

2021W Geometry for Computer Science

Exercise sheet B

Exercise 1. Given real constants r, p , consider the curve $\gamma: [0, 1] \rightarrow \mathbb{R}^3$ defined by

$$\gamma(t) = (r \cos(t), r \sin(t), pt).$$

1. Determine the length of γ ;
2. Determine a unit-speed reparametrization of γ .

Exercise 2. Let $\gamma: [0, 1] \rightarrow \mathbb{R}^3$ be a unit-speed curve, and let n be the principal normal vector of γ . Given $u \in \mathbb{R}$, consider the curve $\alpha: [0, 1] \rightarrow \mathbb{R}^3$ parametrized by

$$\alpha(t) = \gamma(t) + un(t).$$

Is α a regular curve?

Exercise 3. Consider the paraboloid \mathcal{P} of equation $z = x^2 + y^2$. Determine a regular parametrization of \mathcal{P} and compute the coefficients of the first fundamental form of your parametrization.

Exercise 4. Let $\mathbb{S}^2(r)$ be the sphere of radius r centered at the origin. Show that the normal curvature of any curve in $\mathbb{S}^2(r)$ is $\pm 1/r$.