}^{\{a,b,c\}})$

(6) $\forall R.(\overbrace{A \sqcap \neg A}^{\{\}})$

(12 points)

1)
$\{c, b\}$

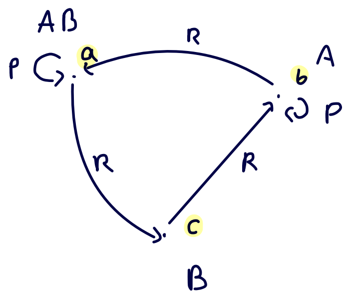
2)
$\{a, b\}$

3)
$\{a, b\}$

4)
$\{a, b\}$

5)
$\{a, b\}$

6)
$\{\}$



a

5.) By defining a suitable interpretation $\mathcal{I}$, show that the concept $B \sqcap (\exists R.(\exists R.\neg B))$ is satisfiable. We additionally require that $|\Delta^{\mathcal{I}}| = 2$, i.e., the domain of $\mathcal{I}$ contains exactly 2 elements. Here B is a concept name and R is a role name. **(12 points)**

$\Delta' = \{a, b\}$

$B' = \{b\}$

$R = \{(a,a), (b,a)$

$(\exists R \neg B)' = \{a, b\}$

$(\exists R.(\exists R \neg B))' = \{a, b\}$

$B \sqcap (\exists R.(\exists R.\neg B)) = \{b\} \cap \{a, b\} = \{b\}$