SUMMARY OF THE COURSE

in master's programme Biomedical Engineering
Course 317.043 Introduction to Biomechanics

Recap questions

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1 Course content- mid-term exam

1.1 Introduction, Definitions (& History)

1. Is biomechanics a small or a large field of science?

Biomechanics is a large field of interdisciplinary science, overlapping with medical and clinical science, mechanical engineering, biology, biophysics, chemistry and computer science.

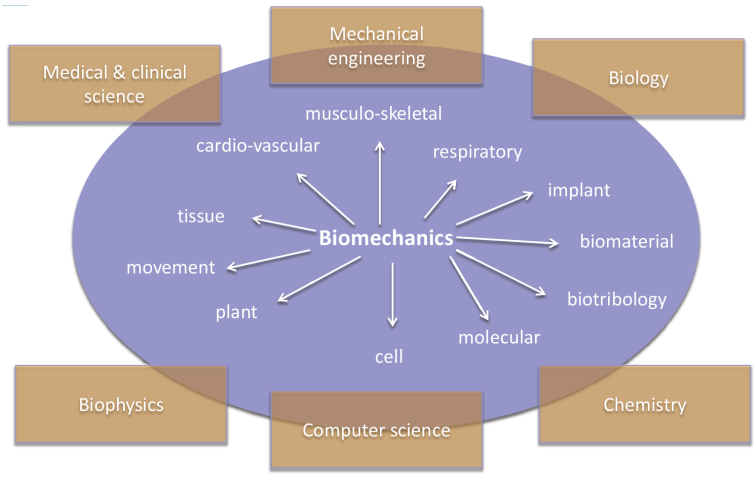


Figure 1: Interdisciplinary behaviour of Biomechanics.

2. What is biomechanics?

Biomechanics is the study of how forces interact with living systems! Biomechanics is the science of motion of living systems, of the forces that animate them, of the laws of equilibrium that govern them and of the behaviour of the living matter. It differs from the classical mechanics by the living nature of the object of study.

3. What are the definitions of (biology, mechanics, evolution), biomechanical engineering, (biomimetics, bionics), mechanobiology?

Biology: Is the sciences that study living systems, their intrinsic nature, functionality, evolution and their interaction with their environment of the vegetal and animal kingdom. Essential properties of a living system are: replication, catalysis and mutability.
Life is the condition that distinguishes animals and plants from inorganic matter, including the capacity for growth, reproduction, functional activity, and continual change preceding death.

Mechanics: Is the study of motion, force, equilibrium as well as the behaviour of materials. It is applied to the analysis of any dynamic system.
In modern physics, mechanics is divided in continuum, statistical, quantum and relativistic theories. Continuum and statistical mechanics are currently the most relevant to living systems.

Evolution: Is the basic principle of biology that gives a meaning to the diversity of the living world. Evolution is the change of genetic information that is transmitted over generation by apparently random mutations and in this process the selection of genetic information favourable to survival and reproduction.

Biomechanical egineering: The principles of biomechanics contribute to understanding, predict and to propose interventions by disease. Biomedical engineering applies principles of mechanical engineering to biological systems and to analyse the performance of a technical system.

Biomimetics: Is the study of the formation, structure, or function of biologically produced substances and materials for the purpose of synthesizing similar products which mimic natural ones.

Bionics: Is the science of systems which have some function copied from nature, or which represent characteristics of natural systems or their analogues.

Mechanobiology: Focuses on how motion, physical forces and changes in the mechanical properties of cells and tissues contributes to growth, development, cell differentiation, physiology, turnover and disease.

4. **Which part of biomechanics are we focusing on in this course?**

   - Musculoskeletal system
     - Function
     - Statics
     - Rheology
     - Movement
     - Dynamics
   - Cardiovascular system
     - Biofluid mechanics
     - Heart
     - Arteries
     - Veins
     - Capillaries

5. **What is collagen and why could collagen mechanics be of interest?**

   Collagen is a family of proteins existent in all tissues that have mechanical function. It is a major structural protein in our bodies-cells attached to it. Collagen forms fibrils, which are ropes with tunable stiffness. The collagen molecules are organized hierarchically in fibrils, fibers and fascicles. The cellular content is dominated by the tenocytes, which are terminally differentiated cells.

6. **Is biomechanics an old or a young discipline?**

   Biomechanics is a fairly young discipline, as it has been recognized as an independent subject for about 30-40 years. On the other hand, it dates back to ancient times, as the history of biomechanics is strongly associated with history of science.

1.2 Scales and Dimensions

1. **Which type of mechanics for biomechanics?**

   Biomechanics is the study of how forces interact with living systems. In this case the most relevant types for living systems are continuum and statistical mechanics. We us in this course only classical mechanics because it is sufficient for all calculations in biomechanics.
   In classical mechanics we are able to show a system at time t, when time and space are homogen and the space also is isotrope.

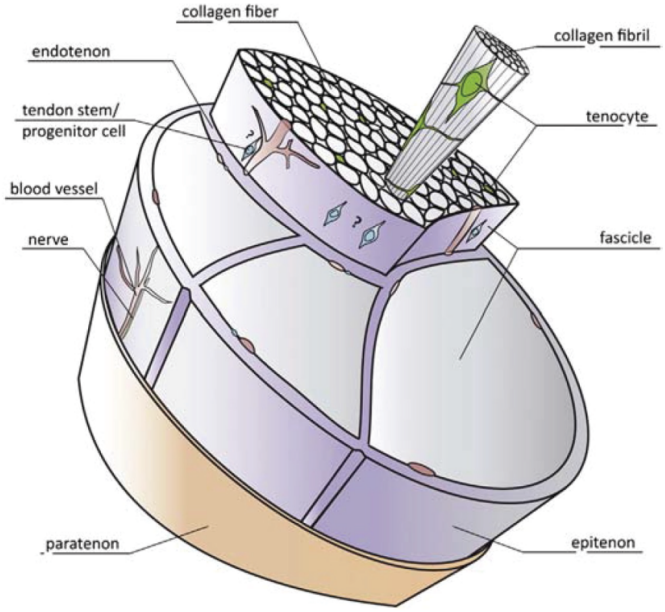


Figure 2: A schematic drawing of basic tendon structure.

2. What is the SI system?

The international system of units is the modern form of the metric system and the world most widely used system of measurement.

Formula symbol	Dimension	Designation	Unit	Symbol
l	L	Length	Meter	m
t	T	Time	Second	s
m	M	Mass	Kilogram	kg
I	I	Current	Ampere	A
T	Θ	Temperature	Kelvin	K
n	N	Amount of substance	Mol	mol
Iv	J	Luminosity	Candela	cd

3. What are scaling laws and what are they useful for?

Scaling law give the relation between different quantities and thus describe the proportional behaviour of one attribute with the relative change of another one. The scaling laws are useful for prognoses of certain biological properties.

4. **For which biological processes is scaling important?**

   Scaling laws are important for every part of biological processes. As example, how high can a human grow includes the scaling laws for the bone size, muscle mass, metabolic rate, etc.

5. **What is dimensional analysis and how it is useful?**

   Is the analysis of the relationship between different physical quantities by identifying their dimension or units. For a variable of interest (x) we can assess its dimensionality and compare this to the dimensional parameters of the system to find our the relation between the variable and the system, as two sides fo an equation must have the same dimension.
   $[x] = [i]^{\alpha} [j]^{\beta} [k]^{\gamma}$

1.3 Rigid Body Mechanics

1. **Rigid body motion can be decomposed in two parts, which are these?**

   - Translation, change of position [x, y, z]
   - Rotation, the change of orientation $[\theta, \varphi]$

   $y(x,t) = R(t)\, x + d(t)$

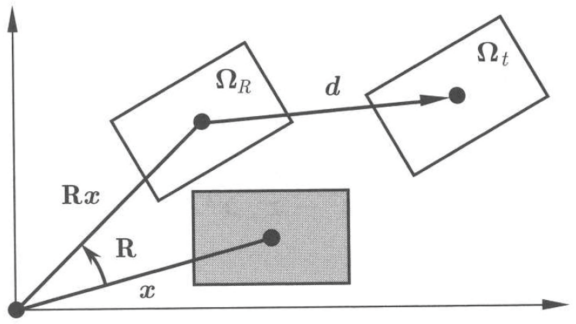


Figure 3: Decomposition in rotation (Ω_R) and translation (Ω_t).

2. **What are the center of mass and the radius of gyration?**

   Center of mass: (COM) Therefore the centre of mass is the point where the sum of all moments generated by a uniformly distributed force cancel. Centre of mass allows reduction of translational acceleration of a rigid body to

point mechanics problem.
$c_m(\Omega) = \frac{\int x\rho(x)dV}{m(\Omega)} \geq 0$

Radius of gyration: Gives locations of a point mass producing the same inertial moment as a rigid body.
$r_g = \sqrt{\frac{I}{m}}$

$r_g \ldots Radius\, of\, gyration\, [m]$
$I \ldots Moment\, of\, intertia\, [kg\, m^2]$
$m \ldots Mass\, [kg]$

3. **Forces acting on rigid bodies can cause which movements?**

   - Translation, is cause by all forces
   - Rotation, is cause by all forces which the line of attack not going through the centre of mass

4. **What are the conditions for static equilibrium for a rigid body?**

   $\Sigma_i F_i = 0$ (The sum of all forces is equal to zero.)
   $\Sigma_i M_i = \Sigma_i (r_i \cdot F_i) = 0$ (The sum of al moments is equal to zero.)
   This formulas are part of Newtons first law. The statical system must be in balance, if not it will broken.

