217) $\int \frac{x^3 - x^2 + 2}{x^3 - 3x + 2}\,\mathrm{d}x$

Partialbruchzerlegung

$(x^3 - x^2 + 2) : (x^3 - 3x + 2) = 1 + \frac{3x - x^2}{x^3 - 3x + 2}$

$-(x^3 - 3x + 2)$

$/\ 3x - x^2$

Nullstellen zählen:

geraten: 1

$(x^3 - 3x + 2) : (x - 1) = x^2 + x - 2$

$-(x^3 - x^2)$

$/\ x^2 - 3x$

$-(x^2 - x)$

$/\ -2x + 2$

$-(-2x + 2)$

$/\ /$

$-\frac{1}{2} \pm \sqrt{\frac{1^2}{4} + 2} = -\frac{1}{2} \pm \frac{3}{2}$

$x_0 = 1$

$x_1 = 1$

$x_2 = -2$

wegen 2facher Nullstelle

$$\frac{3x - x^2}{x^3 - 3x + 2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$3x - x^2 = A \cdot (x^2 + x - 2) + B \cdot (x + 2) + C \cdot (x^2 - 2x + 1)$

$-x^2 + 3x = (A + C)x^2 + (A + B - 2C)x + (-2A + 2B + C)$

LSG

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ -2 & 2 & 1 & 0 \end{array}\right) \xrightarrow{\text{II}-\text{I},\ \text{III}+2\cdot\text{I}} \left(\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -3 & 4 \\ 0 & 2 & 3 & -2 \end{array}\right) \xrightarrow{\text{III}-2\cdot\text{II}} \left(\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & 9 & -10 \end{array}\right)$$

$C = -\frac{10}{9}$

$B = 4 + 3C = \frac{2}{3}$

$A = -1 - C = \frac{1}{9}$

Summenregel

$$\int \frac{x^3 - x^2 + 2}{x^3 - 3x + 2}\,dx = \int 1 + \frac{1}{9(x-1)} + \frac{2}{3(x-1)^2} - \frac{10}{9(x+2)}$$

$\int \frac{1}{(x-1)^2}\,dx$

$= \int u^{-2}\,du = \frac{u^{-1}}{-1}$

$= -\frac{1}{x-1} + C$

$$= x + \frac{1}{9}\ln(x-1) - \frac{2}{3(x-1)} - \frac{10}{9}\ln(x+2) + C$$