Prüfung – VU Discrete Mathematics - WS 2018

NACHNAME: ..... **Vorname:** ..... Matr. Nr.: .....

1. (a) State Euler's formula relating the number of vertices, edges and faces in a graph. For which graphs is it valid?

   (b) Prove the formula by using induction on the number of faces. You may use the formula relating the number of edges and the number of vertices in a tree without proof.

   (c) A connected simple planar graph without triangles has at least twice as many edges as faces. Deduce that $K_{3,3}$ is not planar.

   (d) Draw an explicit embedding of K_4 in the torus and deduce the Euler characteristic of the torus.

6 Punkte (2+2+1+1)

2. Let $G = (V, E, w)$ a weighted directed graph with source s and sink t. **6 Punkte (1+1+1+1+2)**

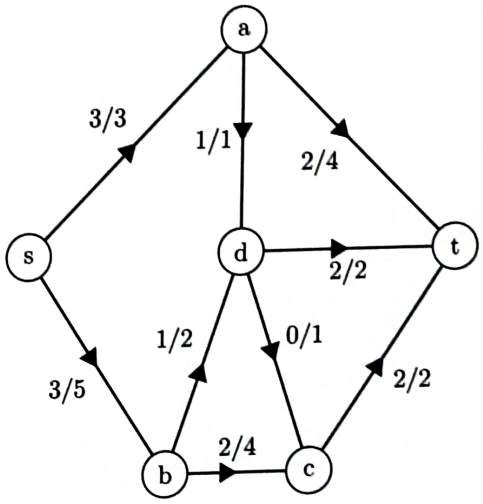
   (a) What is a flow Φ on G?

   (b) What is an augmenting path for Φ?

   (c) What is a cut of G, and what is its capacity?

   (d) Suppose that there is no augmenting path for Φ. Define a cut whose capacity equals the value of Φ.

   (e) Find an augmenting path for the given flow in the graph below. Then indicate a minimal cut of the graph.



3. Let $G = (V_1 \cup V_2, E)$ be a bipartite graph such that for every subset $W \subseteq V_1$, the set of neighbours $\mathcal{N}(W)$ has cardinality at least $|W|$. Let H be the digraph obtained from G by adding a source s and a sink t, and directing all edges from V_1 to V_2. Let the weight of an edge from V_1 to V_2 be ∞, and the weight of all other edges be 1. Let S be a minmial cut and let $W = S \cap V_1$. Prove that

   (a) $S \cap V_2 \supseteq \mathcal{N}(W)$.

   (b) $c(S) = |V_1 \setminus W| + |S \cap V_2|$.

   (c) $c(S) \geq |V_1|$. **4 Punkte (2+1+1)**