5. **What is a free body diagram and what is it useful for?**

   The free body diagram is a schematic drawing of the statical behaviour of a system. It shows all external forces which acts on the system and the resulting counterforces and moments ($F_{ij} = -F_{ji}$ Newtons third law Actio = Reactio). It is used to simplify complex biological images (e.g. elbows) and display them in a simple static system.

1.4 Musculoskeletal Anatomy

1. **What is the musculoskeletal system?**

   Musculoskeletal system = Locomotor system
   Is an organ system that gives humans the ability to move using their muscular and skeletal systems. It has

   - passive structures (ligaments, tendons and cartilage)
   - active structures (striated skeletal muscles)

   It is made up of bone skeleton.

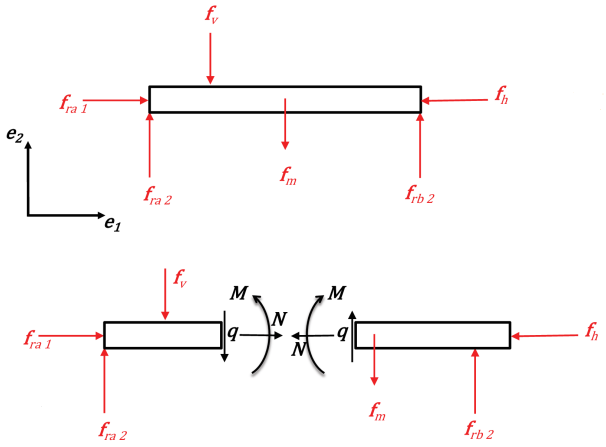


Figure 4: Top: Free body diagram of external forces. Bottom: Free body diagram of external forces and internal forces.

2. Can you name the anatomical references planes and directions?

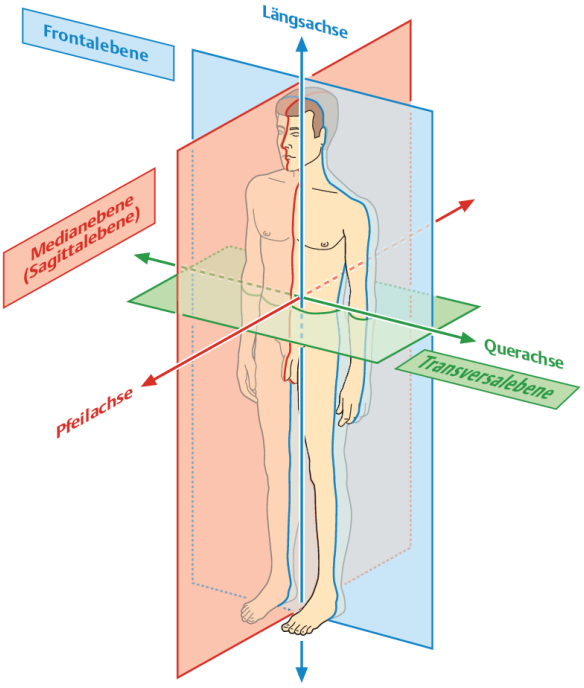


Figure 5: Reference planes.

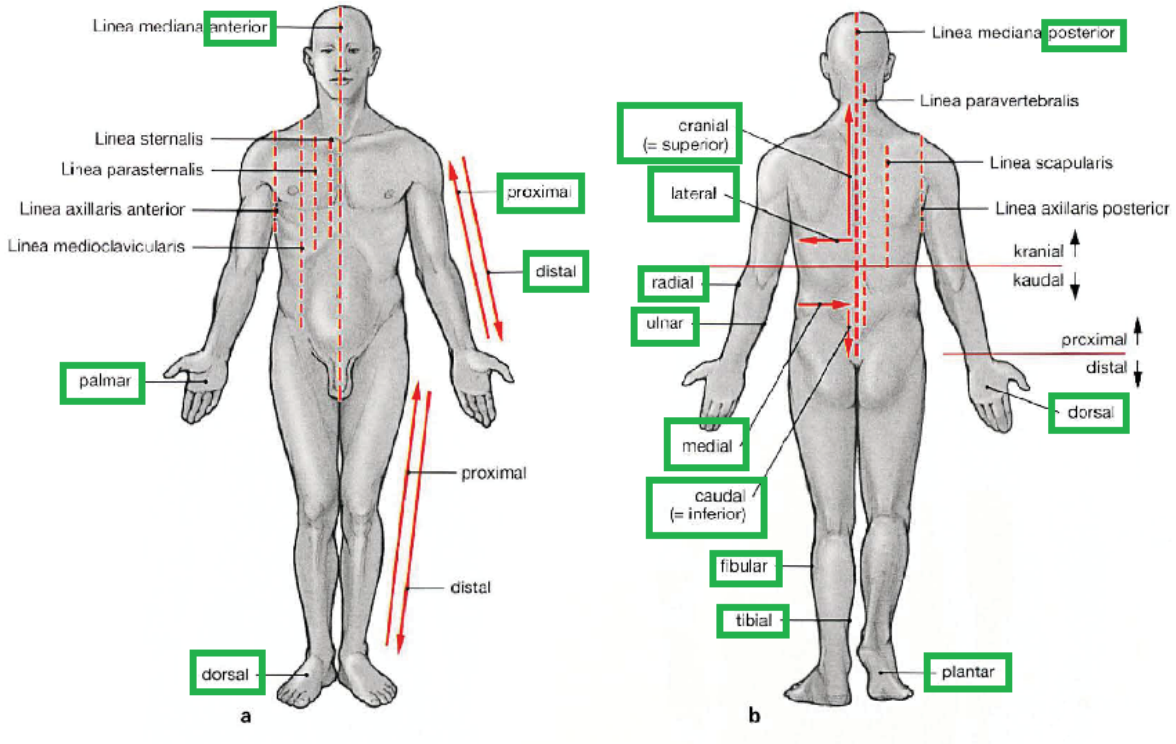


Figure 6: Reference directions.

Anatomical direction	Description
Anterior	Towards the front surface (stomach side)
Posterior	Towards the back surface
Cranial	Towards the head
Caudal	Towards the breech (Steißbein)
Medial	Towards the median plane
Lateral	Away from the median plane, sideways
Proximal	Towards the body
Distal	Towards the end of the extremity
Radial	Towards the spoke side (thumb (Daumen) side)
Ulnar	Towards the ulnar side (side of the little finger)
Dorsal	Towards the back of the hand or foot
Palmar	Towards the inner side of the hand
Fibular	Towards the fibula (Wadenbein) (small toe)
Tibial	Towards the shin (Schienenbein) side (big toe)
Dorsal	Towards the back of the hand or foot
Plantar	Towards the sole of the foot

3. Which joint types do you know?

Type	Description	Examples
Fibrous joints/ synarthroses	Bones held together by fibrous connective tissue and with no joint cavity. No mobility possible.	Suture line of the skull, Peridontal ligament to held tooth in socket
Cartilaginous joints/ amphiarthroses	Bones held together by cartilage, lacking a joint cavity. Some minimal mobility.	Growth plate, fibrocartilage pad in the spine (Wirbelsäule)
Synovial joints/ diarthroses	In which the joint contains a synovial cavity (Gelenkspfane). High mobility.	Knee, shoulder

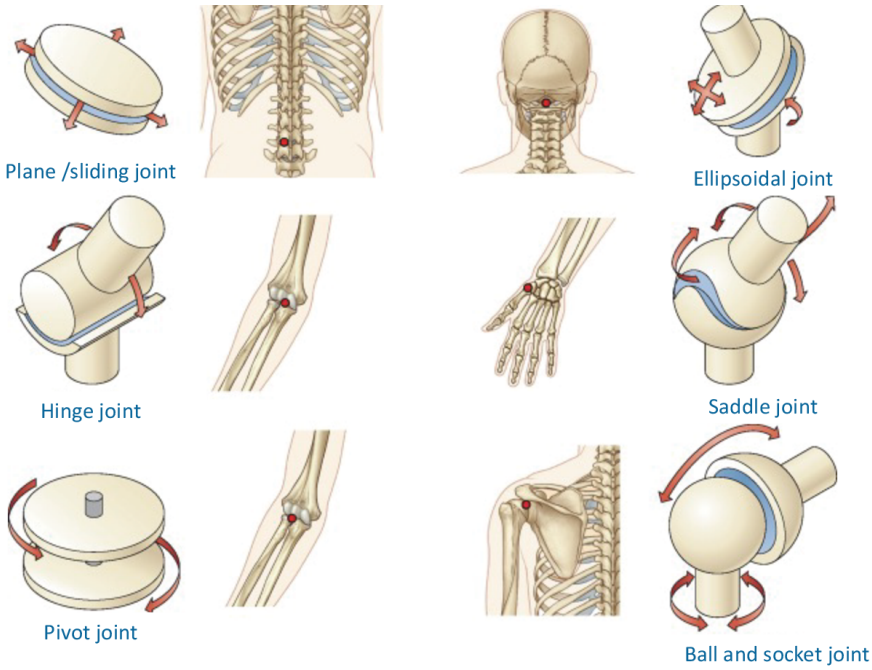


Figure 7: Synovial joints - classification.

4. What is collagen?

Collagen is a tissue made of collagen fiber bundles that provide structural strength. The fibers in those bundles are made of small fibrils of around 100 nm in diameter and up to tens of mm in length, with periodic D-banding (striation) with 67 nm period. The striation comes from the tropocollagen molecules due to their varying overlap. Collagen make up 25% of the total protein mass in the body.

