M 2 f. WINF VU

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① Konsumentenrente für $x(p) = a + bp$

Formel f. Konsumentenrente $\int_{p_0}^{p_h} x(p)\, dp$

$$\int_{p_0}^{p_h} a + bp\ dp =$$

$$= \int_{p_0}^{p_h} a + \int_{p_0}^{p_h} bp\ dp = a \Big|_{p_0}^{p_h} + b\frac{p^2}{2}\Big|_{p_0}^{p_h} =$$

$$= \underline{\underline{a p_h - a p_0 + b\frac{p_h^2}{2} - b\frac{p_0^2}{2}}}$$

② Mittelwert der Grenzkosten für $k(t) = \alpha + \beta t$

$k_0 = k(0)$

$k(T) = k_1$

$$k(\bar{t}) = \frac{1}{T-0}\int_0^T k(t)\ dt$$

$$k(\bar{t}) = \frac{1}{T}\int_0^T \alpha + \beta t\ dt = \frac{1}{T}\left(\int_0^T \alpha\ dt + \int_0^T \beta t\ dt\right) =$$

$$= \frac{1}{T}\left(\alpha\Big|_0^T + \frac{\beta t^2}{2}\Big|_0^T\right) = \frac{1}{T}\left(\alpha T + \frac{\beta T^2}{2}\right) = \underline{\underline{\alpha + \frac{\beta T}{2}}}$$

$k(0) = \alpha$

$k(1) = \alpha + \beta T = k_0 + \beta T$

$\beta T = k_{(1)} - k_{(0)}$

$$\frac{\beta T}{2} = \frac{k(1) - k(0)}{2}$$

→ Entweder die Form wählen

$$\underline{k(\bar{t}) = k_0 + \frac{k(1) - k(0)}{2}}$$

3a) Gegeben: Grenzkosten $k(x)$ in Hinblick auf die Menge x + Fixkosten K_0 → Gesamtkosten $K(x) = \int k(x) + K_0$.
Man berechne die Gesamtkosten für
$k(x) = 6x^2 - 6x + 11 \quad K_0 = 5$

$$K(x) = \int (6x^2 - 6x + 11)\, dx + 5$$

$$2x^3 - 3x^2 + 11x + C + 5$$

$$K_0 = 5 \rightarrow \quad 5 = 0 - 0 + 0 + C + 5 \rightarrow \underline{\underline{C = 0}}$$

$$\underline{\underline{K(x) = 2x^3 - 3x^2 + 11x + 5}}$$

4a) $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\, dx \qquad u = 4 - x^2$

$$\frac{du}{dx} = 2x \quad \rightarrow dx = -\frac{du}{2x}$$

$$-\int \frac{x}{\sqrt{u}} \frac{du}{2x} = -\frac{1}{2}\int \frac{1}{\sqrt{u}} = -\frac{1}{2}\,\frac{u^{\frac{1}{2}}}{\frac{1}{2}} = -\frac{1}{2}\cdot 2\sqrt{u} =$$

$$= -\sqrt{u} = \underline{\underline{-\sqrt{4-x^2} + C}}$$

b) $\displaystyle\int_4^9 \frac{dx}{1+\sqrt{x}} = \int_2^3 \frac{2u}{1+u}\, du =$

$u = \sqrt{x} \quad \sqrt{9} = 3,\ \sqrt{4} = 2$

$\frac{du}{dx} = \frac{1}{2}\cdot x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$dx = du \cdot 2\sqrt{x}$

$dx = 2u\, du$

$$= 2\int_2^3 \frac{1+u-1}{1+u} = 2\int_2^3 1\, du - 2\int_2^3 \frac{1}{u+1}\, du =$$

$$= 2u\Big|_2^3 - 2\ln|u+1|\Big|_2^3 = \cancel{2\sqrt{x}\Big|_2^3 - 2\ln|\sqrt{x}+1|\Big|_2^3} =$$

$$\underline{\underline{6 - 4 - 2\ln|4| - 2\ln|3|}} \approx \underline{\underline{-2{,}97}}$$

5 b) $\int \tan x \, dx = -\int \frac{-\sin x}{\cos x} dx = -\ln|\cos x|$

$$\int \frac{f'}{f} dx = \ln|f|$$

6 b) $\phi(\tau) = ce^{\lambda\tau} \quad (c, \lambda > 0, \lambda \neq u)$

$$R(t) = \int_0^t ce^{\lambda\tau} e^{-u\tau} d\tau =$$

$$= c \int_0^t e^{(\lambda-u)\tau} d\tau = c \left. \frac{e^{(\lambda-u)\tau}}{\lambda-u} \right|_0^t$$

