Neat Stuff: Algorithmics VU

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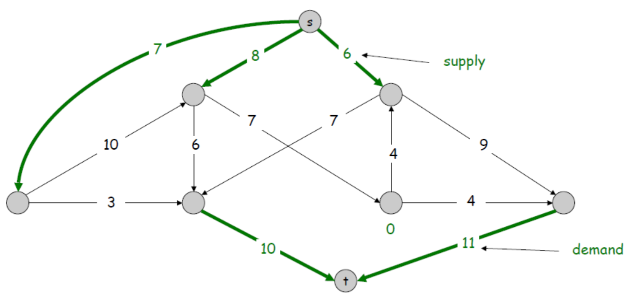
Flow.uωu

Flow value: $v(f) = \sum_{e \text{ out of } s} f(e)$

Capacity of (A, B): $cap(A, B) = \sum_{e \text{ out of } A} c(e)$

Residual: $E_f = \{e \colon f(e) < c(e)\} \cup \{e^R \colon f(e) > 0\}$

Model: Circulation with demands



Model: Circulation with demands and lower bounds

A^*-planarandomzazazazation

Adm. heur. h: $h(x) \leq h^*(x), \forall x \in V$, h^* optimal e.g. $h \in \{0, d_1, d_2\}$

Mon. heur. h: $h(x) \leq w_{xy} + h(y), \forall (x, y) \in E$, $h(t) = 0$ e.g. $h \in \{d_1, d_2\}$

Euler: G connected, planar $\Rightarrow |V| - |E| + f = 2$ and
G simple, planar $\Rightarrow |E| \leq 3|V| - 6$
G simple, planar, bipartite $\Rightarrow |E| \leq 2|V| - 4$

οιδα	running time	correctness
LasVegas	probable	certain
Monte Carlo	certain	probable

Vegas $\rightsquigarrow$ Carlo: Running algo $\mathcal{A}$ for some time and generate random answer when it fails to terminate
Vegas $\rightsquigarrow$ Carlo: no general answer but possible, e.g. Johnson's algo

mSOS

VC: $vc'(C) = \forall e \exists x (Ixe \wedge Cx)$
$\min_{C \subseteq V} |C|, G \models vc(C)$ mit $g = 0 - x_{11}, f_1^G \colon x \mapsto 1$

DS: $ds(D) = \forall x (Dx \vee \exists y (Dy \wedge Exy))$
$\min_{D \subseteq V} |D|, G \models vc(D)$ mit $g = 0 - x_{11}, f_1^G \colon x \mapsto 1$

IS: $is(N) = \forall x, y ((Nx \wedge Ny \wedge x \neq y) \rightarrow \neg Exy)$
$\min_{N \subseteq V} |N|, G \models vc(N)$ mit $g = x_{11}, f_1^G \colon x \mapsto 1$

CLQ: $clq(S) = \forall x, y ((Sx \wedge Sy \wedge x \neq y) \rightarrow Exy)$

Nice properties:
$adjacent(v, w) = \neg(v = w) \wedge (\exists_{e \in E}(Ive \wedge Iwe))$
$conn(X) = \forall_{v,w}((v \neq w \wedge Xv \wedge Xw) \rightarrow \exists_e (Ive \wedge Iwe))$
$induced(S) = \forall_e (Se \leftrightarrow \exists_{v,w}(Sv \wedge Sw \wedge Ive \wedge Iwe))$

Georock

DCEL: each edge of inner (outer) face oriented ↻ (↺)
$\forall\ f$: store incident half-edge e
$\forall\ e$: store next(e), face(e), prev(e), origin(e), twin(e)
$\forall\ v$: store its coordinates and an outgoing e

Voronoi diagram: $Vor(P)$, $\forall q \neq p, p'$ and $x \in \mathbb{R}^2$

Cell: $\mathcal{V}(\{p\}) = \mathcal{V}(p) = \{x : |xp| < |xq|\} = \bigcap_q h(p, q)$

