Einführung in Artificial Intelligence SS 2024, 4.0 VU, 192.027

Exercise Sheet 4 – Knowledge Representation and CSP

For the discussion part of this exercise, mark and upload your solved exercises in **TUWEL** until Wednesday, June 5, 23:55 CEST. The registration for a solution discussion ends on Friday, June 7, 23:55 CEST. Be sure that you tick only those exercises that you can solve and explain!

In the discussion, you will be asked questions about your solutions of examples you checked. The discussion will be evaluated with 0–25 points, which are weighted with the fraction of checked examples and rounded to the next integer. There is *no minimum number of points* needed for a positive grade (i.e., you do not need to participate for a positive grade, but you can get at most ≈80% without doing exercises).
Note, however, that *your registration is binding*. Thus, *if* you register for a solution discussion, then it is *mandatory* to show up. Not coming to the discussion after registration will lead to a reduction of examination attempts from 4 to 2.
Please ask questions in the **TUWEL** forum or visit our tutors during the tutor hours (see **TUWEL**).

Exercise 4.1: A magic square is an $N \times N$ square grid, containing each number from 1 to N^2 exactly once, where the sum of each row, column, and diagonal must be the same. We consider a 4×4 grid in which the sums should all be 34:

8	A	14	B
13	C	7	D
E	16	F	6
G	5	H	I

Some of the numbers are already given, the unknown numbers have been replaced by variables.

(a) Describe the corresponding CSP with its variables and constraints and specify the initial domain of each variable.

(b) Draw the constraint graph.

(c) Find a solution of the puzzle. How many unique solutions exist? You may use automated solvers or program your own, but be sure that you can explain how a solution could be found by hand.

Exercise 4.2:

(a) Given a single ternary constraint $A + B = C$. Transform this constraint into 3 binary constraints achieving the same functionality using auxiliary variables.

(b) Show how unary constraints can be eliminated by altering the domains of variables.

Exercise 4.3: The Joker, the Penguin, and the Riddler meet. They make the following statements:

- JOKER: Penguin tells the truth.

- PENGUIN: Riddler lies.
- RIDDLER: You two either both tell the truth or both lie.

Which of the villains lie and which tell the truth?

To answer this puzzle, represent the statements in propositional logic and use truth tables to determine a model of the conjunction of the three statements. Use the atomic formulas J, P, and R for representing the statements that

- the Joker tells the truth,
- the Penguin tells the truth, and
- the Riddler tells the truth,

respectively.

Exercise 4.4: Prove the following semantic consequences:

- $A, B \models A \wedge B$
- $A, A \vee B \not\models B$
- $A \vee B, A \rightarrow C, B \rightarrow D \models C \vee D$

Exercise 4.5: Consider the following sentences:

1. Every student knows a student who passed the exam.
2. Every student who solved all the exercises also passed the exam.
3. Some student did not solve the exercises, but passed the exam.
4. Some student who passed the exam does not know any student who solved all of the exercises.

(a) Formalise these sentences in first-order logic by using the following predicates:

- $K(x, y)$: "Student x knows student y",
- $P(x)$: "Student x passed the exam",
- $E(x)$: "Student x solved all of the exercises",

(b) Is the set of formulas resulting from the formalisation in (a) satisfiable? If yes, provide a model; if no, give a justification.

Exercise 4.6: Prove the unsatisfiability of the following propositional formula using resolution:

$$(\neg x \vee \neg y \vee \neg z) \wedge (\neg x \vee y \vee \neg z) \wedge (x \vee \neg y \vee z) \wedge \\ (\neg x \vee \neg y \vee z) \wedge (x \vee \neg y \vee \neg z) \wedge (x \vee y \vee \neg z) \wedge z.$$

4.1

$N \times N$ Grid, $N = 4$

$4 \times 4 \Rightarrow 16$ tiles

allowed values per field $= [1, N^2] = [1, 16]$

4.1a

$V = \{A, B, C, D, E, F, G, H, I\}$

already set values $= \{5, 6, 7, 8, 13, 14, 16\}$

Constraints:

- sum of each row $= 34$:
  - $8 + A + 14 + B = 34$
  - $13 + C + 7 + D = 34$
  - $E + 16 + F + 6 = 34$
  - $G + 5 + H + I = 34$
- sum of each column $= 34$:
  - $8 + 13 + E + G = 34$
  - $A + C + 16 + 5 = 34$
  - $14 + 7 + F + H = 34$
  - $B + D + 6 + I = 34$
- sum of each diagonal $= 34$:
  - $8 + C + F + I = 34$
  - $B + 7 + 16 + G = 34$