7 $\int \frac{dx}{(a-x)(b-x)} \Rightarrow \boxed{a \neq b}$

$$\frac{1}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

$$1 = A(x-b) + B(x-a)$$
$$1 = Ax - Ab + Bx - Ba$$

$$0 \cdot x^1 + 1 \cdot x^0 = (A+B)x - (Ab + Ba)x^0$$
$$\Downarrow$$

$$A + B = 0 \Rightarrow A = -B$$
$$-Ab - Ba = 1 \Rightarrow -Ab + Aa = 1 \Rightarrow A(a-b) = 1$$

$$A = \frac{1}{a-b} \qquad B = -\frac{1}{a-b}$$

$$\int \frac{dx}{(a-x)(b-x)} dx = \frac{1}{a-b} \int \frac{1}{x-a} dx - \frac{1}{a-b} \int \frac{1}{x-b} dx = \frac{1}{a-b} \ln\left|\frac{x-a}{x-b}\right|$$

(9) $w = \int_0^\infty z e^{-pt} dt = z \frac{e^{-pt}}{-p} \Big|_0^\infty =$ $\lim_{t \to \infty} e^{-pt} = \emptyset$

$= 0 + \frac{z}{p} = \frac{z}{p}$

(10) a) $f(x,y) = \sqrt{(x+2)^2 - y^2}$ $D = \mathbb{R}^2 \setminus |x+2| \geq y \geq -|x+2|$

$W = \mathbb{R}^+ = [0, \infty)$

Höhenlinien $f(x,y) = C$

$\sqrt{\cancel{x^2}(x+2)^2 - y^2} = C$

$(x+2)^2 - y^2 = C^2$

$\boxed{\frac{(x+2)^2}{C^2} - \frac{y^2}{C^2} = 1}$

b) $f(x,y) = \sqrt{1 - \frac{x^2}{9} - \frac{y^2}{16}}$ $1 - \frac{x^2}{9} - \frac{y^2}{16} \geq 0$

$D = \mathbb{R} \setminus \frac{x^2}{9} + \frac{y^2}{16} \geq 1$

$W = [0, 1]$

$\sqrt{1 - \frac{x^2}{9} - \frac{y^2}{16}} = C$

$1 - \frac{x^2}{9} - \frac{y^2}{16} = C^2$

$\frac{x^2}{9} - \frac{y^2}{16} = 1 - C^2$

$\frac{x^2}{9(1-C^2)} - \frac{y^2}{16(1-C^2)} = 1$

c) $f(x,y) = \arctan \frac{y-2}{x}$ $\quad D = \mathbb{R}$

$W = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \hat{=} (-90°, +90°)$

Höhenlinien: $\arctan \frac{y-2}{x} = C$

$$\frac{y-2}{x} = \tan C$$

$$\boxed{y = (\tan C)x + 2}$$

⑪ $z = f(x,y) = xy^2 - 10x$

Schnitt mit $x = x_0 \Rightarrow z = x_0 y^2 - 10x_0 = \boxed{x_0(y^2 - 10)}$

Schnitt mit $y = y_0 \Rightarrow z = x y_0^2 - 10x \Rightarrow \boxed{z = x(y_0^2 - 10)}$

Schnitt mit $z = z_0 \Rightarrow z_0 = xy^2 - 10x \Rightarrow \boxed{x = \frac{z_0}{y^2-10}}$

⑬ → 2 Seiten weiter

⑭ $x = a - bp_1 \quad ; \quad a = \frac{hp_2}{p_3} \quad ; \quad b = \frac{c + dp_3}{p_2}$

$$x = \frac{hp_2}{p_3} - \frac{p_1(c+dp_3)}{p_2} = x(p_1 p_2 p_3) \quad D = \mathbb{R} \setminus p_3 \geq 0; p_2 \geq 0$$

$$\frac{\partial x}{\partial p_1} = -\frac{c + dp_3}{p_2} < 0$$

$$\frac{\partial x}{\partial p_2} = \frac{h}{p_3} + \frac{(c+dp_3)\, p_1}{p_2^2} > 0$$

$$\frac{\partial x}{\partial p_3} = -\frac{hp_2}{p_3^2} - \frac{dp_1}{p_2} < 0$$

$$\varepsilon_{x_i} = \frac{x_i}{f} \cdot \frac{\partial f}{\partial x_i} \quad \text{für } i = 1, 2, \ldots, n$$

$$\varepsilon_{x, p_1} = \frac{p_1}{x} \cdot \left(-\frac{c+dp_3}{p_2}\right) = -\frac{p_1(c+dp_3)}{p_2 x}$$

(15) $K(v_1, v_2) = v_1 + \ln v_1 + v_2$ $\qquad \dfrac{\partial k}{\partial v_1} = 1 + \dfrac{1}{v_1}$

a) $\varepsilon_{K, v_1} = \dfrac{v_1}{K}\left(1 + \dfrac{1}{v_1}\right) = \dfrac{v_1}{k} + \dfrac{1}{k} = \boxed{\dfrac{v_1 + 1}{v_1 + \ln v_1 + v_2}}$

b) $\dfrac{\partial\left(\frac{K}{v_1}\right)}{\partial v_1} = \dfrac{1 - \ln v_1 - v_2}{v_1^2}$ analog

