285)

$$y' + y\cos x = \sin x \cos x \qquad y(0) = 1$$

homogen

$$y' + y \cdot \cos x = 0$$

$$\frac{y'}{y} = -\cos x$$

$$\ln|y| = \int -\cos x \, dx + C_0$$

$$\ln|y| = -\sin(x) + C_0$$

$$y = e^{-\sin(x)} \cdot C$$

$$1 = e^{-\sin(0)} C + \sin(0) - 1$$

$$1 = C - 1$$

$$C = 2$$

$$\Rightarrow y(t) = 2e^{-\sin(x)} + \sin(x) - 1$$

inhomogen

$$y_p(x) = C(x)\, e^{-\sin(x)}$$

$$C'(x) \cdot e^{-\sin(x)} - \cos x \cdot C(x) \cdot e^{-\sin(x)} + \cos(x)\, C(x) \cdot e^{-\sin x} = \sin x \cos x$$

$$C'(x) = \sin x \cos x \cdot e^{\sin(x)}$$

$$C(x) = \int \frac{\sin x \cos x}{\cancel{\sin(2x)}} e^{\sin(x)} \, dx$$

$$C(x) = \int u \cdot \cos x \cdot e^{u} \frac{du}{\cos x} \qquad | u = \sin(x)$$

$$C(x) = \int \frac{\cos(x)}{\cos(x)} u \cdot e^{u} \, du$$

partiell

$$C(x) = \int u \cdot e^{u} \, du = \left(u \cdot e^{u} - \int e^{u} \, du \right)$$

$$C(x) = \left(u \cdot e^{u} - e^{u} \right) = \left(\sin(x) \cdot e^{\sin x} - e^{\sin x} \right)$$

$$\Rightarrow \text{Lsg} \quad y(x) = e^{-\sin(x)} \cdot C + \left(\sin(x) \cdot e^{\sin x} - e^{\sin x} \right) \cdot e^{-\sin x} = e^{-\sin x} C + \sin x - 1$$

287) $y'' - 3y' - 4y = 2x$

inhomogen

$\lambda^2 - 3\lambda - 4 = 0$ $\qquad \frac{3}{2} \pm \sqrt{\frac{9}{4} + 4} = \frac{3}{2} \pm \sqrt{\frac{25}{4}} = \frac{3}{2} \pm \frac{5}{2}$

$\lambda_1 = -1$

$\lambda_2 = 4$

$C_1 \cdot e^{-x} + C_2 \cdot e^{4x}$

partikulär

Ansatz $Ax + A_0$

$f' = A$ $\qquad -3A - 4Ax - 4A_0 = 2x$

$f'' = 0$ $\qquad A(-3-4x) - 4A_0 = 2x$

$-3A - 4xA - 4A_0 = 2x$

$x(\underbrace{-4A}) - 3A - 4A_0 = 2x$

$2 \Rightarrow A = -\frac{1}{2}$ $\qquad \frac{3}{2} - 4A_0 = 0$

$3 - 8A_0 = 0$

$A_0 = \frac{3}{8}$

$\Rightarrow$ Lsg $y(x) = C_1 \cdot e^{-x} + C_2 \cdot e^{4x} + -\frac{1}{2}x + \frac{3}{8}$

289)

$$y''' - 5y'' + 8y' - 4y = e^{2x}$$

homogen

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

$$\lambda_1 = 1$$

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 : (\lambda - 1) = \lambda^2 - 4\lambda + 4$$

$$\underline{\lambda^3 - \lambda}$$

$$-4\lambda + 8\lambda$$

$$\underline{-4\lambda + 4\lambda}$$

$$4\lambda - 4$$

$$\underline{4\lambda - 4}$$

$$0$$

$$\lambda^2 - 4\lambda + 4 \Rightarrow \lambda_2 = 2$$

$$\underline{C_1 e^x + (C_2 + C_3 x) \cdot e^{2x}}$$

inhomogen

Ansatz Ax^2e^{2x} 2x Resonanz

$$f' = 2Axe^{2x}(1+x)$$

$$f'' = 2Ae^{2x}(1+4x+2x^2)$$

$$f''' = 4Ae^{2x}(3+6x+2x^2)$$

$$4Ae^{2x}(3+6x+2x^2) - 5 \cdot 2Ae^{2x}(1+4x+2x^2) + 8 \cdot 2Axe^{2x}(1+x) - 4Ax^2e^{2x}) = e^{2x}$$

$$2Ae^{2x}(6 + 12x + 4x^2 - 5 - 20x - 10x^2 + 8x + 8x^2 - 2x^2) = e^{2x}$$

$$2Ae^{2x} = e^{2x}$$

$$\Rightarrow \text{Lsg } y(x) = C_1 e^x + (C_2 + C_3 x)e^{2x} + \frac{1}{2}x^2e^{2x} \qquad A = \frac{1}{2}$$

292)

289)

Ableitungen

$$\frac{\partial}{\partial x} Ax^2 \cdot e^{2x} = 2Axe^{2x} + 2Ax^2e^{2x} = 2Axe^{2x}(1+x)$$

$$\frac{\partial}{\partial x} 2Axe^{2x}(1+x) = 2A\left(e^{2x}(1+x) + xe^{2x} + 2xe^{2x}(1+x)\right)$$
$$= 2Ae^{2x}(1+4x+2x^2)$$

$$\frac{\partial}{\partial x} 2Ae^{2x}(1+4x+2x^2) = 2A \cdot \left((4+4x)e^{2x} + 2(1+4x+2x^2)e^{2x}\right)$$
$$= 4Ae^{2x}(3+6x+2x^2)$$

292)

$$\left(4x^3y^3 + \frac{1}{x}\right) dx + \left(3x^4y^2 - \frac{1}{y}\right) dy = 0$$

laut Angabe exakt

Stammfunktion finden

$$\int 4x^3y^3 + \frac{1}{x} \, dx = x^4y^3 + \ln|x| + C$$

$$\frac{d}{dy} x^4y^3 + \ln|x| + C = 3x^4y^2 + \frac{d}{dy} C$$

$$\frac{d}{dy} C = -\frac{1}{y}$$

$$C = \int -\frac{1}{y} \, dy = -\ln|y|$$

Stammfkt = $x^4y^3 + \ln|x| - \ln|y| = C$

Lösung implizit durch diese Glg gegeben

303)

$$(x-y)^2 \, dx + 2xy \, dy = 0$$

Methode des Integierenden Faktors $M = x^{-2}$

$$x^{-2}(x-y)^2 dx + \frac{2y}{x} dy$$

$$\frac{d}{dy}(Mf) = \frac{-2y}{x^2}$$

$$\frac{d}{dx}(Mf) = -\frac{2y}{x^2}$$

Integrabilitätsbedingungen erfüllt

Stammfunktion finden

$$\int \frac{1}{x} - \frac{y^2}{x^2} \, dx = \ln(x) + y^2 \cdot \frac{1}{x} + C$$

$$\frac{d}{dy} = \ln(x) + y^2 \cdot \frac{1}{x} = \frac{2y}{x} \Rightarrow C = 0$$

$$\ln(x) + y^2 \cdot \frac{1}{x} = C$$

$$y = \pm \sqrt{\frac{C - \ln(x)}{x}}$$