Analysis 2 Übungsblatt 4. [redacted]

130)

$$U(x) = \begin{pmatrix} x\, y^2 \\ x^2 - y \end{pmatrix}$$

Über $3y^2 = 4x$ $\quad x = \frac{3y^2}{4} \quad c(t) = \begin{pmatrix} \frac{3t^2}{4} \\ t \end{pmatrix}$

$(0,0) \to (3,2)$

$0 \leq t \leq 2 \quad c'(t) = \begin{pmatrix} \frac{3t}{2} \\ 1 \end{pmatrix}$

$$\int_0^2 u(c(t)) \cdot c'(t)\, dt$$

$$\int_0^2 \begin{pmatrix} \frac{3t^2}{4} \cdot t^2 \\ \left(\frac{3t^2}{4}\right)^2 - t \end{pmatrix} \cdot \begin{pmatrix} \frac{3t}{2} \\ 1 \end{pmatrix} dt$$

$$\int_0^2 \begin{pmatrix} \frac{3t^2}{4} \cdot t^2 \\ \frac{9t^4}{16} - t \end{pmatrix} \cdot \begin{pmatrix} \frac{3t}{2} \\ 1 \end{pmatrix} dt$$

$$\int_0^2 \frac{3t^4}{4} \cdot \frac{3t}{2} + \frac{9t^4}{16} - t\, dt$$

$$\int_0^2 \frac{9t^5}{8} + \frac{9t^4}{16} - t\, dt = \frac{9}{8}\frac{t^6}{6} + \frac{9}{16}\frac{t^5}{5} - \frac{t^2}{2} \Big|_0^2$$

$$= \frac{9}{8} \cdot \frac{2^6}{6} + \frac{9}{16} \cdot \frac{2^5}{5} - \frac{2^2}{2} = 12 + \frac{18}{5} - 2 = 13{,}6$$

Über $(0,0) \to (3,0) \to (3,2)$

$(0,0) \to (3,0) \quad c(t) = \begin{pmatrix} t \\ 0 \end{pmatrix} \quad t \in [0;3]$

$$\int_0^3 \begin{pmatrix} t \cdot 0 \\ t^2 - 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} dt = 0$$

$(3,0) \to (3,2) \quad c(t) = \begin{pmatrix} 3 \\ t \end{pmatrix} \quad t \in [0;2]$

$$\int_0^2 \begin{pmatrix} 3 \cdot t^2 \\ 9 - t \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} dt \quad \int_0^2 9 - t\, dt = 9t - \frac{t^2}{2}\Big|_0^2 = 18 - \frac{4}{2} - (0) = \underline{\underline{16}}$$

Analysis 2 Übungsblatt 4 [redacted]

142)

$$\int_C \cos x \, dx + e^{-y} \, dy + z^2 \, dz \qquad \text{wegunabhängig ?}$$

Integrabilitätskriterien:

$$\frac{\partial f_1}{\partial y} = 0 \qquad \frac{\partial f_2}{\partial x} = 0 \qquad \frac{\partial f_3}{\partial x} = 0$$

$$\frac{\partial f_1}{\partial z} = 0 \qquad \frac{\partial f_2}{\partial z} = 0 \qquad \frac{\partial f_3}{\partial y} = 0 \quad \checkmark$$

Umlaufintegral über den Einheitskreis

es muss gelten $\oint_C f(x) \, dx = F(c(b)) - F(c(a)) = 0$

$$\int_C \cos x \, dx + \int_C e^{-y} \, dy + \int_C z^2 \, dz =$$

$$= \sin x + -e^{-y} + \frac{z^3}{3} \Big|_C$$

$$= \underbrace{\sin(\cos(2\pi))}_{0} + \underbrace{-e^{-\sin(2\pi)}}_{-1} + \underbrace{\frac{1^3}{3}}_{\frac{1}{3}} - \Big(\underbrace{\sin(\cos(0))}_{0} + \underbrace{-e^{-\sin(0)}}_{+1} + \underbrace{\frac{1^3}{3}}_{-\frac{1}{3}} \Big)$$

$$= 0 \checkmark$$

Weg von $(-1,3,4) \rightarrow (6,9,-2)$

$$\left| \sin(-1) - e^{-3} + \frac{4^3}{3} - \left(\sin(6) - e^{-9} + \frac{(-2)^3}{3} \right) \right| = 23{,}39$$