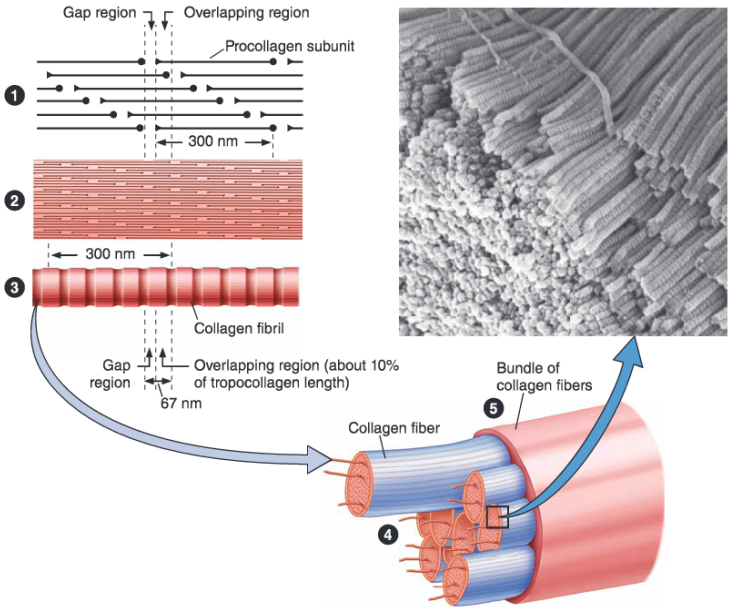


Figure 8: Collagen structure.

5. What is elastin?

Elastin is a tissue that provide elasticity to many tissues. It is made of a network of fibers of cross-linked elastin molecules. Often in combination whit collagen.

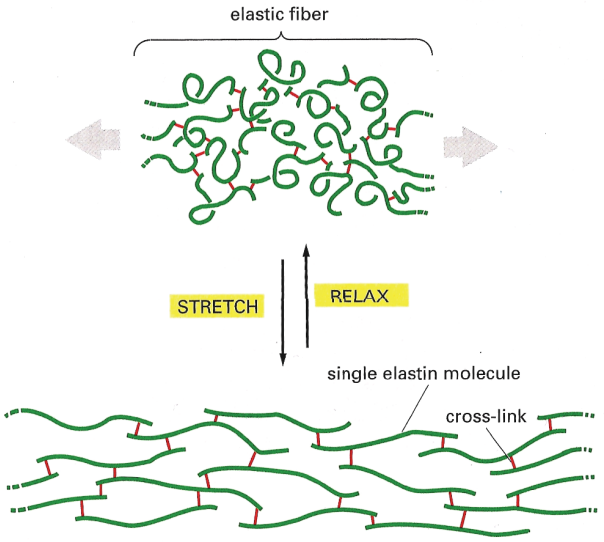


Figure 9: Skeletal tissue - elastin.

6. What are proteoglycans?

Proteoglycan is a protein backbone (hyaluronic acid) with attached side chains (glycosminoglycans = GAGs) which each also have side chains. Those GAGs look like bottle brushes or pipe cleaners and are highly negatively changed. They attract counter ions in solution and water due to osmotic pressure. About 150 GAGs are bound to the backbone, making like also look like a bottle brush or pipe cleaner.

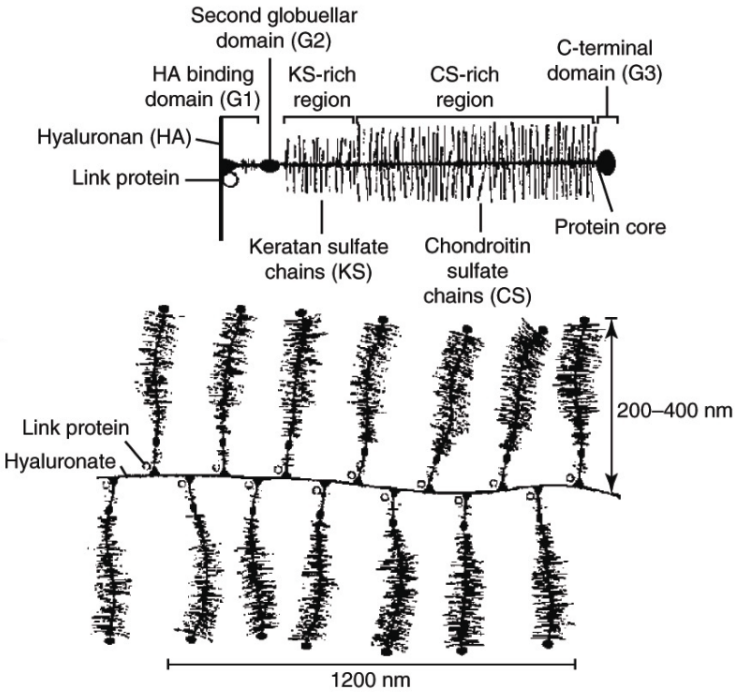


Figure 10: Proteoglycans structure.

7. **What is the role of the following tissues: bone, tendon, ligament, cartilage, muscle?**

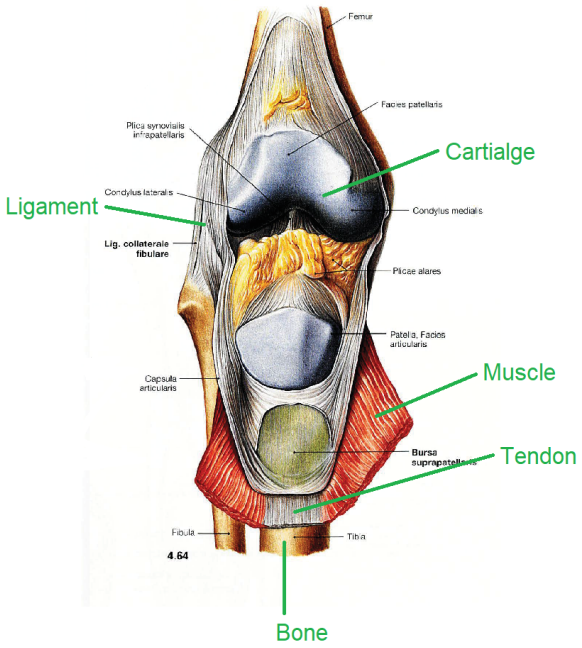


Figure 11: Anatomy of the knee.

Bone: Support body in the gravitational field and protect vital organs. Their function also includes homeostasis of calcium and phosphorus as well as haematopoiesis.

Tendon: Connect muscle extremities to bones through aponeuroses and calcified insertion sites respectively. Their key function is the transmission of muscle forces. They do not support compressive loads along their fibers but support compressive stresses across their fibers.

Ligament: He connect one bone to another bone. They fulfil a stabilization task, maintain joint congruence with a varying constraint and contribute to propioceptivity through presence of nervous fibers at detect stretching. They do not support compressive loads along their fibers.

Cartilage: Is a layer of connective tissue that is few millimetre thick. It covers the articular system and transmits and distributes high stresses form one bone to the other and also allows sliding of the surface with minimal friction.

Muscle: Provide the mobility of the body by their contraction.

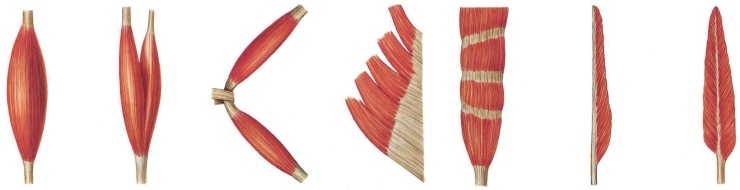


Figure 12: From left to right: fusiform, biceps, digastric, planar, plurigastric, unipennate and bipennate

1.5 Muscle and Joint Loading

1. **How does a statically determinate system help to work out muscle forces in static joints?**

   In a statically determinate system, the number of unknown forces is less or equal to the number of available equations of equilibrium. You can calculate all the unknown forces from these equations alone (without needing to do further experiments).

2. **What are simple biomechanical models for shoulder, elbow and knee?**

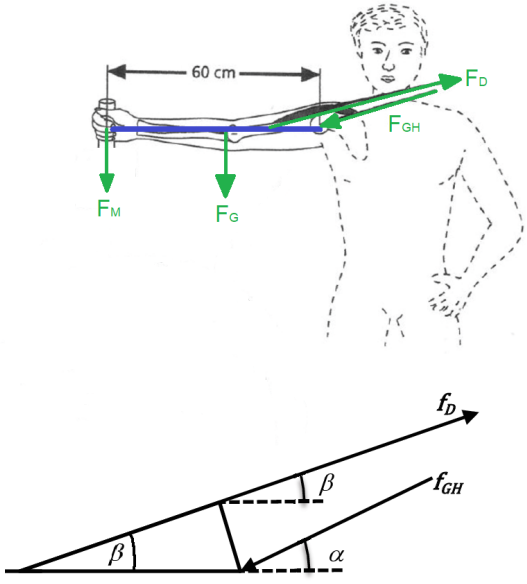


Figure 13: Shoulder is a ball an socket joint.

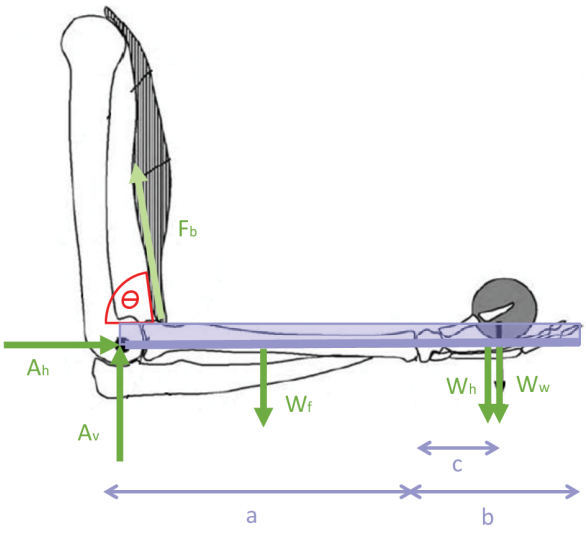


Figure 14: Elbow is a hinge and pivot joint.