$\varepsilon_{\frac{K}{v_1}, v_1} = \dfrac{1 - \ln v_1 - v_2}{K}$

(16) Zeigen dass $dz = \underbrace{(2x + e^y)}_{p}dx + \underbrace{(x+1)e^y}_{q} dy$ total diff. bar

$\dfrac{\partial f}{\partial x} = 2x + e^y \Rightarrow f = \int (2x + e^y)\, dx + \varphi(y) =$

$= x^2 + e^y x + \varphi(y)$

$\dfrac{\partial f}{\partial y} = (x+1)e^y \stackrel{!}{=} e^y \qquad \varphi' = e^y \Rightarrow \int e^y dy = e^y$

$f = x^2 + e^y x + e^y$

Beweis $\dfrac{\partial p}{\partial y} = \dfrac{\partial q}{\partial x} \rightarrow e^y = e^y \rightarrow$ qed

$\dfrac{\partial p}{\partial y} = \dfrac{\partial q}{\partial x}$ Integrabilitäts Bed.

(13) $f(x,y) = x^2 \cos y + \sin(x+2y)$

$$\frac{\partial f}{\partial x} = 2x\cos y + \cos(x+2y)\cdot 1$$

$$\frac{\partial f}{\partial y} = -x^2 \sin y + \cos(x+2y)\cdot 2$$

$$\frac{\partial f}{\partial xy} = -2x\sin y - \sin(x+2y)\cdot 2$$

$$\frac{\partial f}{\partial yx} = -2x\sin y - \sin(x+2y)\cdot 2\cdot 1$$

$$\frac{\partial f}{\partial xx} = 2\cos y - \sin(x+2y)\cdot 1\cdot 1$$

$$\frac{\partial f}{\partial yy} = -x^2\cos y - \sin(x+2y)\cdot 2\cdot 2$$

(5) e) $\int \frac{dx}{\sin x} =$

$u = \tan\frac{x}{2}$

$\frac{du}{dx} = \frac{1}{\cos^2\frac{x}{2}}$

$dx = du \cos^2\frac{x}{2}$

$$= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} =$$

$$= \int \frac{du \cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \int \frac{du}{2}\,\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1}{2}\int\frac{du}{\tan\frac{x}{2}} =$$

$$= \frac{1}{2}\int\frac{du}{u} = \frac{1}{2}\ln\left|\tan\frac{x}{2}\right| + C$$

(18) a) $B = \left\{(x,y) \mid 0 \leq x \leq 1, -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right\}$

$$\underline{\iint_B (x + \cos y)\, dx\, dy}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 x + \cos y \quad dx\, dy =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2}{2}\Big|_0^1 \cancel{dy} + \cos y \; dy =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} + \cos y \quad dy = \frac{y}{2} + \sin y \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \underline{\underline{\frac{\pi}{2} + 2}}$$

b) $B = \left\{(x,y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\right\}$

$$\iint_B \left(x e^y + y e^x\right) dx\, dy$$

$$\int_0^1 \int_0^1 x e^y \, dx\, dy + \int_0^1 \int_0^1 y e^x \, dx\, dy$$

$$\int_0^1 \left(\frac{x^2}{2} e^y \Big|_0^1\right) dy + \int_0^1 \left(y e^x \Big|_0^1\right) dy =$$

$$= \int_0^1 \left(\frac{1}{2} e^y\right) dy + \int_0^1 (ye - y)\, dy =$$

$$= \frac{1}{2} e^y \Big|_0^1 + \frac{y^2}{2} e \Big|_0^1 - \frac{y^2}{2} \Big|_0^1 =$$

$$= \frac{1}{2} e - \frac{1}{2} + \frac{1}{2} e - \frac{1}{2} + 0 = \underline{\underline{e - 1}}$$

Lösung in VE: $\underline{\underline{1}}$?

19 a) $f(x,y) = x^2 + y^2$

$g(x,y) = \cos x + \sin y$

$h(x,y) = 2x + y + 1$

$F(x,y) = f(g,h)$ Kettenregel: $\frac{dF}{dx} = \sum_{i=1}^{n} \frac{\partial f}{\partial g_i} \cdot \frac{dg_i}{dx}$

$$f(g,h) = (\cos x + \sin y)^2 + (2x + y + 1))^2 = \underline{g^2 + h^2}$$

$$\frac{\partial F}{\partial y} = \frac{\partial F}{\partial g} \cdot \frac{\partial g}{\partial y} + \frac{\partial F}{\partial h} \cdot \frac{\partial h}{\partial y}$$

$\frac{\partial g}{\partial y} = \cos y \quad \frac{\partial h}{\partial y} = 1$ "innere Ableitung"

$$\underline{\frac{\partial F}{\partial y} = 2g(x,y)\cos y + 2h(x,y) \cdot 1}$$

$$\frac{\partial F}{\partial y}(0,0) = 2 \cdot 1 \cdot 1 + 2 \cdot 1 \cdot 1 = \underline{\underline{4}}$$

