Analysis Übungsblatt 2 [redacted]

48)

$$f(x,y) = \cos(x+y) + \sin x + \sin y$$

$$f_x = -\sin(x+y) + \cos x$$

$$f_y = -\sin(x+y) + \cos y$$

$$-\sin(x+y) + \cos x = 0 \qquad x = \frac{\pi}{2} \qquad x = \frac{\pi}{6}$$

$$-\sin(x+y) + \cos y = 0 \qquad y = \frac{\pi}{2} \qquad y = \frac{\pi}{6}$$

$$f_{xx} = -\cos(x+y) - \sin(x)$$

$$f_{xy} = -\cos(x+y)$$

$$f_{yy} = -\cos(x+y) - \sin(y)$$

$$\det \begin{pmatrix} -\cos(x+y) - \sin x & -\cos(x+y) \\ -\cos(x+y) & -\cos(x+y) - \sin(y) \end{pmatrix} =$$

$$= (-\cos(x+y) - \sin(x)) \cdot (-\cos(x+y) - \sin y) - (-\cos(x+y))^2$$

für $\left(\frac{\pi}{2}, \frac{\pi}{2}\right) < 0 \Rightarrow$ Sattelpunkt

für $\left(\frac{\pi}{6}, \frac{\pi}{6}\right) > 0$

$f_{xx}\left(\frac{\pi}{6}, \frac{\pi}{6}\right) = -\cos\left(\frac{1}{8}\pi\right) - \sin\left(\frac{\pi}{6}\right) = -1 \Rightarrow$ Maximum

$f\left(\frac{\pi}{6}, \frac{\pi}{6}\right) = 1{,}01811$

Analysis Aufgabenblatt 2 [redacted]

48)

Randstellen

$f(0,y) = \cos(y) + \sin(y) > 1{,}01811$

y max für $\frac{\pi}{4}$ $\Rightarrow (0, \frac{\pi}{4})$ Max

$f(x,0) = \cos(x) + \sin(x) > 1{,}01811$

x Max für $\frac{\pi}{4}$ $\Rightarrow (\frac{\pi}{4}, 0)$ Max

$f(\frac{\pi}{2}, y) = \cos(\frac{\pi}{2} + y) + \sin(\frac{\pi}{2}) + \sin y > 1{,}01811$

$2 \cdot \sin y + 1 > 1{,}01811$

y max für $\frac{\pi}{2}$

$\Rightarrow$ gleich für $f(x, \frac{\pi}{2})$

$\Rightarrow$ Extremstellen: $(0, \frac{\pi}{4})$ $(\frac{\pi}{4}, 0)$ $(\frac{\pi}{2}, \frac{\pi}{2})$ $(\frac{\pi}{6}, \frac{\pi}{6})$

Analysis Aufgabenblatt 2 [redacted]

51) $f(x,y,z) = 2x^2 - 3xz^2 + y^3 + 3z^2 - 3y + 3$

$f_x = 4x - 3z^2 = 0$

$f_y = 3y^2 - 3 = 0$

$f_z = -6xz + 6z = 0$

$y = \pm 1$

$4x - 3z^2 = 0$

$z^2 = \frac{4x}{3}$

$z = \sqrt{\frac{4x}{3}}$

$-6x \cdot \sqrt{\frac{4x}{3}} + 6 \cdot \sqrt{\frac{4x}{3}} = 0$

$\frac{\sqrt{4x}}{\sqrt{3}}(-x+1) = 0$

$x = +1$

$4 \cdot 1 - 3z^2 = 0$

$z^2 = \frac{4}{3}$

$z = \sqrt{\frac{4}{3}}$

$-6xz + 6z = 0 \quad | : 6z$

$-x + 1 = 0$

$x = 1$

$-6xz + 6z = 0$

$z(-6x + 6) = 0$

$z = 0$

$4x = 0$

$x = 0$

stationäre Punkte
$(0, 1, 0)$
$(0, -1, 0)$
$(1, 1, \frac{2}{\sqrt{3}})$
$(1, -1, -\frac{2}{\sqrt{3}})$
$(1, 1, -\frac{2}{\sqrt{3}})$
$(1, -1, \frac{2}{\sqrt{3}})$

Hesse Matrix

$$\begin{pmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{pmatrix}$$

$f_{xx} = 4 \qquad f_{yx} = 0 \qquad f_{zx} = -6z$

$f_{xy} = 0 \qquad f_{yy} = 6y \qquad f_{zy} = 0$

$f_{xz} = -6z \qquad f_{yz} = 0 \qquad f_{zz} = -6x + 6$

Analysis Aufgabenblatt 2 [redacted]

51)

$$\begin{pmatrix} 4 & 0 & -6z \\ 0 & 6y & 0 \\ -6z & 0 & -6x+6 \end{pmatrix}$$

$|H_1| = 4 > 0$

$|H_2| = \begin{vmatrix} 4 & 0 \\ 0 & 6y \end{vmatrix} = 4 \cdot 6y = 24y$

$|H_3| = \begin{vmatrix} 4 & 0 & -6z \\ 0 & 6y & 0 \\ -6z & 0 & -6x+6 \end{vmatrix} \quad 24y(-6x+6) - 6^3 yz^2$