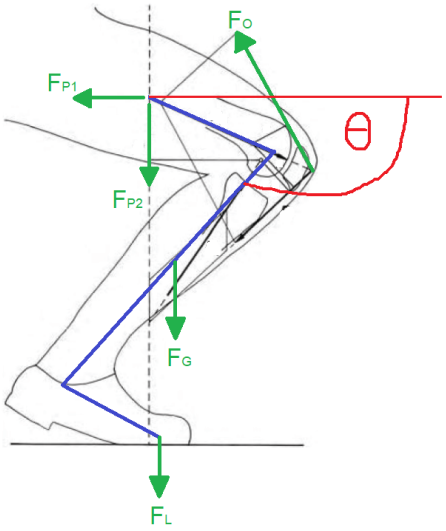


Figure 15: Knee is a hinge+pivot joint = condyloid joint.

3. How are striated (quergestreift) muscles structured?

They are hierarchically structured. The Muscles consist of blood vessels, connective tissues and muscle fibers. The fibers are rod shaped cells with a diameter of 10-100 μm and are up to 30 cm long. These muscle fibers are bundles of myofibrils-contraction elements made from sarcomeres (smallest functional unit of a muscle) encased in sacoplasmic reticulum.

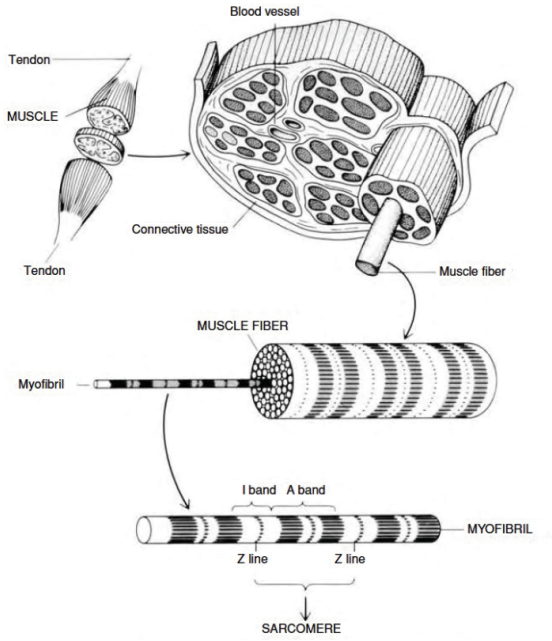


Figure 16: Structure of an striated muscle.

4. What is the sliding filament model?

The model explains the mechanism of muscle contraction based on muscle proteins that slide past each other to generate movement. The movement works by many single steps.

a) No cross-bridge between thick and thin filament. The myosin head is energized (+ADP+PO_3).

b) Action potential from neuromuscular junction arrives the terminal cisternae.

c) Calcium ions are released into the cytosol.

d) Ca^{2+}-ions bind to troponin causing.This conformational change exposes the binding sites for myosin on actin.

e) Now the energized myosin head can bind to to actin forming a crossbridge.

f) Binding of myosin to actin results in a conformational change of the myosin head. As a result ADP and PO3 are released. –> Power stroke

g) Upon binding of ATP the head retracts.

h) Release of myosin head triggers hydrolysis of ATP molecule into ADP and P.

i) Ca^{2+}-ions are pumped back into the sarcoplasmic-reticulum. As Ca^{2+} is removed the troponin-tropomyosin complex again covers binding sites on actin.

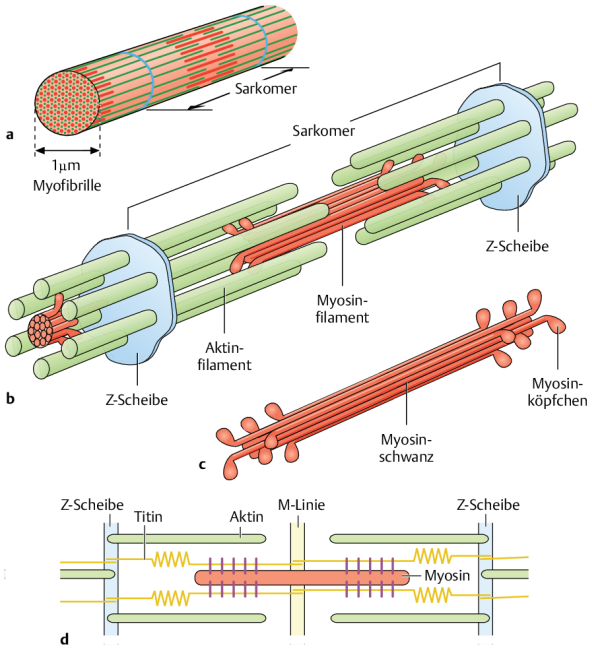


Figure 17: Structure of a sarcomere.

5. What is isotonic, what is isometric muscle contraction?

Isotonic: Constant tension is applied – muscle length is recorded.
Upon on occurrence of a single action potential the muscle will contract although there must be some time delay.

Isometric: Muscle held at constant length – muscle force is recorded.
Before any muscle shortening can occur enough tension must be generated to over come the externally applied load. Latency period increases.

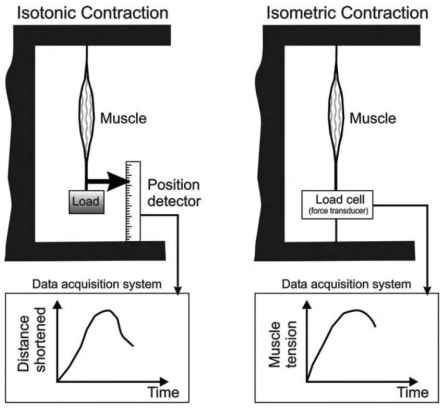


Figure 18: Contraction of muscles.

6. **Can you explain the Hill model of muscle forces and velocity?**

$(a + P)(b + v) = b(P_0 + a)$
$v \ldots velocity\, of\, contraction$
$P \ldots applied\, load$
$P_0 \ldots laod\, applied\, at\, v = 0$
$a, b \ldots constants$
Hill's equation demonstrate that the relationship between P and v is hyperbolic. The muscle tension decrease as the shortening velocity increases.
↑ load applied to muscle ↓ contraction velocity
↓ tension ↑ contraction velocity
This is a good approcximation for isotonic contraction near resting length.

7. **How can muscles be described using a lumped parameter model?**

In the lumped parameter model, a muscle is reduced to a contractile element T_0 (actin-myosin complex), an elastic element k_0 (elastic of connective tissue) and a dashpot η_0 (frictional dissipation in the muscle).

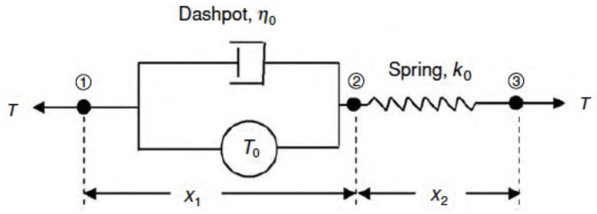


Figure 19: Lumped parameter model.

8. **What are parallel, what are pennant muscle?**

Parallel muscle: Are linear muscles that work along their contraction direction. Their advantage is a large change in length.

Pennate muscle: This muscles are arranged like feathers. This arrangement cannot contract as much in length. Its advantage lies in the much larger contractile forces.

9. **What is the physiological cross-section area (PCSA)?**

$PCSA = \frac{V_{muscle}}{l_{fiber}}$ The PCSA is the area of the cross section of a muscle perpendicular to its fiber, generally at its largest point. It is not the same as the anatomical cross-profile area (ACSA), which is the area of the cross section of a muscle perpendicular to its longitudinal axis.

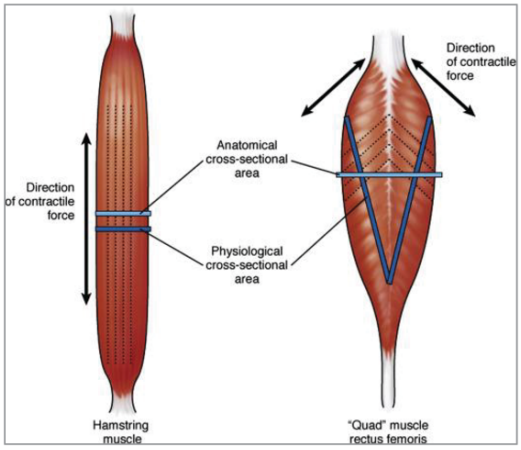


Figure 20: Muscle structures.

2 Course content - final exam

2.1 Movement biomechanics

1. **What is anthropometry and how does it aid movement biomechanics calculations?**

   The study of the physical measurement of the human body is called anthropometry, a branch of anthropology. To analyse human movement much data are required. On part of this data are the segments of the human body. This data are collected by measurements of different bodies $\longrightarrow$ anthropometry.

2. **What is inverse dynamics and which input is required for this process?**

   The inverse Dynamics is the process of using measurements of position, motion and reaction forces as input data to calculate the forces and moments acting on the body.

   The normal way to is the forward dynamics. In this case we have the forces and moments that acts on the body and use this to calculate the reaction forces and moments.

