337)

$$a u_x + b u_y = 1$$

$$\xi = bx + ay \qquad \eta = bx - ay$$

$$x = \frac{\xi + \eta}{2b} \qquad y = \frac{\xi + \eta}{2a}$$

$$U(\xi, \eta) = u(x, y)$$

$$1 = a \cdot (U_\xi \cdot \xi_x + U_\eta \cdot \eta_x) + b \cdot (U_\xi \cdot \xi_y + U_\eta \cdot \eta_y)$$

$$1 = a \cdot (U_\xi \cdot b + U_\eta \cdot b) + b \cdot (U_\xi \cdot a \,\cancel{+}\, U_\eta \cdot a)$$

$$1 = 2ab U_\xi$$

$$U(\xi, \eta) = \frac{1}{2ab} \int 1 \cdot d\xi + G(\eta)$$

$$= \frac{1}{2ab} \cdot \xi + G(\eta)$$

$$u(x,y) = \frac{1}{2ab} \cdot (bx + ay) + G(bx - ay)$$

$\rightarrow$ in Form $\frac{1}{a} x + C\left(y - \frac{b}{a} x\right)$

$$\frac{x}{2a} + \frac{y}{2b} + G(bx - ay)$$

Definiere $G(bx - ay) = C\left(-\left(\frac{bx}{a} - \frac{ay}{a}\right)\right) + \frac{x}{2a} - \frac{y}{2b}$

$$G(t) = C\left(-\frac{t}{a}\right) + \frac{x}{2a} - \frac{y}{2b}$$

$$\underline{\frac{x}{a} + C\left(y - \frac{b}{a} x\right)}$$

337) Anfangsbedingung

$$u(x=0, y) = y^2+1$$

$$u(0,y) = \frac{ay}{2ab} + G(-ay)$$

$$= \frac{y}{2b} + G(-ay)$$

definiere $G(t) = -\frac{ty}{a}$ setze $b = \frac{1}{2}y$

$$\frac{y}{y} + y^2 = \underline{1+y^2}$$

352)

$$x u_x + 2y u_y = 0$$

a) Rumpf DGL
↳ charakteristisches DGLsystem

$$x' = x$$

$$y' = 2y$$

Phasen DGL

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$\int \frac{1}{2y}\, dy = \int \frac{1}{x}\, dx$$

$$\frac{1}{2} \cdot \ln|y| = \ln|x| + C \qquad |\exp$$

$$e^{\frac{1}{2}\ln|y|} = e^{\ln|x|} \cdot C$$

$$\sqrt{y} = x \cdot C$$

$$C = \frac{\sqrt{y}}{x}$$ -> erstes Integral da konstant (C)

$$\Rightarrow u(x,y) = F\left(\frac{\sqrt{y}}{x}\right)$$ F beliebig

b) $$u\left(x, \frac{1}{x}\right) = F\left(\frac{\sqrt{\frac{1}{x}}}{x}\right) = F\left(\frac{1}{x\sqrt{x}}\right)$$

definiere $F(d) = d \cdot x \cdot \sqrt{x}$

$$\Rightarrow F\left(\frac{1}{x\sqrt{x}}\right) = x \qquad \Rightarrow F\left(\frac{\sqrt{y}}{x}\right) = \frac{\sqrt{y}}{x} \cdot x\sqrt{x}$$

$$= \sqrt{y} \cdot \sqrt{x}$$

362)

$$U_x + (y+2z)U_y + zU_z = 0$$

$x' = 1$

$y' = y + 2z$

$z' = z$

-> System der phasen DGL

$$\frac{dx}{dz} = \frac{1}{z}$$

$$\int dx = \int \frac{1}{z}\, dz$$

$x = \ln|z| + C_1$

$C_1 = x - \ln|z|$

$$\frac{dy}{dz} = \frac{y+2z}{z} \to y' = y \cdot \frac{1}{z} + 2$$

homogene Lsg : $y' - \frac{1}{z} y = 0$

$$\int \frac{1}{y}\, dy = \int \frac{1}{z}\, dz$$

$\ln y = \ln z$

$y_h = z \cdot C$

2 erste Integrale

$\varphi_1 = x - \ln|z|$

$$\varphi_2 = \frac{y - 2z\ln|z|}{z}$$

Lsg $U(x,y,z) = F(\varphi_1, \varphi_2)$

partikuläre Lsg $y' - \frac{1}{z} y = 2$

$z \cdot C(z) \to C'(z)\, z + C(z)$

$C'(z)\, z + C(z) - C(z) = 2$

$C'(z) = 2 \cdot \frac{1}{z}$

$C(z) = \int 2 \cdot \frac{1}{z}\, dz$

$C(z) = 2 \cdot \ln|z|$

$y_p = 2z\ln|z|$

$y = z \cdot C + 2z\ln|z|$

$$C = \frac{y - 2z\ln|z|}{z}$$

264)

$$(x^2+1)U_x - 2xy\,U_y + 2xU + 1 = 0$$

siehe 7.59

$$\Big((x^2+1)\xi_x + (-2xy)\xi_y\Big)U_\xi + \Big((x^2+1)\eta_x + (-2xy)\eta_y\Big)U_\eta + 2xU + 1 = 0$$

