Examination for "Logic and Computability"

May 4th, 2021 **4^{th} exam for WS 20/21**

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Task 1:
Consider the following reasoning (involving 'megabarbers', 'bureaucratic barbers' – 'burobarber' in short – and 'megaburobarbers' who are both megabarbers and burobarbers):

(a) From 'every megabarber shaves all those who do not shave themselves' follows 'there does not exist any megabarber'.

(b) From 'every burobarber shaves only those who do not shave themselves' follows 'there does not exist any burobarber'.

(c) From 'every megaburobarber shaves all those and only those who do not shave themselves' follows 'there does not exist any megaburobarber'.

Formalize (a), (b) and (c), and establish which of these consequence claims are correct.
(In each case, provide a proof or a countermodel.)

Task 2:
Prove or disprove the following statement $\models \exists x(\exists y A(y) \to A(x))$.

Task 3:

Recall that a function f is 1-1 if $f(x) = f(y) \Rightarrow x = y$. Is the set

$$\{x \mid \Phi_x \text{ is 1-1 }\}$$

recursive, r.e. or none of them? (Motivate your answer.)

Task 4:
Let g be a total computable function. Is the set

$$H = \{x \mid \neg\exists y \text{ s.t } \Phi_{g(y)} = x)\}$$

recursive, r.e. or none of them?

Task 5:
Show by Robinson-resolution that the clause set $\{C, D, E\}$ is unsatisfiable, where
$C = q(f(x), c) \vee q(y, c)$,
$D = q(f(y), f(x)) \vee \neg q(f(x), y)$
$E = \neg q(f(f(x)), f(c))$.
Specify all used factors, MGUs, and unified literals. (c is a constant, x, y are variables.)

Task 6:
Let G be the modal formula $\Diamond A \vee \Box\Box\neg A$. Prove or refute:
(1) G is valid in every transitive frame.
(2) If $\mathcal{F} \models G$ for a frame $\mathcal{F}$, then $\mathcal{F}$ is transitive.

Task 7:
Use the proof of the ADRF theorem to show that the function $3 \cdot g(f(x), f(x+1))$ is arithmetically definable, if f and g are arithmetically definable. *Hint:* Specify the witnessing formula, following slide 23 of the last set of slides (on incompleteness).