   As input is required:

   - Kinetics (Ground reaction forces / moments), measured whit force plate
   - Kinematics, measured with markers, videography
   - Anthropometric segment model for limb and subject/patient of interest, measurements of the human body

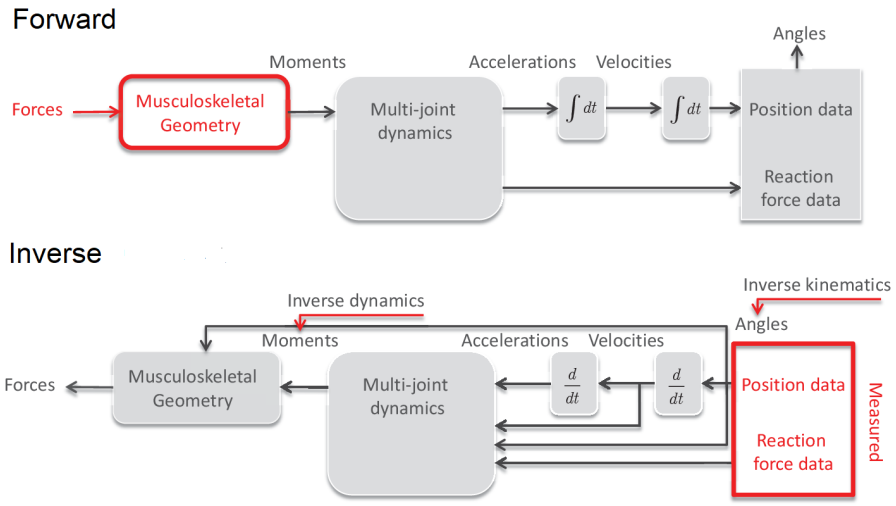


Figure 21: Inverse vs. forward dynamics.

3. What is the difference between joint contact forces and joint reaction forces?

The joint contact force is the actually (*tatsächliche*) force that acts on the articular surface (*Gelenksfläche*) and include the muscle activity.

The joint reaction force is defined as the force generated in a joint as reaction to the forces acting on this joint. It counteract external forces (e.g. gravity, weight) and create a equilibrium.

$$\sum_i F_i = 0$$

$F_i \ldots Force\ [N]$

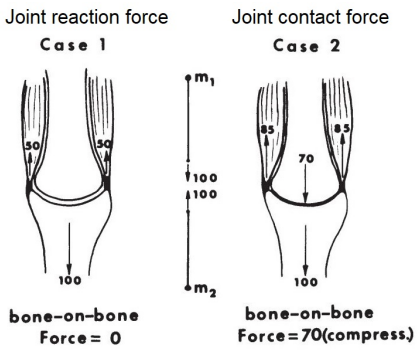


Figure 22: Diagrams to illustrate the differences between joint raction force and contact force. In both cases, the reaction force is 100 N acting upward on M_2 and downward on M_1. With no muscle activity (case 1, lower segment passively hanging downwards), the contact force is zero; with muscle activity (case 2, active muscle contraction) it is 70 N.

The relationship (*Verhältnis*) between joint reaction forces and joint contact forces is very unclear. Generally joint contact forces are higher than joint reaction forces. This results in the muscle activety that takes place by the joint contact force.

4. **What is the only possibility to directly measure joint contact forces?**

   It is only possible through measurements in vivo (in the patient) with telemetrised implants. If the patient have no e.g. smart knee implant you can try to measured the muscle activity with EMG (electromyography).

5. **Which requires a higher metabolic rate (*Stoffwechselrate*): positive work or negative work?**

   Positive work requires a higher metabolic rate. When we starting from equilibrium you do have to increase the muscle tension/force to do positive work. By negative work you only must reduce the muscle tension/force.

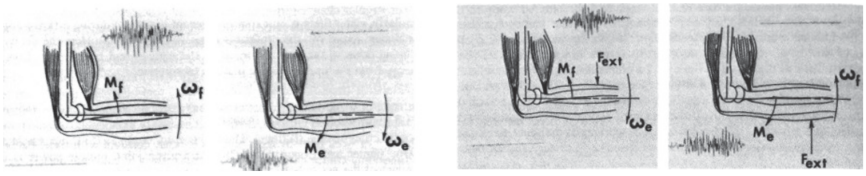


Figure 23: Positive work is done when the muscle moment (M_f) act in the same direction as the angular velocity (ω_f) of the joint. Negative work is done when the muscle moments (M_f) acts in the opposite direction to the moment of the joint ($\omega_f + F_{ext}$). F_{ext} is a force that act from outside on the arm.

6. **How can movement efficiency generally be characterized?**

$$Metabolic\ muscle\ efficiency = \frac{\sum mechanical\ work\ done\ by\ all\ muscles}{metabolic\ work\ of\ muscles}$$

$$Mechanical\ efficiency = \frac{mechanical\ work\ (internal + external)}{metabolic\ cost - resting\ metabolic\ cost}$$

$$Work\ efficiency = \frac{external\ mechanical\ work}{metabolic\ cost - zero\ work\ metabolic\ cost}$$

Resting metabolic cost: not using muscles (e.g. sitting on a bike without cycling).
Zero-work metabolic cost: using muscles with no resting force (e.g. freewheeling (*Freilauf*) on a bike).

7. **What do efficient movements look like?**

   Efficient moments generally look very smooth.

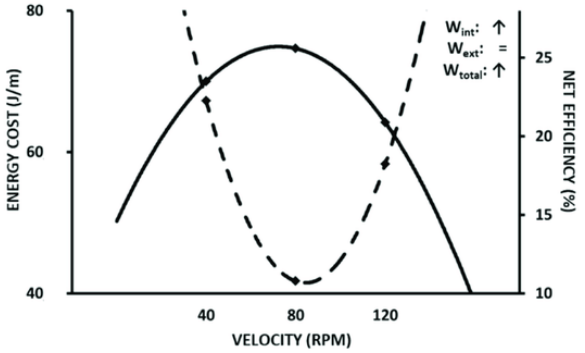


Figure 24: Example of a graph for a metabolic muscle efficiency. Metabolic energy cost and overall efficiency during cycling ergometer test at different velocities (revolutions per minute, RPM).

2.2 Deformable solid mechanics - strain

1. **What is the definition of strain (*Dehung*) using the small strain approximation?**

   Strain is the relative change of length.

$$\varepsilon = \frac{L - L_0}{L_0}$$

   $\varepsilon \ldots Stain\ [\,]\ [\%]$
   $L_0 \ldots length\ to\ beginning\ [m]$
   $L \ldots length\ after\ stretching/compression\ [m]$
   The strain tensor (small strain approximation), defined as

$$\boldsymbol{\varepsilon} = \frac{1}{2}(grad(\mathbf{u}) + grad^T(\mathbf{u}))$$

   $\boldsymbol{\varepsilon} \ldots Stain\ tensor$
   $\mathbf{u} \ldots Displacement\ vector\ (Verschiebungsvektor)$

2. **Which assumptions does the small strain definition entail?**

   The expression (see equation above) was determined under two very important assumptions:

   - Small displacements (*Verschiebung*): the shape (*Form*) of the object does not change significantly during the deformation process.

- Small strains (or linearised strains): each strain component stays small in relation to the unit.

3. **What kind of variable (scalar, vector, etc.) is strain?**

   Strain is a dimensionless second rank tensor.

   A tensor is a multilinear image that images a certain number of vectors onto a vector and fulfils a universal property.

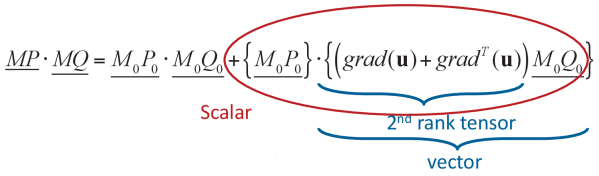


Figure 25: Strain tensor structure.

4. **What is the interpretation of the different strain components (in diagonal - off diagonal)?**

   The in diagonal components ($\varepsilon_{11},\ \varepsilon_{22}$) are the relative stretch between two points close to each other $\longrightarrow$ Axial tension components.

   The off diagonal components (ε_{12}) component of the strain matrix represents the (small) angular deformation $\longrightarrow$ Shear strain components.

$$Strain\ matrix\ components = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21(=12)} & \varepsilon_{22} \end{bmatrix}$$

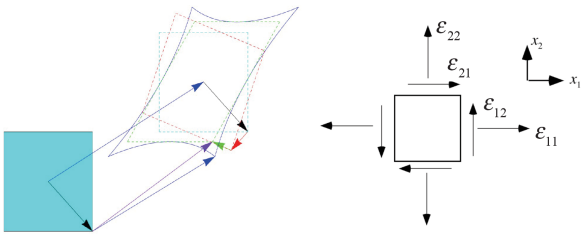


Figure 26: Displacement of a solid.

5. What is the transformation law for strain?

$$\underline{\underline{\varepsilon'}} = \mathbf{R}^T \underline{\underline{\varepsilon}} \mathbf{R}$$

$\underline{\underline{\varepsilon'}} \ldots Transformated\ second\ order\ strain\ tensor$
$\mathbf{R} \ldots Euclidean\ space$
$\underline{\underline{\varepsilon}} \ldots Second\ order\ infinitesimal\ strain\ tensor$

6. What are the principal strains?

For a specific rotation Θ_p the shear component disappears and the resulting normal strains are called principal strains. This principal strains are the eigenvalues of the strain tensor.

$$principal\ strain = \varepsilon_{1,2} = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} \pm \sqrt{\left(\frac{\varepsilon_{11} - \varepsilon_{22}}{2}\right)^2 + \varepsilon_{12}^2}$$

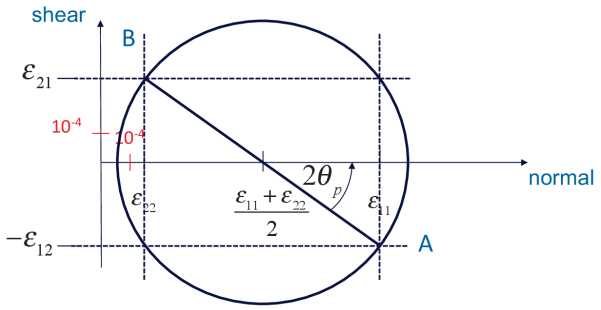


Figure 27: Mohr's Circle.