(19) b) $z^3 - x^2 - y^2 + z^2 - x - y - 1 = 0$

Grenzrate der Substitution

$$GRS_{yx} = \frac{-f_x}{f_y} = \frac{-\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

$$\frac{\partial f}{\partial x} = -2x - 1 \qquad GRS_{yx} = -\frac{2x+1}{2y+1}$$

$$\frac{\partial f}{\partial y} = -2y - 1$$

(20) $Y(A,K) = c\left((1-\alpha)A^{-r} + \alpha K^{-r}\right)^{-\frac{1}{r}}$ $\qquad c, \alpha, r > 0$

a) $S = \frac{dA}{dK} \hat{=} \; v = \frac{A}{K}$

b) Elastizität vers brgl. $v = r+1$

$$s = -\frac{Y_K}{Y_A}$$

$$\frac{\partial Y}{\partial K} = -\frac{c}{r}\left((1-\alpha)A^{-r} + \alpha K^{-r}\right)^{-\frac{1}{r}}\left(-r\alpha K^{-r-1}\right) =$$

$$= c\alpha\left((1-\alpha) + \alpha\left(\frac{A}{K}\right)^{r}\right)^{-\frac{1}{r}+1} \cdot \underbrace{A^{+r-1} \cdot K^{-r-1}}_{v^{r+1}} =$$

$$= c\alpha\left((1-\alpha) + \alpha v^{r}\right)^{-\frac{1}{r}-1} v^{r+1}$$

$$\frac{\partial y}{\partial k} = c\alpha\left((1-\alpha)+\alpha v^{r}\right)^{-\frac{r-1}{r}} v^{r-1}$$

$$\frac{\partial y}{\partial A} = \frac{c}{r}\left((1-\alpha)A^{-r}+\alpha k^{-r}\right)^{-\frac{1}{r}-1}\cdot\left((1-\alpha)A^{-r-1}(-r)\right) =$$

$$= c(1-\alpha)\left((1-\alpha)+\alpha v^{-r}\right)^{-\frac{r+1}{r}}\cdot\underbrace{A^{-r\left(\frac{r+1}{r}\right)}\cdot A^{-r+1}}_{A^0}$$

$$\Rightarrow S = \frac{-c\alpha\left((1-\alpha)+\alpha v^{r}\right)^{-\frac{r+1}{r}} v^{r-1}}{c(1-\alpha)\left((1-\alpha)+\alpha v^{r}\right)^{-\frac{r+1}{r}}} = -\frac{\alpha v^{r+1}}{1-\alpha}$$

$$\boxed{\text{NR: } Y = C\left((1-\alpha)A^{-r}+\alpha k^{-r}\right)^{-\frac{1}{r}}}$$

$$\frac{v}{S}\cdot\frac{dS}{dv} = \frac{v}{\frac{\alpha v^{r+1}}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}(r+1)v^{r}\right) = r+1$$

(21) $f(x,y) = x^2(y-1)+xe^{y^2}$

Formel: $f(x,y) \approx f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0) +$
$+\frac{1}{2}\left(\frac{\partial f}{\partial x^2}(x_0,y_0)(x-x_0)^2 + 2\frac{\partial f}{\partial xy}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y^2}(x_0,y_0)(y-y\right.$

$\frac{\partial f}{\partial x} = 2xy-2x+e^{y^2}$ $\qquad \frac{\partial f}{\partial x}(0,1) = e$

$\frac{\partial f}{\partial x^2} = 2y-2$ $\qquad \frac{\partial f}{\partial x^2}(0,1) = \emptyset$

$\frac{\partial f}{\partial y} = x^2+xe^{y^2}\cdot 2y$ $\qquad \frac{\partial f}{\partial y}(0,1) = \emptyset$

$\frac{\partial f}{\partial_2} = xe^{y^2}\cdot 2 + 4xe^{y^2}\cdot 2y = 2xe^{y^2}+4xy^2e^{y^2}$ $\qquad \frac{\partial f}{\partial x^2}(0,1) = \emptyset$

$$\frac{\partial f}{\partial x \partial y} = 2x + e^{y^2} \cdot 2y \qquad \frac{\partial f}{\partial x \partial y}(0,1) = 2e$$

linear:

$$f(x_0, y_0) \approx 0 + ex + 0 = ex$$

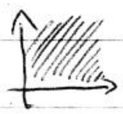
quadratisch:

$$f(x,y) \approx ex + \frac{1}{2}\left[0 + 2 \cdot 2ex(y-1) + 0\right] =$$

$$= \underline{\underline{ex + 2ex(y-1)}}$$

(21) $u = a_1 \ln(1+x_1) + a_2 \ln(1+x_2)$ $\qquad a_1, a_2 \geq 0$, $x_1, x_2 \geq 0$