- each value $[1, 16]$ must occur exactly once

Domains:

$D = \{1, 2, 3, 4, 9, 10, 11, 12, 15\}$

$D(A) = \{1, 2, 3, 4, 9, 10, 11, 12\}$

$D(B) = \{1, 2, 3, 4, 9, 10, 11, 12, 15\}$

$D(C) = \{1, 2, 3, 4, 9, 10, 11, 12, 15\}$

$D(D) = \{1, 2, 3, 4, 9, 10, 11, 12, 15\}$

$D(E) = \{1, 2, 3, 4, 9, 10, 11, 12\}$

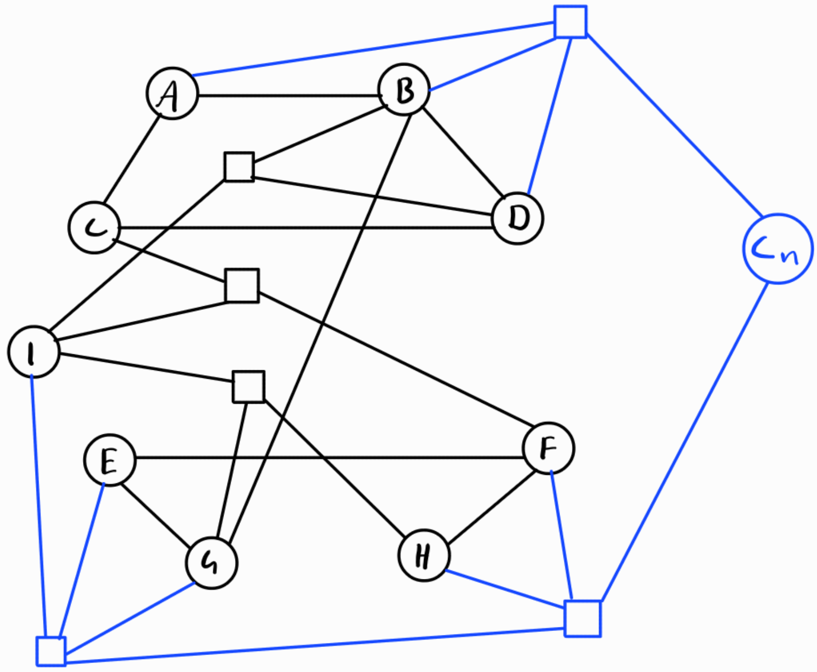
$D(F) = \{1, 2, 3, 4, 9, 10, 11, 12, 15\}$

$D(G) = \{1, 2, 3, 4, 9, 10, 11, 12, 15\}$

$D(H) = \{1, 2, 3, 4, 9, 10, 11, 12, 15\}$

$D(I) = \{1, 2, 3, 4, 9, 10, 11, 12, 15\}$

4.1b



C_n = values must be unique
other = sum constraints

4.1c

using system of linear equations:

A = 15 - H
B = H - 3
C = H - 2
D = 16 - H
E = H - 1
F = 13 - H
G = 14 - H
H = ?
I = 15

Now we look at all possible values for H in the domain D(H) = {1, 2, 3, 4, 9, 10, 11, 12, 15}

- Throw away solutions where values arent in [1, 15]
- Throw away solutions with duplicate values
- Throw away solutions that contain values that are already set {5, 6, 7, 8, 13, 14, 16}

This leaves us with the following 2 solutions

A = 11	A = 3
B = 1	B = 9
C = 2	C = 10
D = 12	D = 4
E = 3	E = 11
F = 9	F = 1
G = 10	G = 2
H = 4	H = 12
I = 15	I = 15

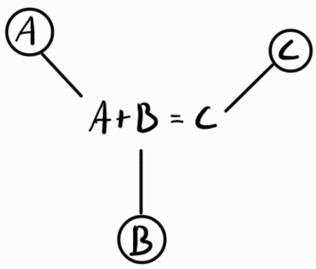
Python code:

```
from sympy import symbols, Eq, solve

# Define the variables
a, b, c, d, e, f, g, h, j = symbols('a b c d e f g h j')

# Define the equations
eq1 = Eq(8 + a + 14 + b, rhs: 34)
eq2 = Eq(13 + c + 7 + d, rhs: 34)
eq3 = Eq(e + 16 + f + 6, rhs: 34)
eq4 = Eq(g + 5 + h + j, rhs: 34)
eq5 = Eq(8 + 13 + e + g, rhs: 34)
eq6 = Eq(a + c + 16 + 5, rhs: 34)
eq7 = Eq(14 + 7 + f + h, rhs: 34)
eq8 = Eq(b + d + 6 + j, rhs: 34)
eq9 = Eq(8 + c + f + j, rhs: 34)
eq10 = Eq(b + 7 + 16 + g, rhs: 34)

# Solve the system of equations
solution = solve( f: (eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10), *symbols: (a, b, c, d, e, f, g, h, j))

# Print the solution
print(solution)
```