7. How can strain at a point be measured using a strain gauge rosette?

A strain gauge (*Dehungsmessstreifen*) can measured the strain along one axis. By applying three gauges on one point of interest, we can measure all three strain components of 2D strain.

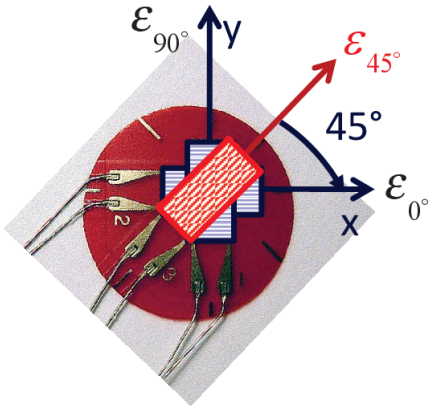


Figure 28: Strain gauge rosette.

2.3 Deformable solid mechanics - stress
Bone structure and mechanics

1. **What is the definition of stress (*Spannung/Druck*)?**

   Stress are the internal forces in a body.

$$\sigma = \frac{F}{A}$$

   $\sigma \ldots Stress\ [N/m^2]$
   $F \ldots\ldots Force\ [N]$
   $A \ldots Area\ [m^2]$

2. **How can stresses be transformed (rotations)?**

   The transformation law for stress tensor is the same as for strain tensor. In this case we take the formula of the transformation law and change the variables.

$$\underline{\underline{\sigma'}} = \mathbf{R}^T \underline{\underline{\sigma}} \mathbf{R}$$

   $\sigma \ldots Stress$
   $\mathbf{R} \ldots Rotation\ matrix$

3. **What are principal stresses?**

   Similar to the principal strains, for a specific rotation Θ_p the shear component disappears and the resulting normal stresses are called principal stresses. This principal stresses are the eigenvalues of the stress tensor. That means that Mohr's circle can also be used to obtain the principal stresses and shear stress.

$$principal\ stress = \sigma_{1,2} = \frac{\sigma_{11} + \sigma_{22}}{2} \pm \sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + \sigma_{12}^2}$$

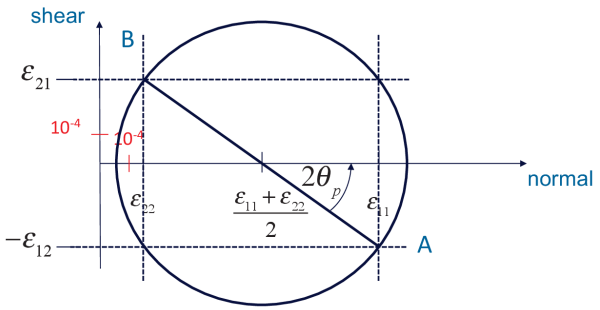


Figure 29: Mohr's Circle.

4. **What are the components that make up bone tissue?**

   - Organic
     - Collagen type I
     - Non-collagenous proteins
     - etc.
   - Inorganic
     - Hydroxyapatite crystals
     - Minerals
   - Water

5. **What are typical values for "elastic" modulus, shear modulus and strength of bone?**

Name	Symbol	Value
Young's (elastic) modulus	E	12-17 GPa
Shear modulus	G	3 GPa
Strength of bone	σ	60-180 MPa

6. **Which kind of stresses and strains arise in bone due to external compression, bending and torsion loads?**

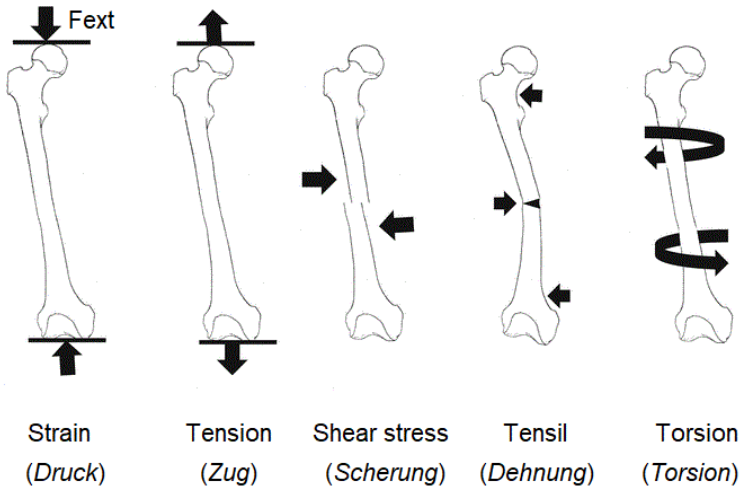


Figure 30: Resulting stresses and strains in case of external forces.

7. **Why are long bones hollow. i.e. what is the result of this geometry on torsional and flexural stiffness?**

Biologically, nature strives to work as efficiently as possible to save energy. For the bone, this means that the following conditions should be met:

- high stability (gravity is constantly at work)
- good elasticity
- as light as possible (we "carry" them with us all the time)
- as little material as necessary (save calcium)

As a result of evolution, a cylinder shape has emerged as the load-bearing structure. Based on this geometry, the following properties result for various externally acting forces.

For the absorption of compressive and tensile forces and their dissipation into the ground, only the cross-sectional area (CSA) is important. The compressive and tensile strength is proportional to the CSA.

$$\sigma = \frac{F}{A} = \frac{m \cdot g}{A}$$

$\sigma \ldots Stress\ [N/m^2]$
$m \ldots Mass\ [kg]$
$g \ldots Gravity\ [m/s^2]$
$A \ldots Area\ [m^2]$

In the case of bending, the wall thickness and radius of the bone are important. A very wide hollow bone with very thin walls is more difficult to bend because of its width, but the thin walls tend to buckle easily and therefore it bends more easily. With a small radius and thicker walls, greater forces can act. Therefore, the second moment of area (I), which represents the resistance of a beam against bending, is higher here.

$$I_{hollow} = \frac{\pi}{4}(R_e^4 - R_i^4) > I_{cylinder} = \frac{\pi}{4}R^4$$

$$\delta_{bending\,hollow} \propto \frac{1}{I} < \delta_{bending\,cylinder}$$

$I \ldots Second\ moment\ of\ area\ [m^4]$
$R_e \ldots Inner\ Radius\ [m]$
$R_e \ldots outer\ Radius\ [m]$
$R \ldots Radius\ [m]$

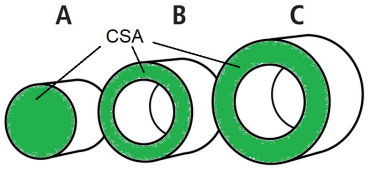
If torsion occurs, the radius of the bone determines its stability against twisting. A bone with a small radius is easier to twist. Hence, a bone with a larger radius has a higher polar second moment of area (J), which represents the resistance of a beam to torsion.

$$J_{hollow} = \frac{\pi}{2}(R_e^4 - R_i^4) > I_{cylinder} = \frac{\pi}{2}R^4$$

$$\tau_{hollow} \propto \frac{1}{J} < \tau_{cylinder}$$

$\tau \ldots Torsion\ stress\ [N/m^2]$
$J \ldots Polar\ second\ moment\ of\ area\ [m^4]$

If we now take the optimum from all these data, we find that the hollow bone with the same CSA has a higher bending and torsional strength (with the same volume or mass).



	A	B	C
Area	1	1	2
Tensile and compressive strength	100%	100%	200%
Bending and torsional strength	100%	210%	459%

Figure 31: Examples of the influence of cross-sectional geometry on the structural strength of three circular bars. Although bar A and B have the same area (1 square unit), bar B has a greater moment of inertia because it has a hollow interior and greater outer diameter. Bar C has twice the mass (or area in the cross-section; 2 square units) and therefore a much greater moment of inertia. Note that the cross-sectional areas of the bars are directly proportional to their tensile and compressive strengths. By contrast, because the moment of inertia is to the 4th power, the bending and torsional strengths become exponentially greater from bar A to C.

Anm.: $\sigma = \frac{Drehmoment(M) \cdot Radius(R_i)}{I}$
$\tau = \frac{Drehmoment(T) \cdot Radius(R_i)}{J}$

2.4 Hemodynamics and the circulatory system Part I

1. What is a non-Newtonian fluid?

Is a fluid whose viscosity does not remain constant when shear forces act on the fluid. The viscosity can increase or decrease. Examples: ketchup, quicksand

$$\tau = \mu_{eff} \dot{\gamma}$$

$$\mu_{eff} \neq const.$$

$\tau \ldots shear\ stress\ [Pa]$
$\mu_{eff} \ldots dynamic\ viscosity\ [Pa \cdot s]$
$\dot{\gamma} \ldots rate\ of\ strain\ [s^{-1}]$

2. Why is blood a non-Newtonian fluid, i.e. which component is responsible?

The main function of blood is to deliver oxygen and nutrients to the tissues. In turn, it delivers carbon dioxide to the lungs and metabolic products to the kidneys. Furthermore, the blood is responsible for regulating the pH value and the

body temperature. For this it is imported that the blood flows in every Vessel of the body, this can only be achieve when the blood is a Non-Newtonian fluid.