$(0,1,0)$ $|H_1| > 0$, $|H_2| > 0$, $|H_3| > 0$ -> positiv definit

↳ Minimum

$(0,-1,0)$ $|H_1| > 0$, $|H_2| < 0$, $|H_3| < 0$ -> indefinit

↳ Sattelpunkt

$(1, 1, \frac{2}{\sqrt{3}})$ $|H_1| > 0$, $|H_2| > 0$, $|H_3| < 0$ -> indefinit

$(1, -1, -\frac{2}{\sqrt{3}})$ $|H_1| > 0$, $|H_2| < 0$, $|H_3| > 0$ -> indefinit

$(1, 1, -\frac{2}{\sqrt{3}})$ $|H_1| > 0$, $|H_2| > 0$, $|H_3| < 0$ -> indefinit

$(1, -1, \frac{2}{\sqrt{3}})$ $|H_1| > 0$, $|H_2| < 0$, $|H_3| > 0$ -> indefinit

Analysis Aufgabenblatt 2 [redacted]

63) $V = \frac{1}{3} r^2 h \pi$

$NB = R^2 = (h-R)^2 + r^2$

$F(r,h,\lambda) = \frac{1}{3} r^2 h \pi - \lambda\left((h-R)^2 + r^2 - R^2\right)$

$F_r = \frac{2}{3} r h \pi - \lambda(2r) = 0 \quad \rightarrow \quad \lambda = \frac{\frac{2}{3} r h \pi}{2r}$

$F_h = \frac{1}{3} r^2 \pi - \lambda(2h - 2R) = 0$

$F_\lambda = -h^2 + 2hR - r^2 = 0 \qquad \rightarrow \frac{\frac{1}{3} r^2 \pi}{2h - 2R} = \lambda$

$$\frac{\frac{2}{3} r h \pi}{2r} = \frac{\frac{1}{3} r^2 \pi}{2h - 2R}$$

$$\frac{h}{3} = \frac{r^2}{6h - 6R}$$

$$r^2 = h \cdot (2h - 2R)$$

$$r = \sqrt{h(2h - 2R)}$$

$\rightarrow F_\lambda$ einsetzen $\quad -h^2 + 2hR - h(2h - 2R) = 0$

$$-3h^2 + 4hR = 0$$

$$-3h^2 + 4Rh + 0 = 0$$

$$\frac{-4R \pm \sqrt{(4R)^2}}{-6} \rightarrow \quad x_1 = 0 \rightarrow \text{geht nicht}$$

$$x_2 = \frac{4}{3} R$$

$\rightarrow r = \sqrt{h(2h - 2R)}$ einsetzen

$$r = \sqrt{\frac{4}{3} R \left(2 \cdot \frac{4}{3} R - 2R\right)}$$

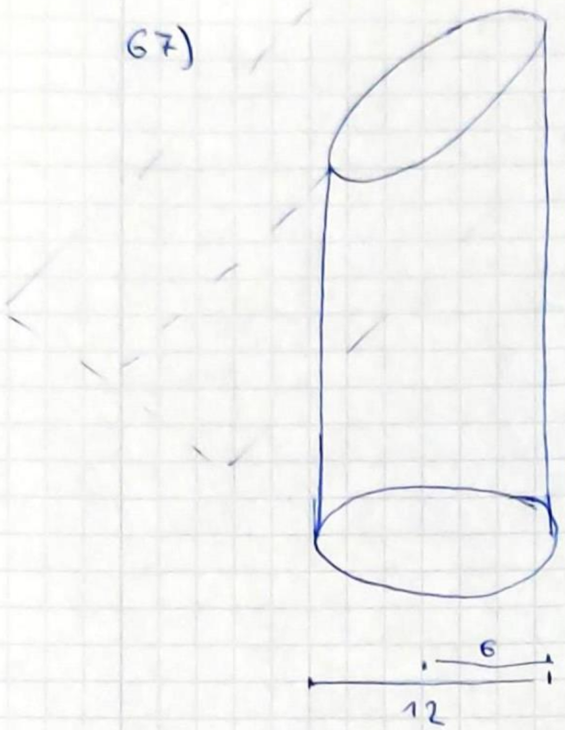
$$r = \sqrt{2 \cdot \frac{16 R^2}{9} - \frac{8}{3} R^2}$$

$$r = \sqrt{\frac{32}{9} R^2 - \frac{8}{3} R^2}$$

$$\underline{\underline{r = \frac{\sqrt{8}\, R}{3}}} \quad \text{und} \quad h = \frac{4}{3} R$$

Analysis Übungsblatt 2 [redacted]

67)