Definitionsbereich $D = \{(x_1, x_2) \mid x_{1,2} \geq 0\}$



$\vec{x} = (x_1, x_2)$, $\vec{y}\,(y_1, y_2)$

$\vec{x} + \lambda(\vec{y} - \vec{x}) \in D$ für $\lambda \in [0,1]$

$x_1 + \lambda(y_1 - x_1)$, $x_2 + \lambda(y_2 - x_2)$

beide gleich, einen ansehen

$$x_1 + \lambda(y_1 - x_1) = x_1 + \lambda y_1 - \lambda x_1 = \underbrace{x_1}_{\geq 0}\underbrace{(1-\lambda)}_{\geq 0} + \lambda \underbrace{y_1}_{\geq 0}$$

$$f = -u = -a_1 \ln(1+x_1) - a_2 \ln(1+x_2)$$

$$f_{x_1} = \frac{-a_1}{1+x_1} \qquad f_{x_2} = \frac{-a_2}{1-x_2}$$

$$f_{x_1x_1} = \frac{a_1}{(1+x_1)^2} \qquad f_{x_2x_2} = \frac{a_2}{(1+x_2)^2}$$

$$f_{x_1x_2} = 0$$

$$|H| = \begin{vmatrix} \frac{a_1}{(1+x_1)^2} & 0 \\ 0 & \frac{a_2}{(1+x_2)^2} \end{vmatrix} = \frac{a_1 a_2}{(1+x_1)^2(1+x_2)^2} \geq 0$$

$\Rightarrow$ u konkav

streng konkav wenn $a_1, a_2 > 0$

streng konkav $\rightarrow$ rel Max $\Rightarrow$ einziges Max.

(23) $f(x,y) = x^4 + y^4 + 2x^2y^2 + x^2 + y^2$

Konvexitätsprüfung

$f_x = 4x^3 + 4xy^2 + 2y$
$f_y = 4y^3 + 4x^2y + 2y$
$f_{xx} = 12x^2 + 4y^2 + 2$
$f_{yy} = 12y^2 + 4x^2 + 2$
$f_{xy} = 8xy$

$$|H| = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} \underbrace{12x^2+4y^2+2}_{>0 \; ①} & 8xy \\ 8xy & 12y^2+4x^2+2 \end{vmatrix} =$$

$$= 96x^2y^2 + 48x^4 + 48y^4 + 32x^2 + 32y^2 + 4 \rightarrow \geq 0 \; ②$$

wegen ① und ② streng ~~konkav~~ konvex

(24) relative Extreme von
$$f(x,y) = x^4 + y^4 + 4x^2y^2 - 4x^2 + 4y^2$$

$f_x = 4x^3 + 8xy^2 - 8x$
$f_y = 4y^3 + 8x^2y + 8y$
$f_{xx} = 12x^2 + 8y^2 - 8$
$f_{yy} = 12y^2 + 8x^2 + 8$

$$\begin{cases} 4x^3 + 8xy^2 - 8x = 0 \rightarrow 4x(x^2 + 2y^2 - 2) = 0 \\ 4y^3 + 8x^2y + 8y = 0 \rightarrow 4y(y^2 + 2x^2 + 2) = 0 \end{cases}$$

$\rightarrow$ (1) = $x = 0$
(2) = $x^2 + 2y^2 - 2 = 0 \rightarrow x^2 = -2y^2 + 2$
(3) = $y = 0$
(4) = $y^2 + 2x^2 + 2 = 0 \rightarrow y^2 = -2x^2 - 2$

(1)+(3) = (0,0)

(1)+(4) = $y^2 = -2$ nicht in $\mathbb{R}$

(2)+(3) = $x^2 = 2 \rightarrow x = \pm\sqrt{2} \mid y = 0$

(2)+(4) = $x^2 = -2(-2x^2-2)+2 \rightarrow 3x^2 = -6$ nicht in $\mathbb{R}$

Punkte $A(\sqrt{2},0)$

$B(0,0)$

$C(-\sqrt{2},0)$

$$|H(0,0)| = \begin{vmatrix} -8 & 0 \\ 0 & 8 \end{vmatrix} = -64 \rightarrow \text{kein Extremum}$$

$$|H(\sqrt{2},0)| = \begin{vmatrix} 16 & 0 \\ 0 & 24 \end{vmatrix} = 16 \cdot 24 \rightarrow \text{rel. Extremum}$$

$$|H(-\sqrt{2},0)| = |H(\sqrt{2},0)|$$

(25) $K(x_1,x_2) = 3x_1^2 + 4x_1x_2 + x_2^2$

$x_1 = \frac{28}{3} - \frac{1}{3}p_1$

$x_2 = 11 - \frac{1}{2}p_2$ ges.: x_1 und x_2 für max. Gewinn

Erlös minus Kosten

$p_1x_1 + p_2x_2 - 3\left(\frac{28}{3} - \frac{1}{3}p_1\right) + 4\left(\frac{28}{3} - \frac{1}{3}p_1\right)\left(11 - \frac{1}{2}p_2\right) + \left(11 - \frac{1}{2}p_2\right)^2 =$

$p_1x_1 + p_2x_2 - 28 + p_1 + \left(\frac{112}{3} - \frac{4}{3}p_1\right)\left(11 - \frac{1}{2}p_2\right) + 121 + \frac{1}{4}p_2^2 - \frac{22}{2}p_2 =$