Rumpfdgl

$$(x^2+1)\,U_x - 2xy\,U_y = 0$$

Dglsystem

$$x' = x^2+1$$
$$y' = -2xy \quad \rightarrow$$

PhasenDgl

$$\frac{dx}{dy} = \frac{x^2+1}{-2xy} \iff x' = \frac{x^2+1}{-2xy}$$

$$x' = \frac{x}{-2y} + \frac{1}{-2xy}$$

$$x' = \frac{1}{-2y}\,x + \frac{1}{-2y}\cdot\frac{1}{x}$$

$$x' = \frac{1}{-2y}\left(x + \frac{1}{x}\right)$$

$\rightarrow$ nichtlineare DGL

$\Rightarrow$ Trennung d. Variablen

$$\frac{x'}{x+\frac{1}{x}} = \frac{1}{-2y} \quad \rightarrow \quad \int \frac{1}{x+\frac{1}{x}}\,dx = \int \frac{1}{-2y}\,dy$$

$$\frac{1}{2}\cdot\ln(x^2+1) = \frac{1}{2}\cdot -\ln(y) + C$$

$$\cancel{\ln}(x^2+1) = -\ln(y) + C \quad |\exp$$

$$x^2+1 = \frac{1}{y}\cdot C$$

$\ulcorner\rightarrow$ erstes Integral

$\Rightarrow$ $\underline{C = (x^2+1)y}$

264) wähle $\xi = (x^2+1)y$

$\xi_x = 2xy \qquad \xi_y = x^2+1$

einsetzen in

$(x^2+1)\xi_x + (-2xy)\xi_y$

$(x^2+1)\,2xy + -2xy\cdot(x^2+1) = 0$

$\Rightarrow$ DGL reduziert sich

$((x^2+1)\eta_x + (-2xy)\eta_y)\cdot U_\eta + 2xU + 1 = 0$

wähle $\eta = y$

$\Rightarrow$ DGL reduziert sich

$$-2xy\cdot U_\eta + 2xU + 1 = 0 \qquad \Big|: -2xy$$
$$U_\eta - \frac{1}{y}U - \frac{1}{2xy} = 0$$

Trennung der Variablen

$U_\eta - \frac{1}{\eta}U = 0$

$\frac{U_\eta}{U} = \frac{1}{\eta}$

$\ln U = \ln \eta$

$U = \eta \cdot C(\xi)$

$\Lsh$!

Variation d. Konstanten

$C'(\xi)y + C(\xi) - C(\xi) = \frac{1}{2xy}$

$C'(\xi) = \frac{1}{2x}$

$C(\xi) = \frac{1}{2x}\xi$

~~$U(x,\xi)$~~ =

$\Rightarrow U = y\cdot C(\xi) + y\cdot\frac{1}{2x}\xi$

Rücksubstitution

$U(x,y) = y\left(C((x^2+1)y) + \frac{1}{2x}(x^2+1)y\right)$

367)

$$u_t + u u_x = 0$$

Ansatz $\quad f(x,t,u) =$ konstant

Ableitung nach t und x von f Kettenregel

$$f_t + f_u \cdot u_t = 0 \qquad f_x + f_u \cdot u_x = 0$$

$$u_t = -\frac{f_t}{f_u} \qquad u_x = -\frac{f_x}{f_u}$$

einsetzen

$$-\frac{f_t}{f_u} + u \cdot -\frac{f_x}{f_u} = 0 \qquad \Big| :-f_u$$

$$f_t + u \cdot f_x = 0 \quad \to \text{Rumpf DGL}$$

charakteristische DGLsystem

$$t' = 1$$
$$x' = u$$

PhasenDGL

$$\frac{dx}{dt} = u \quad \to \quad dx = u \cdot dt \quad \Big| \int$$

$$\int dx = \int u \, dt$$

$$\underline{f(x,t,u) = F\left(x - ut\right) = \text{konst}} \qquad x = ut + c_1$$

$c_1 = x - ut \to$ erstes Integral

(Satz 7.51)