Problem Set 3

Problem 3.1 Consider the pdf

$$f_{\mathsf{x}}(x) = a\left(3e^{-|x|} + 4\delta(x) + 2\delta(x-7)\right), \quad a > 0.$$

Here, $\delta(\cdot)$ denotes the Dirac impulse.

a) Find the constant a.

b) Calculate the probability $\mathrm{P}\{0 < \mathsf{x} \leq 7\}$.

c) Calculate the probability $\mathrm{P}\{\mathsf{x} = 0\}$.

d) Find the variance σ_{x}^2.

Problem 3.2 Consider the joint pdf

$$f_{\mathsf{x},\mathsf{y}}(x,y) = \begin{cases} b \cdot e^{-(x+y)}, & 0 < x < a \quad \text{and} \quad 0 < y < \infty \\ 0, & \text{otherwise} \end{cases}.$$

a) Find b (in terms of a) such that $f_{\mathsf{x},\mathsf{y}}(x,y)$ is a valid joint pdf.

b) Find an expression for the joint cdf $F_{\mathsf{x},\mathsf{y}}(x,y)$.

c) Find the marginal pdfs $f_{\mathsf{x}}(x), f_{\mathsf{y}}(y)$.

Problem 3.3 Let x and y be two random variables with $\mu_{\mathsf{x}} = 2$, $\mu_{\mathsf{y}} = -1$, $P_{\mathsf{x}} = 5$, $C_{\mathsf{x},\mathsf{y}} = 3$. Let $\mathsf{z} = a\mathsf{x} + \mathsf{y}$.

a) Find the correlation $R_{\mathsf{x},\mathsf{z}}$.

b) Find a such that $R_{\mathsf{x},\mathsf{z}} = 0$.

c) Find the covariance $C_{\mathsf{x},\mathsf{z}}$ for the value of a found in the previous subtask.

Problem 3.4 Consider two random variables x, y whose joint pdf $f_{\mathsf{x},\mathsf{y}}(x, y)$ is depicted in the figure below.

a) Provide a mathematical expression for $f_{\mathsf{x},\mathsf{y}}(x, y)$.

b) Find the marginal pdfs $f_{\mathsf{x}}(x), f_{\mathsf{y}}(y)$ and sketch them.

c) Calculate the probability $\mathrm{P}\{(\mathsf{x} \leq 0) \cap (\mathsf{y} \leq 1)\}$.

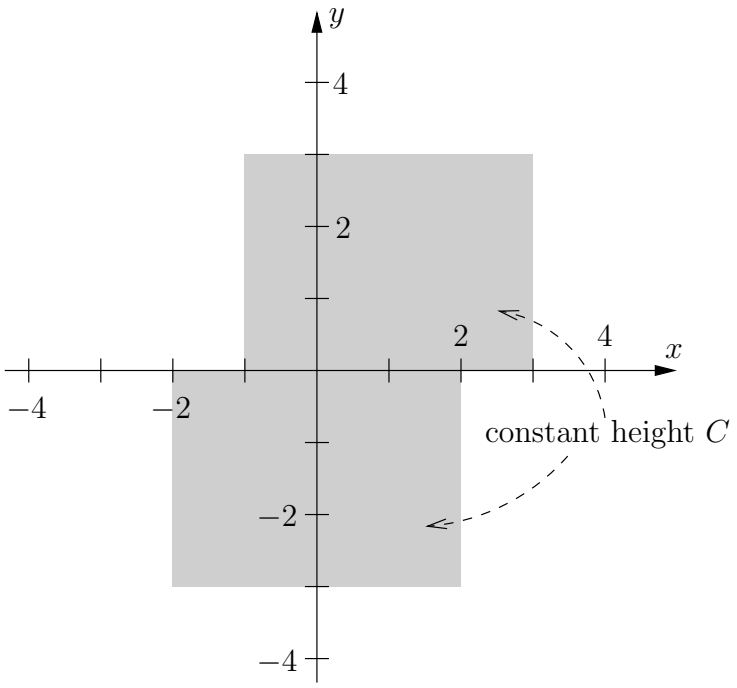

Figure 1: Sketch of the joint pdf $f_{\mathsf{x},\mathsf{y}}(x, y)$.