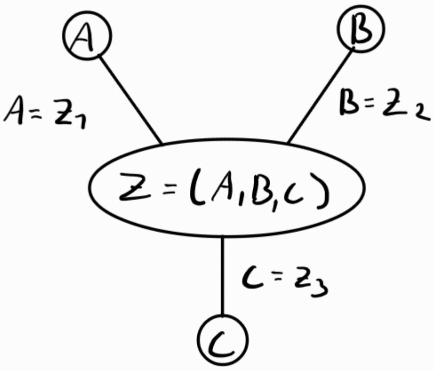
```
unavailable_values = {5, 6, 7, 8, 13, 14, 16}
h_domain = [1, 2, 3, 4, 9, 10, 11, 12, 15]


1 usage
def contains_unavailable_values(values):
    values_set = set(values)
    return len(values_set & unavailable_values) > 0


1 usage
def contains_duplicate_values(values):
    values_set = set(values)
    return len(values_set) != len(values)


1 usage
def contains_out_of_boundary_values(values):
    return any(not (0 < val < 16) for val in values)


for i in h_domain:
    h = i
    a = 15 - h
    b = h - 3
    c = h - 2
    d = 16 - h
    e = h - 1
    f = 13 - h
    g = 14 - h
    i = 15

    values = [a, b, c, d, e, f, g, h, i]
    if contains_duplicate_values(values) or contains_unavailable_values(values) or contains_out_of_boundary_values(values):
        continue

    print(values)
```

4.2a

- Unary constraints restrict the domain D_i of one variable V_i.
  E.g., $C(V_i) = \{1,3,5,7,8\}$.
- Binary constraints restrict the domains $D_i \times D_j$ of a pair of variables V_i, V_j.
  E.g., $C(V_i, V_j) = \{(1,2),(3,5),(7,3),(8,2)\}$.
- Ternary constraints,...



Constraint graph



Binarized Constraint Graph

Variables:

A, B, C, Z

Consider the following domains:

$D(A) = \{0,1\}$

$D(B) = \{0,1\}$

$D(C) = \{0,1,2\}$

$D(Z) = \{1,2,3,4\}$

The constraint $C_{\{A,B,C\}}$ has a corresponding auxiliary variable Z, whose domain can be set as $\{1,2,3,4\}$. (A unique identifier for each of the four tuples in the constraint.

$$D(Z) = \left\{ \begin{array}{l} 1 \to (0,0,0) \\ 2 \to (0,1,1) \\ 3 \to (1,0,1) \\ 4 \to (1,1,2) \end{array} \right\}$$

$\Downarrow$

$A=1$

$B=1$

$C=2$

We introduce a constraint between the pairs of variables $\{A, Z\}, \{B, Z\}, \{C, Z\}$ giving the binary constraints:

$C_{\{A,Z\}} = \{(0,1), (0,2), (1,3), (1,4)\}$

$C_{\{B,Z\}} = \{(0,1), (0,3), (1,2), (1,4)\}$ => if $B=0$ it is compatible with $Z=1$ and $Z=3$

$C_{\{C,Z\}} = \{(0,1), (1,2), (1,3), (2,4)\}$

$\boxed{4.2b}$

To eliminate unary constraints you can adjust the domain of the variable, in such a way that makes the unary constraint unnecessary.

Example:

$D(A) = \{0, 1, 2\}$ with constraint $A \neq 1$

then we can change the domain to $D(A) = \{0, 2\}$

4.3

Joker (J): Joker tells the truth

Penguin (P): Penguin tells the truth

Riddler (R): Riddler tells the truth

- Joker: Penguin tells the truth

  (If the joker tells the truth, then the Penguin tells the truth)

$$J \equiv P$$

- Penguin: Riddler lies

  (If the Penguin tells the truth, then the Riddler lies)

$$P \equiv \overline{R}$$

- Riddler: You either both tell the truth, or both lie

  (If the Riddler tells the truth, then either both tell the truth, or both lie.)

$$R \equiv (J \equiv P)$$

J	P	R	$\overline{R}$	$J \equiv P$	$P \equiv \overline{R}$	$R \equiv (J \equiv P)$	$(J \equiv P) \wedge$ $(P \equiv \overline{R}) \wedge$ $(R \equiv (J \equiv P))$
0	0	0	1	1	0	0	0
0	0	1	0	1	1	1	1
0	1	0	1	0	1	1	0
0	1	1	0	0	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	1	0	0
1	1	0	1	1	1	0	0
1	1	1	0	1	0	1	0

The variables that satisfies the condition:

$J = 0$

$P = 0$

$R = 1$

4.4

Using Tableaux-calculus:

Inference (ctd.)

