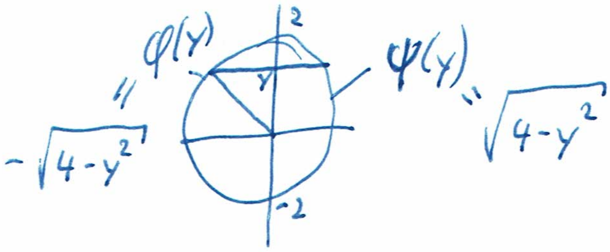
86.

$$I := \iint_B x \cdot e^y \, dx \, dy \quad , \quad B = \{(x,y) \mid x^2 + y^2 \le 2^2 = 4\}$$



$$\Rightarrow I = \int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x \cdot e^y \, dx \, dy = \int_{-2}^{2} e^y \cdot \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x \cdot dx \cdot dy =$$

$$= \underbrace{\int_{-2}^{2} e^y \cdot \int_{0}^{\sqrt{4-y^2}} x \cdot dx \, dy}_{=: I_1} + \underbrace{\int_{-2}^{2} e^y \cdot \int_{-\sqrt{4-y^2}}^{0} x \, dx \cdot dy}_{=: I_2}$$

$u = -x \Rightarrow du = -dx$

x	u
$-\sqrt{4-y^2}$	$\sqrt{4-y^2}$
0	0

$$I_2 = \int_{-2}^{2} e^y \cdot \int_{\sqrt{4-y^2}}^{0} (-u) \cdot (-du) \cdot dy = -\int_{-2}^{2} e^y \int_{0}^{\sqrt{4-y^2}} u \cdot du \, dy = -I_1$$

$$\Rightarrow I = I_1 - I_1 = 0$$

100.)

Ellipse $\mathcal{E}: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Fläche $A = \iint_B 1 \, dx\,dy$, $\quad B = \{(x,y) \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\}$

—

Transformation: $\quad x = a \cdot r \cdot \cos\varphi$
$\quad y = b \cdot r \cdot \sin\varphi$

$\Rightarrow B' = \{(r,\varphi) \mid 0 \leq r \leq 1, 0 \leq \varphi \leq 2\pi\}$

$\varphi(r,\varphi) = a \cdot r \cdot \cos\varphi$
$\psi(r,\varphi) = b \cdot r \cdot \sin\varphi$
$\quad : \begin{pmatrix} \varphi \\ \psi \end{pmatrix} : \tilde{B}' \to \tilde{B}$ bijektiv

$\tilde{B}' = \{(r,\varphi) \mid 0 < r \leq 1, 0 \leq \varphi < 2\pi\}$

$\tilde{B} = \{(x,y) \mid 0 < \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\}$

$\begin{pmatrix} \varphi \\ \psi \end{pmatrix}$ injektiv: klar

$\begin{pmatrix} \varphi \\ \psi \end{pmatrix}$ surjektiv: man kann inverse Abb. leicht angeben:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^2 \cdot \cos^2\varphi + r^2 \cdot \sin^2\varphi = r^2 \Rightarrow r^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$\frac{y}{x} = \frac{b \cdot \sin\varphi}{a \cdot \cos\varphi} = \frac{b}{a} \cdot \tan\varphi \Rightarrow \tan\varphi = \frac{a \cdot y}{b \cdot x}$$

—

Funktionaldet.: $\begin{vmatrix} x_r & x_\varphi \\ y_r & y_\varphi \end{vmatrix} = \begin{vmatrix} a \cdot \cos\varphi & -a \cdot r \cdot \sin\varphi \\ b \cdot \sin\varphi & b \cdot r \cdot \cos\varphi \end{vmatrix} = a b r \cos^2\varphi + a b r \cdot \sin^2\varphi$
$= a \cdot b \cdot r$

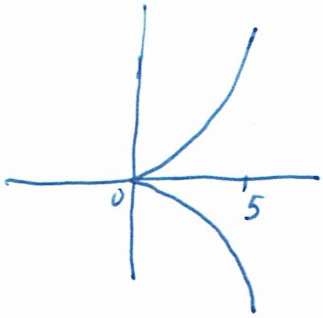
ad 100.1

$$\Rightarrow \quad A = \int_0^1 \int_0^{2\pi} \left| \begin{matrix} x_r & x_\varphi \\ y_r & y_\varphi \end{matrix} \right| \cdot d\varphi \, dr = \int_0^1 \int_0^{2\pi} a\, b \cdot r \cdot d\varphi \, dr =$$

$$= a\, b \cdot 2\pi \cdot \frac{r^2}{2} \Big|_0^1 = a \cdot b \cdot \pi$$

117.1 Bogenlänge der Kurve $\vec{c}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$,

$0 \le x(t) \le 5$, implizit durch $y^2 = x^3$ gegeben.



$$y = \pm x^{\frac{3}{2}}$$

Kurve segmentieren: $\vec{c} = \vec{c}_1 + \vec{c}_2$

$$\vec{c}_1(t) = \begin{pmatrix} t \\ t^{\frac{3}{2}} \end{pmatrix}, \quad 0 \le t \le 5$$

Parametrisierungen

$$\vec{c}_2(t) = \begin{pmatrix} t \\ -t^{\frac{3}{2}} \end{pmatrix}, \quad 0 \le t \le 5$$