Edge: $\mathcal{V}(p, p') = \{x : |xp| = |xp'|, |xp| < |xq|\}$

Vertex: $\mathcal{V}(p, p', p'') = \partial\mathcal{V}(p) \cap \partial\mathcal{V}(p') \cap \partial\mathcal{V}(p'')$

Prop.: $q^{\in \mathbb{R}^2}$ vertex in $Vor(P) \Leftrightarrow |\partial C_P(q) \cap P| \geq 3$

$b(p, q)^{p, q \in P}$ edge $\Leftrightarrow \exists r \in b(p, q) : \partial C_P(r) \cap P = \{p, q\}$

Dual Vor.: $\mathcal{G} = (P, E)$, $E = \{pq : adj(\mathcal{V}(p), \mathcal{V}(q))\}$

Prop.: $p, q \in P, adj(p, q) \Leftrightarrow \exists circle(p, q)^\circ = \emptyset$

$\mathcal{G}$ contains edge e connecting closest pair $p, q \in P$

Hot MILPs in your area

Dual:

$\min\ \boldsymbol{c}'\boldsymbol{x}$		$\max\ \boldsymbol{p}'\boldsymbol{b}$		
s.t. $\boldsymbol{a}_i'\boldsymbol{x} \geq b_i$,	$i \in M_1$,	s.t. $p_i \geq 0$,	$i \in M_1$,
$\boldsymbol{a}_i'\boldsymbol{x} \leq b_i$,	$i \in M_2$,	$p_i \leq 0$,	$i \in M_2$,
$\boldsymbol{a}_i'\boldsymbol{x} = b_i$,	$i \in M_3$,	p_i free,	$i \in M_3$,
$x_j \geq 0$,	$j \in N_1$,	$\boldsymbol{p}'\boldsymbol{A}_j \leq c_j$,	$j \in N_1$,
$x_j \leq 0$,	$j \in N_2$,	$\boldsymbol{p}'\boldsymbol{A}_j \geq c_j$,	$j \in N_2$,
x_j free,	$j \in N_3$,	$\boldsymbol{p}'\boldsymbol{A}_j = c_j$,	$j \in N_3$.

Subtours:
MTZ:
$u_i + x_{ij} \leq u_j + M \cdot (1 - x_{ij}) \quad \forall i, j \in N - \{1\}, i \neq j$
$1 \leq u_i \leq n - 1 \quad \forall i \in N - \{1\}$
where $x_{ij} = 0 \Rightarrow M = n - 2$

SCF:
$\sum_{j, j \neq 1} f_{1j} = n - 1$
$\sum_{i, i \neq j} f_{ij} - \sum_{i, i \neq j} f_{jk} = 1 \quad \forall j \in N - \{1\}$
$0 \leq f_{ij} \leq (n - 1) \cdot x_{ij} \quad \forall i, j \in N, i \neq j$

MCF:
$\sum_{j, j \neq 1} f_{1j}^k - \sum_{j, j \neq 1} f_{j1}^k = 1 \quad \forall k \in N - \{1\}$
$\sum_{i, i \neq k} f_{ik}^k = 1 \quad \forall k \in N - \{1\}$
$\sum_{i, i \neq j} f_{ij}^k - \sum_{i, i \neq j} f_{ji}^k = 0 \quad \forall i, j \in N - \{1\}, j \neq k$
$0 \leq f_{ij}^k \leq x_{ij} \quad \forall i, j \in N, i \neq j, \forall k \in N - \{1\}$

Useful stuff: Projects $X, Y, \ldots$, binary var. $x, y, \ldots$

At most (least) $X, Y, Z \ldots$	$x + y + z + \ldots \leq (\geq) N$
If X then Y	$x \leq y$
Not X	$\overline{x} = 1 - x$
If X then not Y	$x + y \leq 1$
If not X then Y	$1 \leq x + y$
If X or Y then Z	$x \leq z$ and $y \leq z$
If X then Y or Z	$x \leq y + z$
If X and Y then Z	$x + y - 1 \leq z$