Analysis Übungsblatt 3 [redacted]

86)

Kreisgleichung $x^2+y^2=r^2$ $\quad$ $x^2+y^2=4$

$x = r\cdot\cos\varphi$

$y = r\cdot\sin\varphi$

$$\iint_{x^2+y^2=4} x\cdot e^{y}\,dx\,dy =$$

$$\int_0^4\int_0^{2\pi} r\cdot\cos\varphi\cdot e^{r\cdot\sin\varphi}\cdot\underbrace{\left|\det\begin{pmatrix}\cos\varphi & -r\cdot\sin\varphi\\ r\sin\varphi & r\cdot\cos\varphi\end{pmatrix}\right|}_{r}\,d\varphi\,dr$$

$$\int_0^4\int_0^{2\pi} r^2\cdot\cos\varphi\cdot e^{r\cdot\sin\varphi}\,d\varphi\,dr \qquad \text{subs} \quad \sin\varphi = u$$

$$\int_0^4\int_0^{2\pi} r^2\cdot\cos\varphi\cdot e^{ru}\,\frac{du}{\cos\varphi}\,dr$$

$$\int_0^4\int_0^{2\pi} r^2\cdot e^{ru}\,du\,dr \qquad \text{subs} \quad ru = z$$

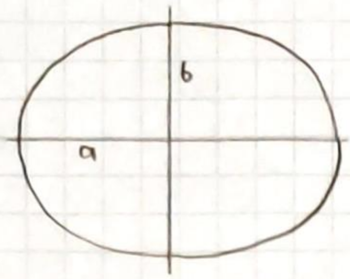
$$\int_0^4 r^2\int_0^{2\pi} e^{z}\,\frac{dz}{r}\,dr$$

$$\int_0^4 r\left(e^{r\sin\varphi}\Big|_0^{2\pi}\right)dr$$

$$\int_0^4 r\cdot(1-1)\,dr = 0$$

Analysis Übungsblatt 3 [redacted]

100)



Fläche Ellipse

$B\left\{(x,y) \mid \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1\right\}$

$B\left\{(r,\varphi) \mid 0 \le r \le 1,\ 0 \le \varphi \le 2\pi\right\}$ $\quad ar\cos\varphi = x$

$br\sin\varphi = y$

$$\int_0^1\int_0^{2\pi} 1 \cdot \left|\det\begin{pmatrix} a\cos\varphi & -ar\sin\varphi \\ b\sin\varphi & br\cos\varphi \end{pmatrix}\right| \, d\varphi \, dr$$

$$\int_0^1\int_0^{2\pi} a\cos\varphi\, br\cos\varphi + ar\sin\varphi\, b\sin\varphi \, d\varphi \, dr$$

$$\int_0^1\int_0^{2\pi} abr\left(\cos^2\varphi + \sin^2\varphi\right) \, d\varphi \, dr$$

$$\int_0^1 abr \int_0^{2\pi} 1 \, d\varphi \, dr$$

$$\int_0^1 abr \, 2\pi \, dr$$

$$ab\frac{r^2}{2} 2\pi \Big|_0^1 = ab\pi$$

Analysis Aufgabenblatt 3 [redacted]

108) $\sqrt{r^2 - x^2} = y$

$$2 \cdot 2\pi \int_0^r \sqrt{r^2 - x^2} \cdot \sqrt{1 + \left(\frac{-1}{2\sqrt{r^2 - x^2}} \cdot 2x\right)^2} \, dx$$

$$4\pi \int_0^r \sqrt{r^2 - x^2 \cdot 1 + \frac{x^2}{r^2 - x^2}} \, dx$$

$$4\pi \int_0^r \sqrt{r^2 - x^2 + \frac{r^2 x^2 - x^4}{r^2 - x^2}} \, dx$$

$$4\pi \int_0^r \sqrt{\frac{(r^2 - x^2)^2 + x^2 \cdot (r^2 - x^2)}{r^2 - x^2}} \, dx$$

$$4\pi \int_0^r \sqrt{(r^2 - x^2) + x^2} \, dx$$

$$4\pi \int_0^r \sqrt{r^2} \, dx$$

$$4\pi r \cdot x \Big|_0^r = \underline{\underline{4\pi r^2}}$$

Analysis Übungsblatt 3 [redacted]

117)

$$L = \int_0^5 \| c'(t) \| \, dt$$

$y(t) = \sqrt{t^3}$ $\quad x(t) = t$

$y' = \frac{3}{2} \cdot \sqrt{t}$ $\quad x' = 1$

$$\int_0^5 \sqrt{1 + \left(\frac{3}{2}\sqrt{t}\right)^2} \, dt$$

$$\int_0^5 \sqrt{1 + \frac{9}{4} t} \, dt \qquad \text{subs } u = 1 + \frac{9}{4} t$$

$$\frac{4}{9} \int_0^5 u^{\frac{1}{2}} = \frac{4}{9} \cdot \left(\frac{2u^{\frac{3}{2}}}{3} \Big|_0^5 \right) = \frac{4}{9} \cdot \left(\frac{2 \cdot 5^{\frac{3}{2}}}{3} - \frac{2}{3} \right) \approx \underline{12{,}407}$$

Analysis Übungsblatt 3 [redacted]

120)

Parametrisierung nach Bogenlänge

$x'(t) = 2 \cdot \sqrt{t+1}$

$y'(t) = t$

$$l(u) = \int_0^u \sqrt{4t+4+t^2}$$

$$l(u) = \int_0^u t+2 \quad dt$$

$$l(u) = \left(\frac{t^2}{2} + 2t \right)_0^u$$

$$l(u) = \frac{u^2}{2} + 2u \qquad u \in [0, \infty]$$

$$l = \frac{u^2}{2} + 2u \qquad u^{-1}(s) \to \quad s(l) = \sqrt{2l+4} - 2$$

$$2l = u^2 + 4u \quad | +4$$

$$2l+4 = u^2 + 4u + 4$$

$$2l+4 = (u+2)^2$$

$$u = \sqrt{2l+4} - 2$$

$$x(l) = \begin{pmatrix} \frac{4}{3} (s(l) + 1)^{\frac{3}{2}} \\ \frac{s(l)}{2} \end{pmatrix}$$

Analysis Übungsblatt 3 [redacted]

123 $f(x,y) = \frac{xy}{x^2+y^2}$ $\quad c(t) = (\cos t, \sin t) \quad 0 \le t \le \frac{\pi}{2}$

Kurven integral berechnen

$$\int_0^{\frac{\pi}{2}} \cos t \cdot \sin t \underbrace{\sqrt{\cos^2 t + \sin^2 t}}_{1} \, dt$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin(2t)}{2} \, dt$$

$$\frac{1}{2} \int_0^{\frac{\pi}{2}} \sin(2t) \, dt \qquad \text{subs } u = 2t$$

$$\frac{1}{4} \int_0^{\frac{\pi}{2}} \sin(u) \, du$$

$$\frac{1}{4} \cdot \left(-\cos(2t) \Big|_0^{\frac{\pi}{2}} \right) = \frac{1}{4} \cdot (1+1) = \frac{1}{2}$$