$z = x + 2y + 55$

Nebenbedingung: $x^2 + y^2 = r^2$

$x^2 + y^2 = 36$

$g(x,y) = x^2 + y^2 - 36$

$F(x,y) = x + 2y + 55 - \lambda(x^2 + y^2 - 36) = F(x,y,\lambda)$

$F_x = 1 - \lambda\, 2x \nearrow \frac{1}{2x}$

$F_y = 2 - \lambda\, 2y \searrow \frac{1}{y}$

$F_\lambda = -(x^2 + y^2 - 36) = -x^2 - y^2 + 36$

$\frac{1}{2x} = \frac{1}{y}$

$y = 2x$

$-x^2 - 4x^2 + 36 = 0$

$-5x^2 + 36 = 0$

$x = \sqrt{\frac{36}{5}}$ $\quad y = 2 \cdot \frac{6}{\sqrt{5}}$

$x = \frac{6}{\sqrt{5}}$ $\quad y = \frac{12}{\sqrt{5}}$

$z = \frac{6}{\sqrt{5}} + 2 \cdot \frac{12}{\sqrt{5}} + 55$

$z = \frac{30}{\sqrt{5}} + 55$

$z = 6 \cdot \sqrt{5} + 55$

$\underline{\underline{z \approx 68{,}42}}$ = Höhe

Analysis ~~[illegible]~~ Aufgabenblatt 2 [redacted]

72) $f(x\,y\,z) = x+y+z^2 \qquad x^2-y^2+z^2=1 \qquad x+y=1$

$g_1 = x^2-y^2+z^2-1$

$g_2 = x+y-1$

$F(x,y,z,\lambda_1,\lambda_2) = x+y+z^2 - \lambda_1(x^2-y^2+z^2-1) - \lambda_2(x+y-1)$

$F_x = 1-\lambda_1 \cdot 2x - \lambda_2 = 0$

$F_y = 1-\lambda_1 \cdot 2y - \lambda_2 = 0$

$F_z = 2z - \lambda_1 \cdot 2z = 0$

$g_1 = x^2-y^2+z^2-1 = 0$

$g_2 = x+y-1 = 0$

$x = \frac{1}{2}, \frac{1}{2}, 1$ Punkte: $(\frac{1}{2}, \frac{1}{2}, -1)$

$y = \frac{1}{2}, \frac{1}{2}, 0$ $(\frac{1}{2}, \frac{1}{2}, 1)$

$z = -1, 1, 0$

$\lambda_1 = 1$ $(1, 0, 0)$

$\lambda_2 = 0$

$F_x = F_y \Rightarrow x = y$

$x+x-1=0 \qquad y+y-1=0$

$x = \frac{1}{2} \qquad y = \frac{1}{2}$

$x^2-y^2+z^2-1=0$

$z = \pm 1$

$\frac{1}{2}, \frac{1}{2}, 1$

$\frac{1}{2}, \frac{1}{2}, -1$

[redacted]

$1 - \lambda_1 \cdot 2x - \lambda_2 = 0$

$-\lambda_1 \cdot 2x = \lambda_2 - 1$

$\lambda_1 = \frac{1-\lambda_2}{2x}$

$2z - \frac{1-\lambda_2}{2x} \cdot 2z = 0$

$1 - \frac{1-\lambda_2}{2x} = 0$

$2x - 1 + \lambda_2 = 0$

$x = \frac{1+\lambda_2}{2}$

$\frac{1+\lambda_2}{2} + y - 1 = 0$

$1 + \lambda_2 = (1-y) \cdot 2$

$\lambda_2 = (1-y) \cdot 2 - 1$

$1 - \lambda_1 \cdot 2y - ((1-y) \cdot 2 - 1) = 0$

$2 - \lambda_1 \cdot 2y - 2 + 2y = 0$

$2y(1 - \lambda_1) = 0$

$y = 0$

$x + 0 - 1 = 0$

$x = 1$

$1 - 0^2 + z^2 - 1 = 0$

$z = 0$

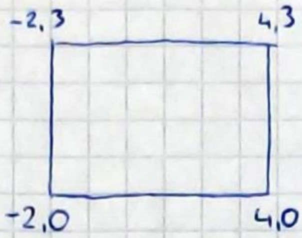
$\Rightarrow (1, 0, 0)$

Analysis Übungsblatt 2 [redacted]

76)

$$\iint_B e^{2x}\,(y+1) \qquad (-2,0),\ (4,0),\ (4,3),\ (-2,3)$$

$$B = (-2,4) \times (0,3)$$



$$\int_0^3 \int_{-2}^4 e^{2x}\,(y+1)\,dx\,dy$$

$$\int_0^3 \frac{1}{2}\cdot e^{2x}\cdot(y+1)\Big|_{-2}^{4}\,dy$$

$$\int_0^3 \frac{1}{2}\cdot e^{8}\cdot(y+1) - \left(\frac{1}{2}\cdot e^{-4}\cdot(y+1)\right)\,dy$$

$$\frac{1}{2}\cdot e^{8}\cdot\int_0^3 y+1\,dy - \frac{1}{2}\cdot e^{-4}\cdot\int_0^3 y+1\,dy \rightarrow \frac{y^2}{2}+y$$

$$\frac{1}{2}\cdot e^{8}\left(\frac{9}{2}+3\right) - \frac{1}{2}\cdot e^{-4}\left(\frac{9}{2}+3\right)$$

$$\underline{\frac{15}{4}\cdot\left(e^{8}-e^{-4}\right)}$$