Analysis 2 Übungsblatt 4 [redacted]

144)

$$u\begin{pmatrix}x\\y\end{pmatrix} = \left(\frac{2x}{1+(x^2+y)^2} \; ; \; \frac{1}{1+(x^2+y)^2} \right)$$

$$\left.\begin{aligned} \frac{du(x)}{dy} &= -\frac{4x(x^2+y)}{(1+(x^2+y)^2)^2} \\ \frac{du(y)}{dx} &= -\frac{4x(x^2+y)}{(1+(x^2+y)^2)^2} \end{aligned}\right\} \text{besitzt Stammfkt}$$

$$F = \int \frac{2x}{1+(x^2+y)^2}\,dx = \int \frac{2x}{1+u^2}\,\frac{du}{2x} = \int \frac{1}{1+u^2}\,du = \operatorname{atan}(x^2+y)$$

$$F = \operatorname{atan}(x^2+y) + d(y)$$

$$F_y = \frac{1}{1+(x^2+y)^2} \cdot 1 \quad \Rightarrow \quad d(y) = 0$$

$$F(c(t)) \rightarrow \quad F(c(2\pi)) - F(c(0)) \qquad c(t) = \begin{pmatrix}\cos t\\ \sin t\end{pmatrix}$$

$$\operatorname{atan}(\cos^2(2\pi) + \sin(2\pi)) - \operatorname{atan}(\cos^2(0) + \sin(0))$$

$$\operatorname{atan}(1 + 0) - \operatorname{atan}(1 + 0) = \underline{\underline{0}}$$

$\Rightarrow$ wegunabhängig

da Vektorfeld überall definiert ist in $\mathbb{R}^2$

$\rightarrow$ einfach zusammenhängend und überall wegunabhängig

Analysis 2 Übungsblatt 4 [redacted]

156)

$f(t) = t$ 2π periodisch $\rightarrow$ Fourierreihe

$$a_n = \frac{2}{T} \cdot \int_0^T f(t) \cdot \cos(n\omega t)\, dt$$

$$a_n = \frac{1}{\pi} \cdot \int_0^{2\pi} t \cdot \cos(nt)\, dt$$

$$a_n = \frac{1}{\pi} \cdot \left(t \cdot \frac{\sin(nt)}{n} - \int_0^{2\pi} \frac{\sin(nt)}{n}\, dt \right)$$

$$a_n = \frac{1}{\pi} \cdot \left(t \cdot \frac{\sin(nt)}{n} + \frac{\cos(nt)}{n^2} \right) \Big|_0^{2\pi}$$

$$a_n = 0$$

$$b_n = \frac{2}{T} \cdot \int_0^T f(t) \cdot \sin(n\omega t)\, dt$$

$$b_n = \frac{1}{\pi} \cdot \left(t \cdot \frac{-\cos(nt)}{n} + \frac{\sin(nt)}{n^2} \right) \Big|_0^{2\pi}$$

$$b_n = -\frac{2}{n}$$

$$f(t) = \sum_{n=1}^{N} -\frac{2}{n} \cdot \sin(nt)$$

$$c_k = \frac{a_k - i \cdot b_k}{2} = \frac{-i \cdot \frac{-2}{k}}{2} = \frac{i}{k}$$

$$f(t) = \sum_{k=-N}^{N} \frac{i}{k} \cdot e^{ikt}$$

Analysis 2 Übungsblatt 4 [redacted]