- Important properties:
  - *Soundness*:
    - i is sound if $KB \vdash_i \alpha$ implies $KB \models \alpha$.
  - *Completeness*:
    - i is complete if $KB \models \alpha$ implies $KB \vdash_i \alpha$.
- Many different sound and complete proof systems for various logics have been defined in the literature, like
  - Hilbert-type systems,
  - sequent-type calculi,
  - tableau calculi,
  - resolution calculi,
  - natural deduction systems, etc.

☞ Important in computer science are sequent-type calculi, tableau calculi, and resolution calculi.

• Prove $A, B \models A \wedge B$

(1) t: A, B Annahme

(2) f: $A \wedge B$ Annahme

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(3) t: A von 1

(4) t: B von 1

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(5) f: A von 2	(6) f: B von 2

Contradiction 3/5

• Prove $A, A \vee B \not\models B$

(1) t: A Annahme

(2) t: $A \vee B$ Annahme

(3) t: $\neg B$ Annahme

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(4) t: A von 2	(5) t: B von 2
(6) f: B von 3	

Contradiction 5/6

• Prove $A \vee B, A \to C, B \to D \models C \vee D$

(1) t: $A \vee B$

(2) t: $A \to C$

(3) t: $B \to D$

(4) f: $C \vee D$

---

(5) f: C von 4

(6) f: D von 4

---

(7) f: A von 2	(8) t: C von 2	
	(9) f: B von 3	(10) t: D von 3

Contradiction 6/10

$\boxed{4.5a}$

1. $\forall x (\exists y\, P(y) \wedge K(x,y))$

2. $\forall x (E(x) \supset P(x))$

3. $\exists x (\neg E(x) \wedge P(x)$

4. $\exists x (P(x) \wedge \forall y (E(y) \supset K(x,y)))$

OR

$\exists x (P(x) \wedge \neg \exists y (K(x,y) \wedge E(y)))$

$\boxed{4.5b}$

$D = \{a, b\}$

$P(a) = 1 \quad P(b) = 1$

$E(a) = 0 \quad E(b) = 1$

$K(a,b) = 0 \quad K(b,a) = 1$

1. b knows a, a passed ✓ a does not know b
2. b solved all the exercises and passed the exam ✓
   a did not solve all the exercises ✓
3. a did not solve all the exercises, but passed the exam ✓
4. a passed the exam. He does not know b (who solved all the exercises) ✓

$\boxed{4.6}$

THIS IS WRONG!

Because the formula is in CNF, we can write it down in short form:

$$(\bar{x}\,\bar{y}\,\bar{z})(\bar{x}\,y\,\bar{z})(x\,\bar{y}\,z)(\bar{x}\,\bar{y}\,z)(x\,\bar{y}\,\bar{z})(x\,y\,\bar{z})z$$

Because we have the simple literal z, we know that in order to satisfy the formula we know $z=1$ must be set.

partially resolve using $z=1$

$$\underbrace{(\bar{x}\,\bar{y}\,\bar{z})}_{(\bar{x}\bar{y})}\underbrace{(\bar{x}\,y\,\bar{z})}_{(\bar{x}y)}\underbrace{(x\,\bar{y}\,z)}_{1}\underbrace{(\bar{x}\,\bar{y}\,z)}_{1}\underbrace{(x\,\bar{y}\,\bar{z})}_{(x\bar{y})}\underbrace{(x\,y\,\bar{z})}_{(xy)}\underbrace{z}_{1}$$

$$(\bar{x}\bar{y})(\bar{x}y)(x\bar{y})(xy)$$

Consider $(\bar{x}\bar{y})(xy)$: To satisfy this part we must set $x \neq y$. So we can only interpret x/y as $\begin{matrix} x=0 \\ y=1 \end{matrix}$ OR $\begin{matrix} x=1 \\ y=0 \end{matrix}$ $\Rightarrow (x,y) = \{(0,1), (1,0)\}$

Consider $(\bar{x}y)(x\bar{y})$: To satisfy this part we must set $x = y$. So we can only interpret x/y as $\begin{matrix} x=0 \\ y=0 \end{matrix}$ OR $\begin{matrix} x=1 \\ y=1 \end{matrix}$ $\Rightarrow (x,y) = \{(0,0), (1,1)\}$

These values contradict each other $\Rightarrow$ no possible solution.