The vessel diameter is decisive for the viscosity.
In large arteries (shear rate > 100 s^{-1}) blood is mostly Newtonian. In very small capillaries (diameter of 6 - 8 μm), the viscosity of the blood increases. The blood shows Non-Newtonian properties.

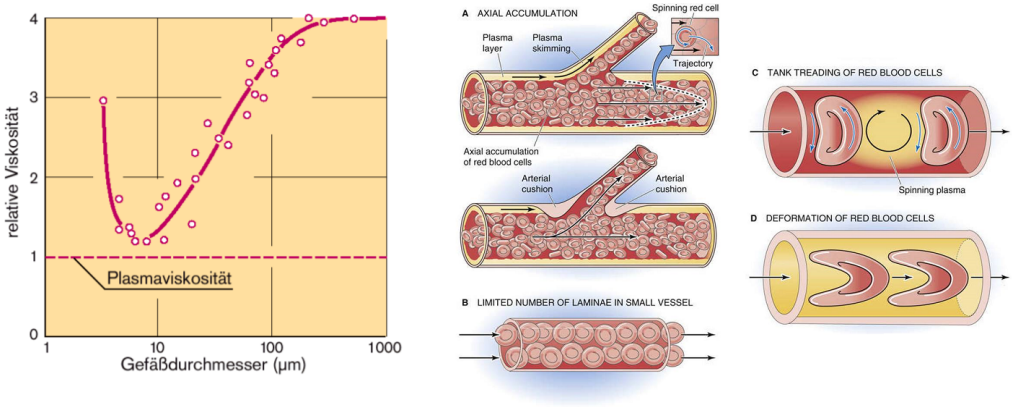


Figure 32: Flow of blood in small vessels. Red blood cells pass through such capillaries in a row. Graph left shows the Fahraeus-Lindqvist effect.

3. Is blood shear thinning or shear-thickening?

Blood is pseudo plastic or shear thinning.

Effective viscosity decreasing with strain rate is called shear thinning or pseudo plastic. Strain rate is the change in strain (deformation) of a material with respect to time.

4. Is blood flow mostly laminar or turbulent?

Blood flow is mostly laminar. Exceptions are the proximal aorta and aortic arch.

5. Which law can we use to describe steady blood flow?

The Poiseuille's law for Newtonian fluids:

$$u(r) = -\frac{R^2}{4\mu}\frac{dp}{dx}\left(1 - \frac{r^2}{R^2}\right)$$

$$Q = -\frac{\pi R^4}{8\mu}\frac{\Delta p}{L}$$

Law of Casson fluid (Non-Newtonian for $R_c \leq r \leq R$):

$$u(r) = -\frac{1}{4\mu}\frac{dp}{dx}\left[(R^2 - r^2) - \frac{8}{3}\sqrt{R_c}\left(R^{\frac{3}{2}} - r^{\frac{3}{2}}\right) + 2R_c(R - r)\right]$$

$$Q = -\frac{\pi R^4}{8\mu}\frac{\Delta p}{L} \cdot 1 - \frac{16}{7}\sqrt{\xi} + \frac{4}{3}\xi - \frac{1}{21}\xi^4$$

Newtonian (Poiseuille)
Non-Newtonian (Casson)

$u(r),\ Q \ldots Axial\ velocity\ [m^3 \cdot s^{-1}]$
$R \ldots Radius\ [m]$
$\mu \ldots Dynamic\ viscosity\ [Pa \cdot s]$
$r \ldots Distance\ to\ the\ pipe\ axis\ [m]$
$dp,\ p \ldots Pressur\ difference\ [Pa]$
$dx,\ L \ldots Length\ [m]$
$R_c \ldots Radius\ of\ the\ non - flowing\ "core"\ [m]$
$\xi \ldots \frac{R_c}{R}\ []$

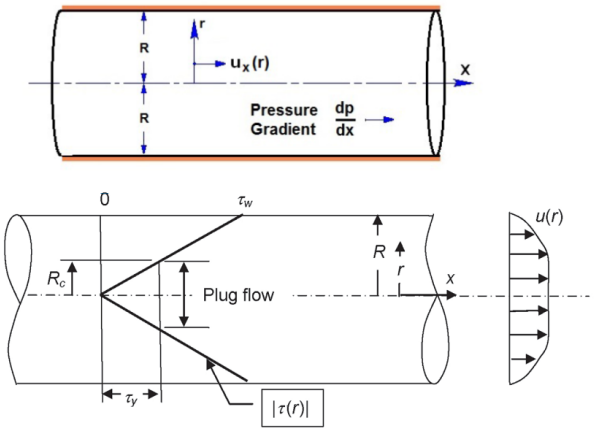


Figure 33: Top:Newtonian flow; Bottom:Non-Newtonian flow.

6. What is the Fahraeus-Lindqvist and inverse F.-L. effect?

Fahreus-Lindqist effect: Is the decreasing in the viscosity of blood as the diameter of the vessel decrease.
Observations:

- Whit decreasing tube radius the effective viscosity reduced.
- The relative haematocrit in the capillary is always lower as one the feed reservoir.

inverse Fahreus-Lindqist effect: For very small capillaries $6 - 8\,\mu$m diameter the blood viscosity increases. Because the deformability limit of the erythrocytes is reached (*erreicht*) and the viscosity rises sharply again.

7. How does the heart pump the blood?

The heart creates a pressure gradient by contracting its muscles and prevents flow in the wrong direction by closing of the heart valves.

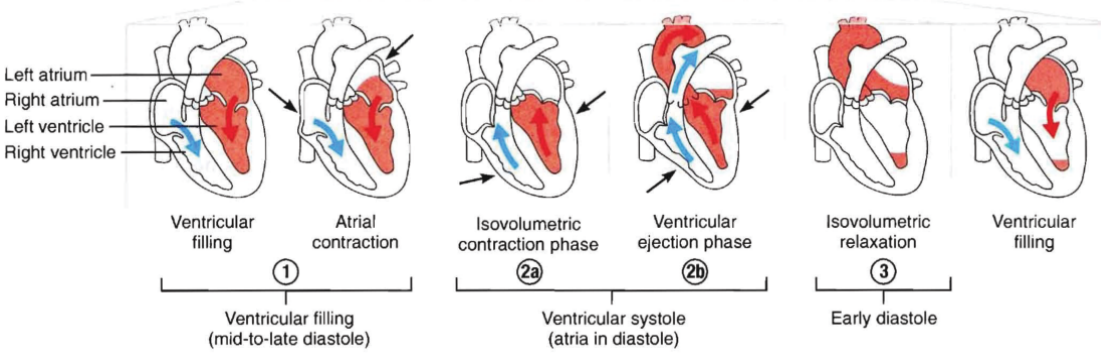


Figure 34: Heart Cycle.

3+1	Isovolumetric relaxation + Ventricular filling	Relaxation of the ventricular myocardium, leaflet valves are opened, Chambers are filled
1	Atrial contraction	Ventricles filled, leaflet valves open, pocket valves closed, ventricular pressure almost 0 mmHg
2a	Isovolumetric contraction phase	isometric contraction of the ventricular myocardium, leaflet valves are closed
2b	Ventricular ejection phase	isotonic contraction of the ventricular myocardium, pocket valves are opened, stroke volume is ejected, atria are filled

8. Why are there two vascular circuits serviced by the heart?

By separating into two circuits, not only the body but also the lungs are supplied with blood at full power. For this we need in the to vascular systems different pressures.

Pulmonary circuit: Lungs, exchange CO_2 with O_2. Low pressure circulation.

Systemic Circuit: Provision of body tissues with O_2 rich blood. High pressure circulation.

9. **Which diagram often used in thermodynamics can be used to estimate work done and power output of heart?**

   The pressure-volume diagram. The work done can be estimated as the area in the diagram for both ventricles respectively.

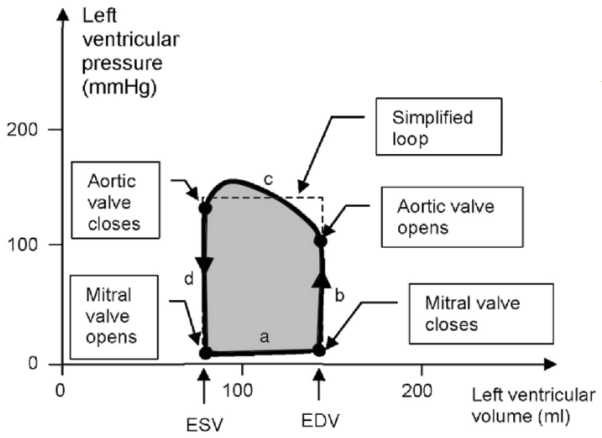


Figure 35: Pressure-volume digram. EDV: end diastolic volume; ESV: end systolic volume.

2.5 Circulatory system Part II

1. **What is the function of arteries?**

   She transport blood away from the heart into the body and smoothen the blood flow and pressure in the process.

2. **What is the structure of an artery (layers) and what are the main components?**

   The artery have tree layers:

   - Tunica intima
     - Endothelium
     - Subendothelial layer
   - Tunica media
   - Tunica externa

   The main components of an artery are Elastin, collagen fibers, smooth muscles and elastic fibers.