$p_1x_1 + p_2x_2 + 93 + \frac{1}{4}p_2^2 - 11p_2 + \frac{1232}{3} - \frac{44}{3}p_1 - \frac{112}{6}p_2 + \frac{4}{6}p_1p_2 =$

$p_1x_1 + p_2x_2 + \frac{1511}{3} + \frac{1}{4}p_2^2 - \frac{88}{3}p_2 + \frac{2}{3}p_1p_2 - \frac{44}{3}p_1$

25) $K(x_1, x_2) = 3x_1^2 + 4x_1x_2 + x_2^2$

$x_1 = \frac{28}{3} - \frac{1}{3}p_1$

$x_2 = 11 - \frac{1}{2}p_2$

$G(p_1, p_2) = p_1x_1 + p_2x_2 - K(x_1, x_2)$

$\frac{1}{3}p_1 = \frac{28}{3} - x_1 \rightarrow p_1 = 28 - 3x_1$

$\frac{1}{2}p_2 = 11 - x_2 \rightarrow p_2 = 22 - 2x_2$

$G(x_1, x_2) = (28 - 3x_1)x_1 + (22 - 2x_2)x_2 - K(x_1, x_2) =$

$= 28x_1 - 3x_1^2 + 22x_2 - 2x_2^2 - 3x_1^2 - 4x_1x_2 - x_2^2$

$\underline{G(x_1, x_2) = 28x_1 - 6x_1^2 + 22x_2 - 3x_2^2 - 4x_1x_2}$

$\frac{\partial f}{\partial x_1} = 28 - 12x_1 - 4x_2$ $\quad \frac{\partial f}{\partial x_1x_1} = -12$

$\frac{\partial f}{\partial x_2} = 22 - 6x_2 - 4x_1$ $\quad \frac{\partial f}{\partial x_2x_2} = -6$

$\frac{\partial f}{\partial x_1x_2} = -4$ $\quad \frac{\partial f}{\partial x_2x_1} = -4$

$28 - 12x_1 - 4x_2 = 0$

$4x_1 = 22 - 6x_2$

$28 - 66 + 18x_2 - 4x_2 = 0$

$14x_2 = 38$

$x_2 = \frac{18}{7}$

$x_1 = \frac{10}{7}$

$|H| = \begin{vmatrix} f_{x_1x_1} & f_{x_1x_2} \\ f_{x_2x_1} & f_{x_2x_2} \end{vmatrix} =$

$= \begin{vmatrix} -12 & -4 \\ -4 & -6 \end{vmatrix} = 72 - 16 = \underline{56} \rightarrow \underline{\text{pos}}$

(2b) $f(p_1,p_2) = (a_1 - b_1 p_1)\, p_2$ $\qquad a_1, a_2, b_1, b_2 > 0$

$f_2(p_1,p_2) = (a_2 - b_2 p_2)\, p_1$

$G = f_1 p_1 + f_2 p_2 = p_1 p_2 (a_1 + a_2 - b_1 p_1 - b_2 p_2)$

$\frac{\partial G}{\partial p_1} = 0 = p_2(a_1 + a_2 - 2 b_1 p_1 - b_2 p_2)$

$\frac{\partial G}{\partial p_2} = 0 = p_1(a_1 + a_2 - b_1 p_1 - 2 b_2 p_2)$

(1) $p_2 = 0$
(2) $a_1 + a_2 - 2 b_1 p_1 - b_2 p_2 = 0$
(3) $p_1 = 0$
(4) $a_1 + a_2 - b_1 p_1 - 2 b_2 p_2 = 0$

(1,3) sinnlos

(2)÷(4) $1 = \frac{b_1 p_1}{b_2 p_2} \rightarrow p_2 = \frac{b_1 p_1}{b_2}$ einsetzen

$p_1 = \frac{a_1 + a_2}{3 b_1} \qquad p_2 = \frac{a_1 + a_2}{3 b_2}$

$\cancel{G_{p_1 p_1} = p_2(-2b_1)}$

$\cancel{G_{p_2 p_2} = p_1(-2b_2)}$

$\cancel{G_{p_1 p_2} = (a_1 + a_2 - 2 b_1 p_1 - b_2 p_2) + p_2(-b_2)}$

$\cancel{|H| = \begin{vmatrix} p_2(-2b_1) & (a_1 \ldots) \\ (a_1 \ldots) & p_1(-2b_2) \end{vmatrix} = 4 b_1 b_2 - \frac{1}{p}(a_1 + a_2)^2}$

$G_{p_1^2}(p_1,p_2) = 2b_1 \frac{e_1+e_2}{b_2}$ $\qquad G_{p_2^2}(p_1,p_2) = -2b_2 \frac{e_1+e_2}{3b_1}$

