Status	Beendet
Begonnen	Donnerstag, 1. Mai 2025, 14:00
Abgeschlossen	Donnerstag, 1. Mai 2025, 14:18
Dauer	18 Minuten 15 Sekunden
Bewertung	**9,63** von 10,00 (**96,25**%)

Frage 1

Teilweise richtig

Erreichte Punkte 0,63 von 1,00

Assume that $e(j(d, h)) \land \forall o(m(l(o), n(d)) \to \forall i(k(l(o))))$ is a well-formed formula. Classify each symbol as either a function or a predicate by drag-and-dropping it to the correct location.

Important: The symbols should be specified in alphabetical order. For instance, in $a(b) \land c(d)$, the predicate symbols should be specified as a, c and the function symbols as b, d. Furthermore, note that constants are 0-ary functions.

- The predicate symbols are e, k, m.
- The function symbols are d, h, l, n, j.

Frage 2

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{a\ (arity\ 0), b\ (arity\ 0), f\ (arity\ 2), g\ (arity\ 1)\} \subseteq \mathcal{F}$ and $\{x, y\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. Which formulas are **not** well-formed formulas in First-Order Logic?

Important: Note that there can be more than one correct answer.

- [ ] a. $\forall x(Q(a, b) \to P(x, b)) \to P(f(a, b), g(a))$
- [x] b. $(P(b, b) \to Q(g(b))) \to \exists(x \land y)P(f(x, b), g(x))$
- [x] c. $\forall xQ(f(x, b), g(x)) \land \neg(P(f(a), b) \to Q(g(a)))$
- [x] d. $\exists xQ(f(x, b), g(f(x, b))) \land \neg(f(b, a) \lor Q(g(b)))$

Frage 3

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{a\ (arity\ 0), g\ (arity\ 1)\} \subseteq \mathcal{F}$ and $\{x\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. How many **different terms** occur in the formula $P(a, g(a)) \vee \exists x(P(a, x) \rightarrow Q(a, x))$? Note that if a term occurs twice, it only counts as one.

Example: The formula $P(f(x), g(x), f(a))$ contains 4 unique terms - x, $f(x)$, $g(x)$, $f(a)$.

Antwort: 3

Frage 4

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2), Q\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{f\ (arity\ 2), g\ (arity\ 1)\} \subseteq \mathcal{F}$ and $\{z, x, y\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. Which variables have free occurrences in the formula $\forall x(P(f(x, x), f(x, y)) \wedge \exists x Q(f(x, z), g(x)))$? Note that variables that have both free and bound occurrences should also be listed.

Important: In order for your answer to be properly validated, input your variables comma-separated and with no white-spaces in between. For instance, for the formula $\forall y P(x, y) \wedge P(x, y)$ your answer should be: *x,y*

Antwort: y,z

Frage 5

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the signature $\mathcal{S}$ with $\{P\ (arity\ 2)\} \subseteq \mathcal{P}$, $\{g\ (arity\ 1), a\ (arity\ 0), f\ (arity\ 2), b\ (arity\ 0)\} \subseteq \mathcal{F}$ and $\{x\} \subseteq \mathcal{V}$ predicates, functions and variables, respectively. How many **subformulas** does the formula $(\exists x P(g(x), x) \wedge P(a, g(a))) \wedge P(f(a, b), g(a))$ have? Note that if a subformula occurs twice, it only counts as one.

Antwort: 6

Frage 6

Richtig

Erreichte Punkte 1,00 von 1,00

How is the correct formalization of the natural language sentence "All Viennese people are friendly."?

- a. $\forall x\ \neg Viennese(x) \lor \forall x\ Friendly(x)$
- b. $\forall x(Viennese(x) \to Friendly(x))$
- c. $Viennese(x) \to \forall x\ Friendly(x)$
- d. $\forall x\ Viennese(x) \to \forall x\ Friendly(x)$

Frage 7

Richtig

Erreichte Punkte 1,00 von 1,00

Which formula is equivalent to the **negation** of $\forall x\ P(x)$?

- a. $\exists x\ \neg P(x)$
- b. $\exists x\ P(x)$
- c. $\neg\exists x\ \neg P(x)$
- d. $\neg\exists x\ P(x)$

Frage 8

Richtig

Erreichte Punkte 1,00 von 1,00

Which one of the following entailments holds?

- a. $\forall x(P(x) \lor Q(x)) \models \exists y\ Q(y)$
- b. $\forall x(P(x) \to Q(x)), P(a) \models \exists y\ Q(y)$
- c. $\exists x(P(x) \to Q(x)), \exists x\ P(x) \models \forall y\ Q(y)$
- d. $\forall x(P(x) \to Q(x)), P(a) \models \forall y\ Q(y)$

Frage 9

Richtig

Erreichte Punkte 1,00 von 1,00

Consider the formula $\varphi := \exists x(\neg\forall y P(x, y) \wedge \forall y P(y, x))$. Given the structure $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with domain $D_{\mathcal{M}} = \{a, b\}$, your task is to specify the relation $P^{\mathcal{M}}$ associated to the predicate symbol P by $I_{\mathcal{M}}$, such that the formula φ is satisfied by $\mathcal{M}$.

Note: List the tuples in $P^{\mathcal{M}}$ comma-separated, with no spaces in between. In case $P^{\mathcal{M}}$ should not contain any elements, input "empty" (without the quotes) in the field below. Furthermore, note that multiple correct answers may be possible.

Example: For $\exists x P(x, x)$ and $\mathcal{M} = (\{a, b\}, I_{\mathcal{M}})$, a possible input could be: *(a,a),(a,b)*

Antwort: (a,a),(b,a)

Frage 10

Richtig

Erreichte Punkte 1,00 von 1,00

Which of the following interpretations is a model of the formula $\forall x P(x, x)$? Note that more than one answer may be correct.

Note: We consider 0 as a natural number, i.e. $\mathbb{N} = \{0, 1, 2, \dots\}$

- [x] a. $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with $D_{\mathcal{M}} = \mathbb{N}$ and $P^{\mathcal{M}} = \{(n, n') \mid gcd(n, n') = n\}$, where $gcd(\cdot, \cdot)$ is the greatest common divisor
- [ ] b. $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with $D_{\mathcal{M}} = \mathbb{N}$ and $P^{\mathcal{M}} = \{(n, n') \mid n \cdot n' = 0\}$
- [ ] c. $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with $D_{\mathcal{M}} = \mathbb{N}$ and $P^{\mathcal{M}} = \{(n, n') \mid gcd(n, n') = 1\}$, where $gcd(\cdot, \cdot)$ is the greatest common divisor
- [x] d. $\mathcal{M} = (D_{\mathcal{M}}, I_{\mathcal{M}})$ with $D_{\mathcal{M}} = \mathbb{N}$ and $P^{\mathcal{M}} = \{(n, n') \mid n \mid n'\}$, where $a \mid b$ can be interpreted as b divides a