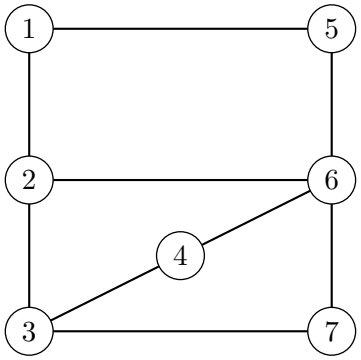
5. ÜBUNG
104.283 Diskrete Mathematik für Informatik

(34) (a) The line graph $\bar{G}$ of a simple undirected graph $G = (V, E)$ is the (simple) graph with vertex set E, with an edge between two vertices of $\bar{G}$, if and only if the corresponding edges are incident to a common vertex of G. Show that the line graph of an Eulerian graph is Eulerian and Hamiltonian, and that the line graph of a Hamiltonian graph is also Hamiltonian. If $\bar{G}$ is Hamiltonian, can we conclude that G is Hamiltonian?

(b) Is a subdivision of an Eulerian graph Eulerian? Is a subdivision of a Hamiltonian graph Hamiltonian?

(35) Compute the closure of the following graph:



(36) Show that the n-dimensional hypercube is Hamiltonian for $n \geq 2$.

(37) For which m and n does the complete bipartite graph $K_{m,n}$ have a Hamiltonian cycle?

(38) Given a subset $A \subseteq \mathbb{R}^2$ with area a and two decomposition of A into subsets $A_1, A_2, \ldots, A_m$ and $B_1, B_2, \ldots, B_m$ such that all the sets A_i and B_i have the same area a/m. Prove that there exists a permutation π of $\{1, 2, \ldots, m\}$ such that for all $i = 1, \ldots, m$ we have $A_i \cap B_{\pi(i)} \neq \emptyset$.

(39) Let G be an undirected simple graph and H a subgraph of G satisfying $H \cong K_n$ for some n. What can be said about the relation between $\chi(G)$ and $\chi(H)$? Find a graph G with $\chi(G) = 3$ which does not have a K_3 as a subgraph.

(40) Show that a planar graph G has as many spanning trees as its dual.

(41) Let $X = \{1, 2, 3, 4, 5, 6\}$. Which of the following sets are topologies on X?

(a) $\{\{1\}, \{2\}, \{1, 2\}, \{1, 4, 5\}, \{1, 2, 4, 5\}, X\}$

(b) $\{\emptyset, \{1\}, \{1, 2, 3\}, \{2, 3, 4\}, \{4, 5, 6\}, X\}$

(c) $\{\emptyset, \{1\}, \{3\}, \{5\}, \{1, 3\}, \{1, 5\}, \{5, 6\}, \{1, 3, 5\}, \{1, 5, 6\}, \{3, 4, 5\}, \{3, 5, 6\}, \{1, 3, 4, 5\}, \{1, 3, 5, 6\}, \{1, 3, 4, 5, 6\}, X\}$

(d) $\{\emptyset, \{1\}, X\}$