$G_{p_1p_2} = -\frac{1}{3}(e_1+e_2)^2$

$$|H| = \frac{1}{3}(e_1+e_2)^2 \quad \underline{\underline{>0}}$$

27) $f(x,y) = xy(5-x-y) = 5xy - x^2y - xy^2$

$f_x = 5y - 2x^2y - y^2$
$f_y = 5x - x^2 - 2xy$

(1) $5-2x-y=0$
(2) $5-x-2y=0$
(3) $y=0$
(4) $x=0$
(1)+(2) $x=y=\frac{5}{3}$
(1)+(4) $x=0 \quad y=5 \qquad (0,5)$
(2)+(3) $(5,0)$
(3)+(4) $(0,0)$

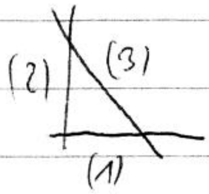
$f_{xx} = -2y \qquad f_{yy} = -2x \qquad f_{yx} = f_{xy} = 5-2x-2y$

$$|H| = \begin{vmatrix} -2y & 5-2x-2y \\ 5-2x-2y & -2x \end{vmatrix} \Rightarrow |H|\left(\frac{5}{3},\frac{5}{3}\right) = \frac{75}{9} > 0$$

$|H|(0,5) < 0 \to$ kein Extrempunkt
$|H|(0,0) < 0 \to$ kein Extrempunkt $\quad |H|(5,0) < 0 \to$ kein Extrempunkt

für $f_{xx}\left(\frac{5}{3},\frac{5}{3}\right) = -\frac{10}{3} \to$ Maximum

Nun noch Betrachtung der Ränder gem. Definitions-bereich:



(1) $f(x,0)=0$
(2) $f(0,y)=0$
(3) $f(x,5-x)=0$

keine Maxima an Rändern → abs. Max.

(28) Absolute Extrema von

$$f(x,y) = 3x^2-2x(y+1)+3y-1 =$$
$$= 3x^2-2xy+2x+3y-1 \qquad D=\{(x,y)\mid 0\le x,y\le 1\}$$

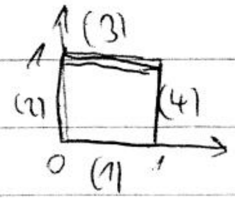
$f_x = 6x-2y+2$ $\qquad f_{xx} = 6$
$f_y = -2x+3$ $\qquad f_{yy} = 0$
$\qquad f_{xy}=f_{yx} = -2$

(1) $6x-2y+2=0$
$3x-y+1=0$

$|H| = \begin{vmatrix} 6 & -2 \\ -2 & 0 \end{vmatrix} = -4$ → kein rel. Extremum

(2) $2x+3=0$
$x=\frac{3}{2}$

→ Ränder ansehen!



	min	max.
(1)	$-\frac{4}{3}$	0
(2)	-1	2
(3)	$\frac{2}{3}$	2
(4)	0	1

(1) $y=0$

$g(x) = 3x^2 - 2x - 1$

$g' = 6x - 2 = 0 \quad \rightarrow x = \frac{1}{3}$

$g'' = 6 \qquad f\left(\frac{1}{3}, 0\right) = -\frac{4}{3}$

(2) $x = 0$

$f(0,y) = 3y - 1 = 0 \qquad y = \frac{1}{3}$

$f' = 3 \qquad f\left(0, \frac{1}{3}\right) =$

(3) $y = 1$

$f = 3x^2 - 2x + 2$

$f' = 6x - 4 = 0 \rightarrow x = \frac{2}{3} \qquad f\left(\frac{2}{3}, 1\right) = 3 \cdot \frac{4}{9} - 2\frac{2}{3} + 2 = \frac{12}{9} - \frac{8}{9} + 2 = \frac{2}{3} + 2$

$f'' = 6$

(4) $x = 1$

$f(1,y) = y \quad y = 0 \qquad f(1,0) = 0$

aus Tabelle

$f\left(\frac{1}{3}, 0\right) \Rightarrow$ MINIMUM

$f(0,1) \Rightarrow$ MAXIMUM

(24) Produktionsfunktion

$$y = f(x_1, x_2) = 5 - \frac{1}{\sqrt{x_1}} - \frac{1}{\sqrt{x_2}} \qquad x_1, x_2 > 0$$

$$G(x_1, x_2, p_0) = y p_0 - x_1 p_1 - x_2 p_2 \qquad p_0 = 3,\ p_1 = 1,\ p_2 = 12$$

$$G(x_1, x_2, 3) = 3\left(5 - \frac{1}{\sqrt{x_1}} - \frac{1}{\sqrt{x_2}}\right) - x_1 - 12x_2$$

$$\frac{\partial G}{\partial x_1} = \frac{3}{2} x_1^{-\frac{3}{2}} - 1 = 0 \rightarrow x_1 = \left(\frac{3}{2}\right)^{\frac{2}{3}}$$

$$\frac{\partial G}{\partial x_2} = \frac{3}{2} x_2^{-\frac{3}{2}} - 12 = 0 \rightarrow x_2 = \left(\frac{1}{8}\right)^{\frac{2}{3}}$$