Symmetrie:

$$L = 2 \cdot \int_0^5 \| \vec{c}_1{}'(t) \| \cdot dt =$$

$$c_1'(t) = \begin{pmatrix} 1 \\ \frac{3}{2} \cdot \sqrt{t} \end{pmatrix}$$

$$\|c_1'(t)\| = \sqrt{1 + \frac{9t}{4}}$$

$$= 2 \cdot \int_0^5 \sqrt{1 + \frac{9t}{4}} \cdot dt =$$

$$u = 1 + \frac{9t}{4}$$

$$du = \frac{9}{4} dt \Rightarrow dt = \frac{4}{9} du$$

t	u
0	1
5	$1 + \frac{45}{4} = \frac{49}{4}$

$$= 2 \cdot \int_1^{\frac{49}{4}} \sqrt{u} \cdot \frac{4}{9} du = \frac{8}{9} \cdot \int_1^{\frac{49}{4}} \sqrt{u}\, du =$$

$$= \frac{8}{9} \cdot \frac{2}{3} \cdot u^{\frac{3}{2}} \Big|_1^{\frac{49}{4}} = \frac{16}{27} \cdot \left(\left(\frac{49}{4} \right)^{\frac{3}{2}} - 1 \right) =$$

$$= \frac{16}{27} \cdot \left(\left(\frac{7}{2} \right)^3 - 1 \right) = \frac{16}{27} \cdot \left(\frac{343}{8} - 1 \right) = \frac{16}{27} \cdot \frac{335}{8} = \frac{670}{27} \approx 24.81$$

120.) $\vec{x}(t) = \begin{pmatrix} \frac{4}{3}\cdot(t+1)^{\frac{3}{2}} \\ t^2/2 \end{pmatrix}, \quad t \geq 0$

Parametrisieren nach der Bogenlänge

Bogenlänge bist:

$$s(t) = \int_0^t \|\vec{x}'(\tau)\| \cdot d\tau =$$

$$\vec{x}'(\tau) = \begin{pmatrix} 2\cdot\sqrt{\tau+1} \\ \tau \end{pmatrix}$$

$$\|\vec{x}'(\tau)\| = \sqrt{\tau^2 + 4\cdot(\tau+1)}$$

$$= \int_0^t \sqrt{\tau^2+4\cdot(\tau+1)}\cdot d\tau = \int_0^t \sqrt{(\tau+2)^2}\, d\tau =$$

$$= \int_0^t (\tau+2)\cdot d\tau = \frac{\tau^2}{2} + 2\tau \Big|_0^t = \frac{t^2}{2} + 2t$$

also: $s = \frac{t^2}{2} + 2t \quad \Rightarrow \quad t^2 + 4t - 2s = 0$

$$t_{1,2} = -2 \pm \sqrt{4+2s}$$

$$t = -2 + \sqrt{4+2s}, \quad s \geq 0$$

$$\Rightarrow \tilde{x}(s) = \begin{pmatrix} \frac{4}{3}\cdot\left(\sqrt{4+2s}-1\right)^{\frac{3}{2}} \\ \frac{\left(\sqrt{4+2s}-2\right)^2}{2} \end{pmatrix}, \quad s \geq 0$$

Parametrisierung nach Bogenlänge

123.) Kurvenintegral der skalarwert. Fkt. $f(x,y) = \frac{x \cdot y}{x^2+y^2}$,

längs Kurve $\vec{c}(t) = (\cos t, \sin t)$, $0 \leq t \leq \frac{\pi}{2}$.

$$I = \int_0^{\frac{\pi}{2}} f(x(t), y(t)) \cdot \|\vec{c}\,'(t)\| \cdot dt = \qquad \left| \quad \vec{c}\,'(t) = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} \right.$$

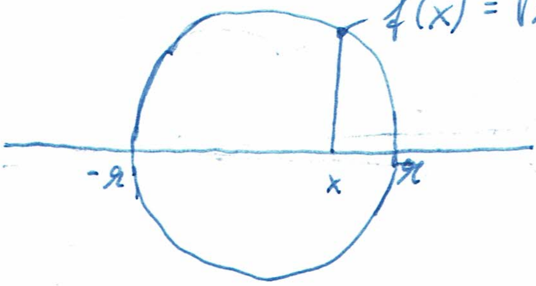
$$\|\vec{c}\,'(t)\| = \sqrt{\sin^2 t + \cos^2 t} = 1$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos t \cdot \sin t}{\cos^2 t + \sin^2 t}\, dt = \int_0^{\frac{\pi}{2}} \sin t \cdot \cos t \cdot dt =$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2} \cdot \sin(2t) \cdot dt = \frac{1}{2} \cdot \frac{(-\cos(2t))}{2} \Bigg|_0^{\frac{\pi}{2}} = -\frac{1}{4} \cdot \cos(2t) \Bigg|_0^{\frac{\pi}{2}} =$$

$$= -\frac{1}{4} \cdot \left(-1-1\right) = \frac{1}{2}$$

108.) Kugel mit Radius r als Rotationskörper interpretieren und Oberfläche berechnen



$$f(x) = \sqrt{r^2 - x^2} \Rightarrow f'(x) = \frac{-2x}{2 \cdot \sqrt{r^2 - x^2}} = -\frac{x}{\sqrt{r^2 - x^2}}$$

$$O = 2\pi \cdot \int_{-r}^{r} f(x) \cdot \sqrt{1 + (f'(x))^2} \cdot dx =$$

$$= 2\pi \cdot \int_{-r}^{r} \sqrt{r^2 - x^2} \cdot \sqrt{1 + \frac{x^2}{r^2 - x^2}} \cdot dx = 2\pi \cdot \int_{-r}^{r} \sqrt{r^2 - x^2} \cdot \frac{\sqrt{r^2}}{\sqrt{r^2 - x^2}} \, dx =$$

$$= 2\pi \cdot \int_{-r}^{r} r \cdot dx = 2\pi \cdot r \cdot x \Big|_{-r}^{r} = 2\pi \cdot r \cdot 2r = 4r^2 \cdot \pi$$