159) $f(t) = \cos(t) + |\cos(t)|$ 2π periodisch gerade Fkt

$\Rightarrow b_n = 0$

$$a_n = \frac{1}{\pi} \cdot \int_0^{2\pi} (\cos(t) + |\cos(t)|) \cdot \cos(nt)\, dt$$

$$a_n = \frac{1}{\pi} \cdot \int_0^{2\pi} \cos(t) \cdot \cos(nt) + |\cos(t)| \cdot \cos(nt)\, dt$$

$$a_n = \frac{1}{\pi} \cdot \Big(\underbrace{\int_0^{2\pi} \cos(t) \cdot \cos(nt)\, dt}_{\text{Teil 1}} + \underbrace{\int_0^{2\pi} |\cos(t)| \cdot \cos(nt)\, dt}_{\text{Teil 2}} \Big)$$

Teil 1:

$$\int_0^{2\pi} \cos(t) \cdot \cos(nt)\, dt$$

$$= \int_0^{2\pi} \frac{1}{2} \cdot (\cos(nt+t) + \cos(nt-t)\, dt$$

$$= \frac{1}{2} \cdot \left(\frac{\sin(nt+t)}{n+1} + \frac{\sin(nt-t)}{n-1} \right) \Bigg|_0^{2\pi}$$

$$= \frac{1}{2} \cdot \left(\frac{\sin(2\pi n + 2\pi)}{n+1} + \frac{\sin(2\pi n - 2\pi)}{n-1} - \frac{\sin(0)}{n+1} - \frac{\sin(0)}{n-1} \right)$$

$$= \frac{1}{2} \cdot \left(\frac{\sin(2\pi n) \cdot (n+1+n-1)}{n^2-1} \right) =$$

$$= \frac{n \cdot \sin(2\pi n)}{n^2-1}$$

Teil 2

$$\int_0^{2\pi} |\cos(t)| \cdot \cos(nt)\, dt$$

$$= \int_0^{\frac{\pi}{2}} \underbrace{\cos(t) \cdot \cos(nt)\, dt}_{1} - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \underbrace{\cos(t) \cdot \cos(nt)\, dt}_{2} + \int_{\frac{3\pi}{2}}^{2\pi} \underbrace{\cos(t) \cdot \cos(nt)\, dt}_{3}$$

1)

$$\frac{1}{2} \cdot \left(\frac{\sin(nt+t)}{n+1} + \frac{\sin(nt-t)}{n-1} \right) \Big|_0^{\frac{\pi}{2}}$$

$$\frac{1}{2} \cdot \left(\frac{\sin(\frac{\pi}{2}n + \frac{\pi}{2})}{n+1} + \frac{\sin(\frac{\pi}{2}n - \frac{\pi}{2})}{n-1} \right)$$

$$\frac{1}{2} \cdot \left(\frac{\cos(\frac{\pi}{2}n)}{n+1} - \frac{\cos(\frac{\pi}{2}n)}{n-1} \right)$$

$$\frac{1}{2} \cdot \left(\frac{\cos(\frac{\pi}{2}n) \cdot (n-1-(n+1))}{n^2-1} \right) = -\frac{\cos(\frac{\pi}{2}n)}{n^2-1}$$

2)

$$\frac{1}{2} \cdot \left(\frac{\sin(nt+t)}{n+1} + \frac{\sin(nt-t)}{n-1} \right) \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$\frac{1}{2} \cdot \left(\frac{\sin(\frac{3\pi}{2}n + \frac{3\pi}{2})}{n+1} + \frac{\sin(\frac{3\pi}{2}n - \frac{3\pi}{2})}{n-1} \right) + \frac{\cos(\frac{\pi}{2}n)}{n^2-1}$$

$$\frac{1}{2} \cdot \left(\frac{-\cos(\frac{3\pi}{2}n)}{n+1} + \frac{\cos(\frac{3\pi}{2}n)}{n-1} \right) + \frac{\cos(\frac{\pi}{2}n)}{n^2-1}$$

$$\frac{1}{2} \cdot \left(\frac{\cos(\frac{3\pi}{2}n) \cdot (-n+1+n+1)}{n^2-1} \right) + \frac{\cos(\frac{\pi}{2}n)}{n^2-1}$$

$$\frac{\cos(\frac{3\pi}{2}n) + \cos(\frac{\pi}{2}n)}{n^2-1}$$

3)