$$\frac{\partial G}{\partial x_1^2} = -\frac{9}{4} x_1^{-\frac{5}{2}}$$

$$\frac{\partial G}{\partial x_2^2} = \frac{9}{4} x_2^{-\frac{5}{2}}$$

$$\frac{\partial G}{\partial x_1 x_2} = \emptyset$$

$$|H| = \begin{vmatrix} -\frac{3}{4} x_1^{-\frac{5}{2}} & 0 \\ 0 & -\frac{3}{4} x_2^{-\frac{5}{2}} \end{vmatrix} = \frac{81}{16} (x_1 x_2)^{-\frac{5}{2}} \overset{!}{>} 0$$

dann betrachten wir noch $\frac{\partial G}{\partial x^2}(x_1, x_2) < 0 \rightarrow$ Minimum

$$G(x_1, x_2) \approx 2{,}08$$

(3) Lagrange: $f(x,y) = x - y$

$NB: x^2 + y^2 = 1 \rightarrow x^2 + y^2 - 1 = 0$

$$\Phi(x,y,\lambda) = x - y + \lambda(x^2 + y^2 - 1)$$

$\Phi_x = 1 + 2x\lambda \rightarrow x = -\frac{1}{2\lambda}$

$\Phi_y = -1 + 2y\lambda \rightarrow y = \frac{1}{2\lambda}$

$\Phi_\lambda = x^2 + y^2 - 1$

$$x^2 + y^2 - 1 = 0$$

$$\left(-\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 - 1 = 0$$

$$\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 1$$

$$\frac{1}{2\lambda^2} = 1$$

$$\lambda = \pm \frac{1}{\sqrt{2}}$$

$x = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$y = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$P_1 = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$P_2 = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

③ $K(x_1, x_2) = 12x_1 + 6x_2 + 10$

NB: $2x_1^2 x_2 - 400 = \emptyset$

$\Phi = 12x_1 + 6x_2 + 10 + \lambda(2x_1x_2 - 400)$

$\Phi_\lambda = 2x_1^2 x_2 - 400$

$\Phi_{x_1} = 12 + 4\lambda x_1 x_2 = 0 \qquad \lambda x_1 x_2 = 3$

$\Phi_{x_2} = 6 + 2\lambda x_1^2 = 0 \qquad \lambda x_1^2 = -3$

$2x_1^2 x_1 = 400$

$x_1^3 = 200$

$x_1 = \sqrt[3]{200}$

$K = 12\sqrt[3]{200} + 6\sqrt[3]{200} + 10 = \underline{\underline{18\sqrt[3]{200} + 10}}$

(37) $u(x,y)$ und p, q

$px + qy = e$

$\frac{u_x}{u_y} \overset{!}{=} \frac{p}{q}$

$\Phi(x,y) = u(x,y) + \lambda(px + qy - e)$

$\Phi_x = U_x(x,y) + \lambda p = 0 \quad \rightarrow u_x = -\lambda p$

$\Phi_y = U_y(x,y) + \lambda e = 0 \quad \rightarrow u_y = -\lambda q$

$\Rightarrow \boxed{\frac{u_x}{u_y} = \frac{p}{q}}$

$\Phi_\lambda = p\lambda + qy = e$

(33) $y' = \sqrt[3]{y^2}$ Singuläre Lösung zeigen.

$y' = \frac{dy}{dx}$ $\frac{dy}{dx} = y^{\frac{2}{3}}$

$$\int y^{\frac{2}{3}} dy = \int dx$$

$$3y^{\frac{1}{3}} = x + C \Rightarrow y = \frac{1}{27}(x+C)^3$$

$y(x) = \emptyset$ auch Lösung

$y(x) = \emptyset$ kann nicht durch Spezialisierung von C erreicht werden → singuläre Lösung

D.h. man findet eine Lösung, die auch gilt, aber nicht hergeleitet werden kann.

(34) $y' = (1-y)\,y$ Gleichgewichtslösungen & Stabilität

$y_A(x) = 1 + \vartheta(x)$ $\vartheta(x)$ ... Auslenkung

$y_A'(x) = \vartheta'(x)$

in Gleichung einsetzen: $\vartheta'(x) = -\vartheta(x)\left(1+\vartheta(x)\right) \Rightarrow$

$$\vartheta'(x) = -\vartheta(x) - \vartheta^2(x)$$

Es genügt der Beweis der Gleichung f ϑ, weil $\vartheta^2(x)$ noch kleiner ist. Daher zeigen wir nur, dass $\vartheta(x)$ beliebig klein wird.
$\to\infty$

$$\frac{d\vartheta}{dx} = \vartheta \Rightarrow \int \frac{d\vartheta}{\vartheta} = \int dx$$

$$\ln|\vartheta| = -x + \ln|C| \Rightarrow \vartheta = e^{-x+\ln|C|} \Rightarrow$$

$$\underline{\vartheta = Ce^{-x}}$$

$$y_A(x) = 1 + Ce^{-x}$$

$$\lim_{x\to\infty} y_A(x) = \lim_{x\to\infty}\left(1 + \underbrace{Ce^{-x}}_{=0}\right) = 1 = y_1^*$$

asymptotisch stabil