Einführung in Wissensbasierte Systeme, 3.0 VU, 184.737
Exercise Sheet 3 - Nonmonotonic Reasoning

You have to tick the prepared exercises in TUWEL at the latest before

Friday, Dec 7 2012, 13:00 CET (Kreuzerlübung Exercise Sheet 3).

Be sure that you tick only subtasks which you can solve and explain on the blackboard!

Exercise 1 (1 pts.): Let T be a theory consisting of the formulas

$$\forall x\,(p(x) \vee r(x)) \tag{1.1}$$
$$\forall x\,(\neg q(x) \rightarrow r(x)) \tag{1.2}$$
$$p(a) \wedge r(b) \tag{1.3}$$
$$\neg q(a) \tag{1.4}$$

over the alphabet containing unary predicate symbols p, q, and r, and only the object constants a and b.

- Which of the literals $p(b), q(b), r(b), \neg p(b), \neg q(b)$ and $\neg r(b)$ are contained in $\mathrm{CWA}(T)$?
- Is $\mathrm{CWA}(T)$ consistent? Justify your answer.
- Are the following formulas contained in $\mathrm{CWA}(T)$?

$$p(a) \wedge p(b) \tag{1.5}$$
$$\exists x\,(p(x) \wedge r(x)) \tag{1.6}$$
$$\neg q(a) \rightarrow r(a) \tag{1.7}$$

Exercise 2 (2 pts.): Let $T = (W, \Delta)$ be a propositional default theory over atoms a, b, and c, where $W = \emptyset$ and Δ is the set of defaults

$$\frac{: \neg b}{a} \tag{δ_1}$$
$$\frac{a : \neg b}{c} \tag{δ_2}$$
$$\frac{c : \neg a}{b} \quad . \tag{δ_3}$$

- Determine the defaults applicable to $Cn(W)$ relative to E, as well as the classical reduct Δ_E for

$$E = Cn(\{a\}), \tag{2.1}$$
$$E = Cn(\{b\}),\ \text{and} \tag{2.2}$$
$$E = Cn(\emptyset). \tag{2.3}$$

- Determine $\Gamma_T(E)$ for (2.1)–(2.3).

- Determine the extensions of T.

Exercise 3 (2 pts.): Show that any closed normal default theory (W, D) has an extension.
Hint: First deal with the trivial case that W is inconsistent and show that the default theory has an inconsistent extension. For the general case, where W is consistent, modify the semi-recursive characterisation of extensions such that it constructs an extension without assuming a given set E (substitute E with E_i in the definition of the steps). Show that the set constructed by this procedure is in fact an extension of the given default theory by showing that it reproduces itself in the semi-recursive characterisation.

Exercise 4 (3 pts.): You have the following information:
(1) There is a party.
(2) When there is a party, Paris is usually attending.
(3) When there is a party, Britney is usually attending.
(4) When there is a party and Paris is attending, then Britney does definitely not participate.

(a) Formalise this situation in terms of a default theory. It should have an extension indicating that Paris attends, as well as an extension that states that Britney attends. Use $Party$ for "there is a party," $Comes\_Paris$ for "Paris is attending the party," and $Comes\_Britney$ for "Britney is attending the party." (1 pt.)

(b) Replace information (4) from above by a formalisation of

(4') If there is a party and Paris attends, then Britney usually does not attend.

How does the default theory change? Which extensions emerge after this replacement? Are all of them plausible? (1 pt.)

(c) Change your defaults in a way that only intuitive extensions emerge.

More precisely: the rule expressing information (4'), determining what Britney does when there is a party that Paris is attending, should have priority over the less specific rule expressing information (3), indicating what Britney is doing when there is a party. You are only allowed to change default justifications. (1 pt.)

Exercise 5 (2 pts.): Let T be a consistent propositional theory. Show that $\mathrm{CWA}(T)$ is inconsistent iff there are atomic formulas $a_1, \ldots, a_n$ such that $T \models a_1 \vee \cdots \vee a_n$ and $T \not\models a_i$ for $i = 1, \ldots, n$.

Hint: to show the only-if direction (left to right implication) you might first prove that if $\mathrm{CWA}(T)$ is inconsistent, then T_{asm} is non-empty. Further on, you might find it useful to consider the propositional variables appearing in T_{asm} and to use the Contradiction Theorem.