Aufgabe 6

Man berechne alle Ableitungen erster und zweiter Ordnung sowie die Jacobi-Matrix für die folgenden Funktionen:

a) $\boldsymbol{f(x,y,z) = x^2 sin(yz) + e^{x+y+z}}$

$$f_x = 2x\sin(yz) + e^{x+y+z}$$
$$f_y = x^2 z\cos(yz) + e^{x+y+z}$$
$$f_z = x^2 y\cos(yz) + e^{x+y+z}$$
$$f_{xx} = 2\sin(yz) + e^{x+y+z}$$
$$f_{yy} = -x^2 z^2\sin(yz) + e^{x+y+z}$$
$$f_{zz} = -x^2 y^2\sin(yz) + e^{x+y+z}$$
$$f_{xy} = 2xz\cos(yz) + e^{x+y+z}$$
$$f_{xz} = 2xy\cos(yz) + e^{x+y+z}$$
$$f_{yz} = x^2\cos(yz) - x^2 yz\sin(yz) + e^{x+y+z}$$

$$A = \begin{pmatrix} \frac{\delta f}{\delta x} \\ \frac{\delta f}{\delta y} \\ \frac{\delta f}{\delta z} \end{pmatrix} = \begin{pmatrix} 2x\sin(yz) + e^{x+y+z} \\ x^2 z\cos(yz) + e^{x+y+z} \\ x^2 y\cos(yz) + e^{x+y+z} \end{pmatrix}$$

b) $\boldsymbol{\vec{g}(t) = \begin{pmatrix} t^2 sin\, t \\ e^{3t} \\ \frac{1}{1+t^2} \end{pmatrix} = \begin{pmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{pmatrix}}$

$$\frac{\delta g_1}{\delta t} = 2t\sin t + t^2\cos t$$
$$\frac{\delta g_2}{\delta t} = 3e^{3t}$$
$$\frac{\delta g_3}{\delta t} = -\frac{2t}{(1+t^2)^2}$$
$$\frac{\delta^2 g_1}{\delta t^2} = 2\sin t + 2t\cos t + 2t\cos t - t^2\sin t$$
$$\frac{\delta^2 g_2}{\delta t^2} = 9e^{3t}$$
$$\frac{\delta^2 g_3}{\delta t^2} = -2(1+t^2)^2 - (-2t) * \frac{-2}{(1+t^2)^3} * 2t = -2(1+t^2)^2 - \frac{8t^2}{(1+t^2)^3}$$

$$A = \begin{pmatrix} \frac{\delta g_1}{\delta t} & \frac{\delta g_2}{\delta t} & \frac{\delta g_3}{\delta t} \end{pmatrix} = \begin{pmatrix} 2t\sin t + t^2\cos t & 3e^{3t} & -\frac{2t}{(1+t^2)^2} \end{pmatrix}$$

c) $\vec{\boldsymbol{h}}(\boldsymbol{x},\boldsymbol{y}) = \begin{pmatrix} \boldsymbol{x^2 sin\, y} \\ \boldsymbol{e^{x+2y}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{h_1(x,y)} \\ \boldsymbol{h_2(x,y)} \end{pmatrix}$

$$\frac{\delta h_1}{\delta x} = 2x \sin y$$
$$\frac{\delta h_1}{\delta y} = x^2 \cos y$$
$$\frac{\delta h_2}{\delta x} = e^{x+2y}$$
$$\frac{\delta h_2}{\delta y} = 2e^{x+2y}$$
$$\frac{\delta^2 h_1}{\delta x^2} = 2 \sin y$$
$$\frac{\delta^2 h_1}{\delta y^2} = -x^2 \sin y$$
$$\frac{\delta^2 h_1}{\delta x \delta y} = 2x \cos y$$
$$\frac{\delta^2 h_2}{\delta x^2} = e^{x+2y}$$
$$\frac{\delta^2 h_2}{\delta y^2} = 4e^{x+2y}$$
$$\frac{\delta^2 h_2}{\delta x \delta y} = 2e^{x+2y}$$

$$A(\vec{h}) = \begin{pmatrix} \frac{\delta h_1}{\delta x} & \frac{\delta h_2}{\delta x} \\ \frac{\delta h_1}{\delta y} & \frac{\delta h_2}{\delta y} \end{pmatrix} = \begin{pmatrix} 2x \sin y & e^{x+2y} \\ x^2 \cos y & 2e^{x+2y} \end{pmatrix}$$