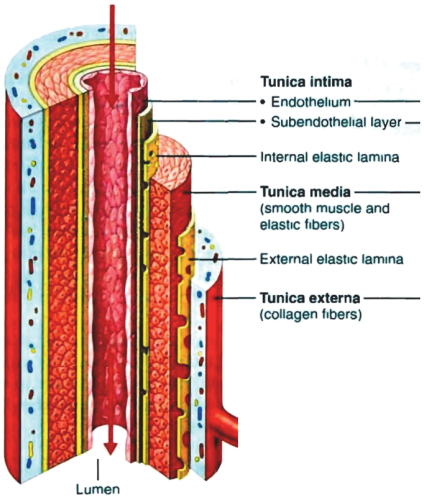


Figure 36: Structure of blood vessels - arteries.

3. **Which behaviour do arteries show under pulsatile pressure?**

   The arteries bulge during a systole to smoothen the blood flow and pressure. Simplest model is the Windkessel model who describe this behaviour.

4. **What is the Korteweg Moens wave speed?**

   It describes the speed of the pressure pulse wave.

$$c_0 = \sqrt{\frac{Et}{\rho D}}$$

$c_0 \ldots Korteweg - Moens\ wave\ speed\ [m/s]$
$E \ldots Elastic\ modulus\ [Pa]$
$t \ldots Wall\ thickness\ [m]$
$\rho \ldots Blood\ density\ [kg/m^3]$
$D \ldots Vessel\ diameter\ [m]$

5. **What is the function of capillaries?**

   The provides an interface (*Schnittstelle*) between bloodstream and organs. The deliver gases, nutrients and hormones to the e.g. muscles, organs and take away wast products.

6. **What is the Windkessel Model?**

   This model is a simplification of the arterial system to describe the blood volume flow, accounting for the interaction of the blood pressure waves with the compliance and resistance of elastic arteries and arterioles.

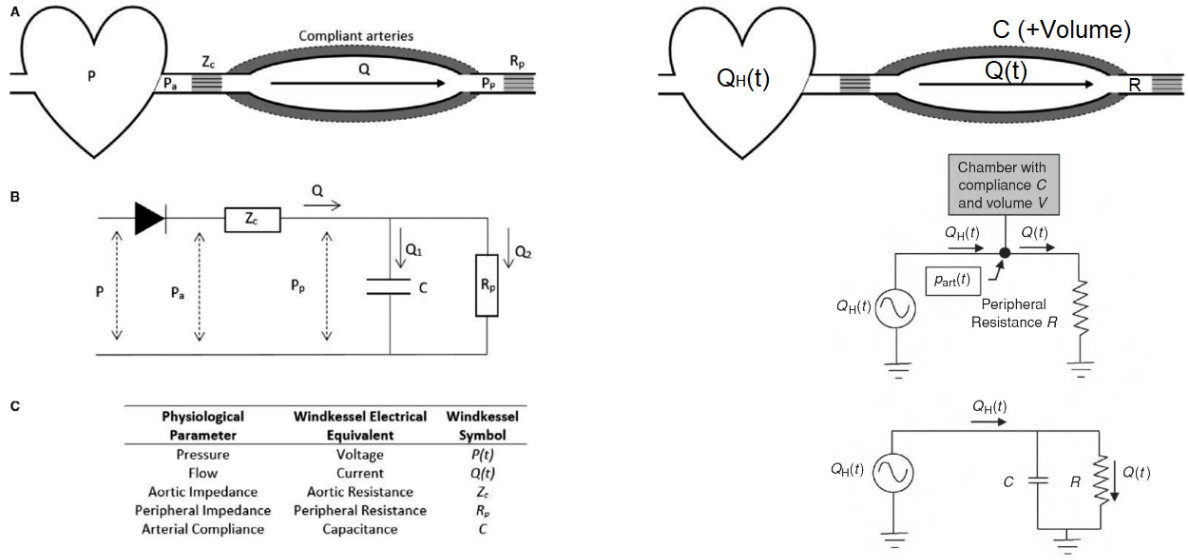


Figure 37: Mechanical and electrical representation of a 3-element Windkessel system. (A) A diagram of the Windkessel arterial system. (B) A diagram of the electrical circuit representing a 3-element Windkessel model. (C) Relation of Windkessel parameters to their physiological equivalents.

7. What is the structure of a capillary?

The capillary have only one layer:

- Tunica intima

The main components of an capillary are basement membrane and endothelial cells.

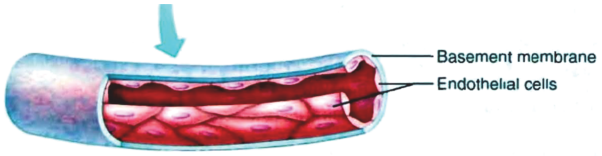


Figure 38: Structure of Capillaries.

8. Are capillaries always open?

It depends. If it is necessary to supply more blood to other important organs, some of the capillary beds can be excluded from the blood flow. This is done by the metarterioles of which some have muscles called precapillary sphincters. By contracting these precapillary sphincters, the blood flow into the capillary bed can be stopped.

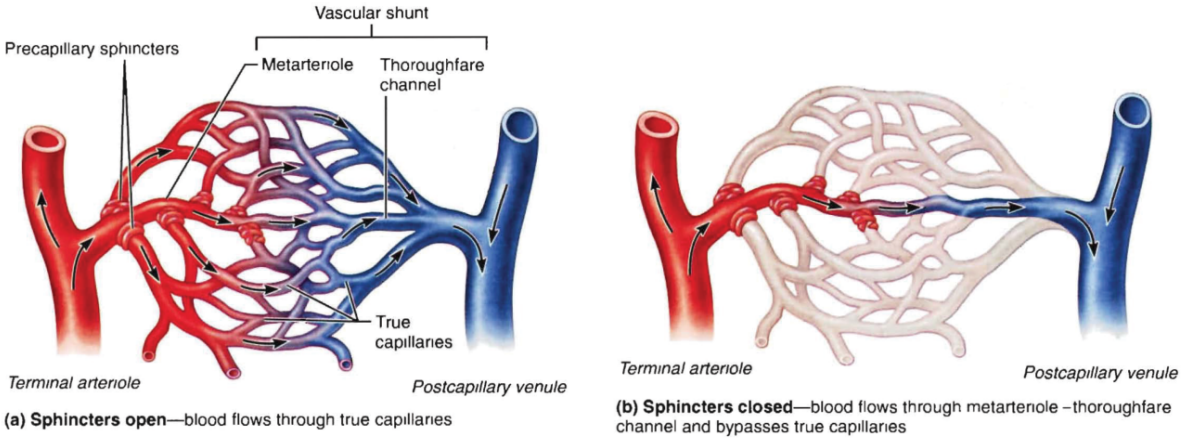


Figure 39: Anatomy of a Capillary Bed with blood flow during open/closing process.

9. Why is osmotic pressure important for blood flow in capillaries?

It regulates and drives solvent flow even when fluid pressure (Δp) alone would not allow any flow $\longrightarrow$ Equilibrium between filtration and resorption. Osmotic pressure ($\Delta \pi$) is crucial for resorption of solvent in capillaries.

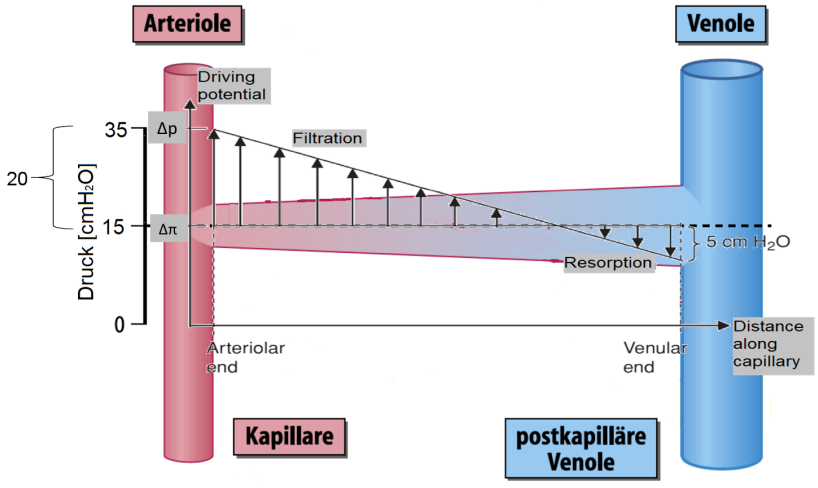


Figure 40: Pressure gradient of osmotic pressure in the capillaries. Δp is fluid pressure difference between capillary and tissue; $\Delta \pi$ is osmotic pressure difference between capillary and tissue.

10. What is the function of veins?

There collect low pressure blood and transport it back to the heart. She also are important blood reservoir, she contains 65 % of the total blood.

11. Why do veins contain valves?

The valves aid transport blood back towards the heart by restricting (*einschränken*) flow back away form the heart. For this they ned valves that are autonomous closed when blood want to flow back.

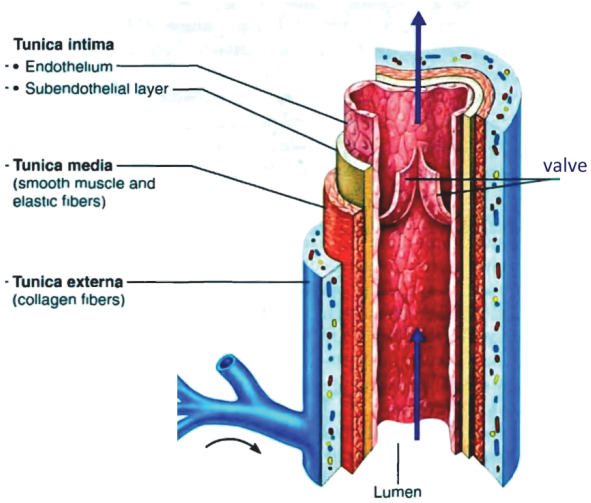


Figure 41: Structure of Veins.