This is the fourth homework assignment. Students should tick in TUWEL problems they solved and upload their solutions by **20:00 on November 14, 2022**. There will be no exercises on November 15, 2022. Instead, randomly chosen solutions will be graded, and results will be discussed in the following week, on **November 22, 2022**.

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(1) `R` **-functions**

(a) `trees` is the `R` Dataset Package containing Diameter, Height and Volume for 31 Black Cherry trees. Using `R` define a vector h that contains the values of the column `Height` and define a vector v that contains the values of the column `Volume` from the dataset `trees`. Use the command `table(h)` to obtain the frequency table of the vector h. What are the outputs of the commands `summary(h)` and `factor(h)`? Compute the sample means `mean()` and the sample variances `var()` for both vectors. Use the command `plot()` to plot the points (h, v).

(b) Consider the curve given by the parametrization

$$t \mapsto \begin{pmatrix} e^{-\frac{t}{8}} \cdot \cos 2t \\ e^{\frac{t}{5}} \cdot \sin 3t \end{pmatrix} \quad \text{for} \quad 0 \leq t \leq 5\pi\,.$$

Use `plot()` to plot the curve and `dev.print()` to save the result as a pdf file.

(2) **Coffee and doughnuts**
At a certain coffee shop, all the customers can buy a cup of coffee and also a doughnut. The shop owner believes that the number of cups she sells each day is normally distributed with an expectation of 320 cups and a standard deviation of 20 cups. She also believes that the number of doughnuts she sells each day is independent of the coffee sales and is normally distributed with an expectation of 150 doughnuts and a standard deviation of 12.

(a) The shop is open every day but Sunday. Assuming day-to-day sales are independent, what is the probability she will sell more than 1900 cups of coffee in a week?

(b) If she makes a profit of 1.2 euro on each cup of coffee and 0.7 euro on each doughnut, can she reasonably expect to have a day's profit over 550 euro? Justify your answer.

(3) **Mail order company**
A mail order company provides free examination of its products for 7 days. If not completely satisfied, a customer can return the product within that period and get a full refund. According to past records of the company, an average of 2 of every 10 products sold by this company are returned for a refund.

(a) Compute the probability that no more than 6 of the 40 products sold by this company on a given day will be returned for a refund.

(b) Use a Poisson distribution to approximate the probability in (a). What can be said about the accuracy of this approximation?

(4) **Inversion**
Let F be a continuous cumulative distribution function (cdf).

(a) Let $U \sim \mathcal{U}(0,1)$ and $Y = F^{-1}(U)$. Find the cdf of Y.

(b) Let X be a random variable with cdf F and $Z = F(X)$. Find the cdf of Z.

(5) **Generating random numbers from a Weibull distribution**
The cumulative distribution function of a random variable $X \sim Weibull(k, \lambda)$ is given by

$$F_X(x) = \begin{cases} 1 - e^{-\left(\frac{x}{\lambda}\right)^k}, & x \geq 0 \\ 0, & x < 0 \end{cases},$$

where $k > 0$ is the shape parameter and $\lambda > 0$ is the scale parameter of the distribution.

Example (4) shows that if $U \sim \mathcal{U}(0,1)$, then $X = F^{-1}(u)$ generates a random number X from any continuous distribution with the specified cdf F. This gives us the following algorithm to generate random numbers from a random variable X with the given cdf F.

Algorithm: Inversion method

1° compute the inverse F^{-1} of F

2° generate n independent random numbers $u_1, u_2, \ldots, u_n$ from $\mathcal{U}(0,1)$

3° compute $x_1 = F^{-1}(u_1), x_2 = F^{-1}(u_2), \ldots, x_n = F^{-1}(u_n)$.

Then, $x_1, x_2, \ldots, x_n$ are independent random observations of the random variable X.

In `R`, generate 30 observations from a random variable X with the $Weibull(2,1)$ by applying the inversion method.