$$\frac{1}{2}\cdot\left(\frac{\sin(nt+t)}{n+1}+\frac{\sin(nt-t)}{n-1}\right)\Bigg|_{\frac{3\pi}{2}}^{2\pi}$$

$$=\frac{n\cdot\sin(2\pi n)}{n^2-1}-\frac{\cos\left(\frac{3\pi}{2}n\right)}{n^2-1}$$

$$a_n=\frac{1}{\pi}\cdot(\text{Teil } 1+\text{Teil } 2)=\frac{1}{\pi}\cdot(\text{Teil } 1+1-2+3)$$

$$-\frac{\cos\left(\frac{3\pi}{2}n\right)}{n^2-1}$$

$$=\frac{1}{\pi}\cdot\left(\frac{n\cdot\sin(2\pi n)}{n^2-1}-\frac{\cos\left(\frac{\pi}{2}n\right)}{n^2-1}-\frac{\cos\left(\frac{3\pi}{2}n\right)+\cos\left(\frac{\pi}{2}n\right)}{n^2-1}+\frac{n\cdot\sin(2\pi n)}{n^2-1}\right)$$

$$=\frac{2}{\pi}\cdot\left(\underbrace{\frac{n\cdot\sin(2\pi n)}{n^2-1}}_{\text{für ganze } n=0}-\frac{\cos\left(\frac{\pi}{2}n\right)-\cos\left(\frac{3\pi}{2}n\right)}{n^2-1}\right)$$

$$\Rightarrow a_n=\frac{2}{\pi}\cdot\frac{\cos\left(\frac{\pi}{2}n\right)+\cos\left(\frac{3\pi}{2}n\right)}{1-n^2}$$

$$\Rightarrow \quad f(t)=\sum_{n=1}^{N}\frac{2}{\pi}\cdot\frac{\cos\left(\frac{3\pi}{2}n\right)+\cos\left(\frac{\pi}{2}n\right)}{1-n^2}\cdot\cos(nt)$$

$$c_k=\frac{a_k-i\cdot b_k}{2}=\frac{a_k}{2}=\frac{\cos\left(\frac{3\pi}{2}k\right)+\cos\left(\frac{\pi}{2}k\right)}{\pi\cdot(1-n^2)}$$

$$f(t)=\sum_{k=-N}^{N}\frac{\cos\left(\frac{3\pi}{2}k\right)+\cos\left(\frac{\pi}{2}k\right)}{\pi\cdot(1-n^2)}\cdot e^{ikt}$$

Analysis 2 Übungsblatt 4 [redacted]

170)

i) $f(t) = \text{gerade}$

$$b_n = \frac{2}{T} \int_0^T \underbrace{f(t) \cdot \sin(n\omega t)\, dt}_{\text{ungerade}}$$

$$= \frac{1}{\pi} \cdot \int_0^{2\pi} f(t) \cdot \sin(nt)\, dt$$

$$= \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(t) \cdot \sin(nt)\, dt$$

$$= \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} f(t) \cdot \sin(nt)\, dt + \int_0^{\pi} f(t) \cdot \sin(nt)\, dt \right)$$

da $-f(t) = f(-t) \rightarrow f(t) = -f(-t)$

$$= \frac{1}{\pi} \cdot \left(- \int_0^{\pi} f(t) \cdot \sin(nt)\, dt + \int_0^{\pi} f(t) \cdot \sin(nt)\, dt \right)$$

$= 0$ ✓

ii) $f(t)$ ungerade

$$a_n = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} \underbrace{f(t) \cdot \cos(nt)\, dt}_{\text{ungerade}}$$

$$= \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} f(t) \cdot \cos(nt)\, dt + \int_0^{\pi} f(t) \cdot \cos(nt)\, dt \right)$$

da $-f(-t) = f(t) \rightarrow -f(t) = f(-t)$

$$= \frac{1}{\pi} \cdot \left(- \int_0^{\pi} f(t) \cdot \cos(nt)\, dt + \int_0^{\pi} f(t) \cdot \cos(nt)\, dt \right)$$

$= 0$ ✓