183)

(a) $\lim_{x \to 1-} \ln(1-x) \cdot \ln(x)$

(b) $\lim_{x \to \infty} x \ln\left(1 + \frac{1}{x}\right)$

$$Regel\ de\ l`Hospital:$$

$Gilt: \lim_{x \to x_0} \frac{f(x)}{g(x)}\ und\ \lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x) = 0, oder\ \lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x) = \infty$

$dann\ gilt: \lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$

$(weitere\ unbestimmte\ Formen\ bei\ denen\ l'Hospital\ einsetzbar\ ist: 0 * \infty, \infty - \infty, 1^\infty, 0^0)$

(a)

$\lim_{x \to 1-} \ln(1-x) * \ln(x) = \lim_{x \to 1-} \frac{\ln(1-x)}{\ln(x)^{-1}}, die\ Division\ ist\ für\ de\ l`Hospital\ notwendig!$

$\frac{\ln(1-1)}{\ln(1)^{-1}} = \frac{\ln(0)}{0^{-1}} => \ln(0)\ unbestimmt, de\ l`Hospital\ anwendbar! Ableiten!$

$\lim_{x \to 1-} \frac{\frac{1}{1-x}\cancel{(-1)}}{\cancel{-1}(\ln(x))^{-2} * \frac{1}{x}} = \lim_{x \to 1-} \frac{\frac{1}{1-x}}{\frac{1}{(\ln(x))^2 * x}} = \lim_{x \to 1-} \frac{(\ln(x))^2 * x}{1-x}$

$\frac{(\ln(1))^2 * 1}{1-1} = \frac{0}{0} => de\ l`Hospital! (Ketten-und\ Produktregel)$

$\lim_{x \to 1-} \frac{2(\ln(x))^1 * \frac{1}{\cancel{x}} * \cancel{x} + (\ln(x))^2 * 1}{1} = \lim_{x \to 1-} \frac{2 * \ln(x) + (\ln(x))^2}{1} = \lim_{x \to 1-} \ln(x)(2 + \ln(x))$

$\ln(1) * (2 + \ln(1)) = 0 * (2 + 0) = 0$

(b)

$\boldsymbol{für\ die\ elegante\ Lösung\ thx\ an\ Dr. Rotz}!$

$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

$$\lim_{x \to \infty} x ln\left(1 + \frac{1}{x}\right) = \lim_{x \to \infty} ln\left(1 + \frac{1}{x}\right)^x = \ln \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = \ln e^1 = 1$$