25 - 4 - 17 - Aufgabe 2

$n=40$

$$\tilde{x} = \frac{x_{(k)} + x_{(k+1)}}{2}$$ n gerade

$$\tilde{x} = \frac{72+72}{2} = 72 \quad Q_2$$

$$\text{lower Hinge} = \frac{64+64}{2} = 64 \quad Q_1$$

$$\text{upper Hinge} = \frac{76+76}{2} = 76 \quad Q_3$$

$$LF = Q_1 - 1{,}5\,(Q_3 - Q_1)$$
$$= 64 - 1{,}5\,(76-64)$$
$$= 46$$

$$UF = Q_3 + 1{,}5\,(Q_3 - Q_1)$$
$$UF = 76 + 1{,}5 \cdot (76-64)$$
$$= 94$$

25 - 4 - 17 - Aufgabe 3

$$F(x) = \frac{x^2+2}{6} = \frac{x^2}{6} + \frac{2}{6}$$

$$f(x) = F(x)' = \frac{2x}{6} = \frac{x}{3}$$

$$f(x) = \begin{cases} \frac{x}{3} & 0 \leq x \leq 2 \\ 0 & \text{sonst} \end{cases}$$

$$E(x) = \int_0^2 \left(x \cdot \frac{x}{3}\right) dx = \int_0^2 \left(\frac{x^2}{3}\right) dx = \frac{x^3}{3 \cdot 3} \Big|_0^2$$

$$\frac{2^3}{9} - 0 = \frac{8}{9}$$

$$\text{Varianz} = E(x^2) - [E(x)]^2$$

$$E(X^2) = \int_0^2 \left(x^2 \cdot \frac{x}{3}\right) dx = \int_0^2 \left(\frac{x^3}{3}\right) dx = \frac{x^4}{3 \cdot 4} \Big|_0^2$$

$$= \frac{2^4}{12} = \frac{16}{12} = \frac{4}{3}$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{4}{3} - \left(\frac{8}{9}\right)^2 = 0{,}5432$$

Aufgabe 4 – 25-4-17

$$F_{max}(y) = \prod_{i=1}^{n} F_i(y) \qquad Y_n = \max\{X_1, X_2\}$$

$$F_{max} = (1-e^{-x})(1-e^{-x}) = 1 - e^{-x} + e^{-2x} - e^{-x}$$

$$= 1 - 2e^{-x} + e^{-2x}$$

$$F_{min}(y) = 1 - \prod_{i=1}^{n} [1 - F_i(y)]$$

$$= 1 - \left[1 - (1 - 2e^{-x} + e^{-2x})\right] \cdot (1 - (1 - e^{-x}))$$

$$= 1 - \left[(2e^{-x} - e^{-2x}) \cdot (1 - (1 - e^{-x}))\right]$$

$$1 - \left[(2e^{-x} - e^{-2x}) \cdot e^{-x}\right]$$

$$1 - (2e^{-2x} - e^{-3x})$$

$$1 - 2e^{-2x} + e^{-3x}$$

Dichte: $f(x) \; -2 \cdot (-2) e^{-2x} + (-3) e^{-3x} = +4e^{-2x} - 3e^{-3x}$

Aufgabe 4 - 25-4-17

Dichte: $f(x) = 4e^{-2x} - 3e^{-3x}$

$$E(x) = \int_0^\infty x \left(4e^{-2x} - 3e^{-3x}\right) dx =$$

$$= 2\int_0^\infty x \cdot \underbrace{2e^{-2x}}_{\lambda = \frac{1}{2}} dx - \int_0^\infty x \cdot \underbrace{3e^{-3x}}_{\lambda = \frac{1}{3}} dx$$

$$E(x) = 2 \cdot \frac{1}{2} - \frac{1}{3} = \frac{2}{3}$$

25-4-17 – Aufgabe 5

Radius eines Kreises.

$A = r^2 \pi \qquad f_R = \begin{cases} \frac{1}{3} & 1 < r < 3 \\ 0 & \text{sonst} \end{cases}$

$E(Y) = E(r^2\pi) = \int_1^3 \left(r^2\pi \cdot \frac{1}{3}\right) dr = \frac{r^3}{3} \frac{\pi}{3} \Big|_1^3$

$\frac{3^3}{3} \frac{\pi}{3} - \frac{1^3}{3} \frac{\pi}{3} = \frac{26\pi}{9}$

R – Command

$\tau = 3 \qquad \boxed{E(x) = \frac{1}{\lambda} = \tau}$

$\lambda = \frac{1}{\tau}$

$F(x) = 1 - e^{-\lambda x}$

$y = 1 - e^{-\lambda x}$

$y - 1 = -e^{-\lambda x}$

$-y + 1 = e^{-\lambda x} \qquad | \ln$

$\ln(1-y) = -\lambda x \qquad | : (-\lambda)$

$\ln(1-y) = -\frac{1}{3} x \qquad | \cdot (-3)$

$-3 \ln(1-y) = x$

Aufgabe 6 – 25-4-17

$N(\mu, \sigma^2) \qquad x \sim N(100, 5)$

$\qquad\qquad\quad y \sim N(120, 10)$

$D \sim Y - X$

$D \sim N(120 - 100; 10 + 5) = D \sim N(20; 15)$

$E(Y - X) = E(\cancel{X}) - E(X)$

$Var(Y - X) = V(Y + (-1)(X)) = V(Y) + V((-1) \cdot X) = V(Y) + (-1)^2 \cdot V(Y)$

Aufgabe 6 – 25-4-17

$$\boxed{P(Y_n \leq x) \rightarrow \phi\left(\frac{X-\mu}{\sigma}\right)}$$

$\mu = 3 \quad \sigma = 2$

$$\sum_{i=1}^{16} x_i \sim N(n\mu ; n\sigma^2) \qquad (3 \cdot 16 ; 16 \cdot 2^2 \qquad n = 16$$

$$N(48 ; 64) \qquad \mu = 48 \quad \sigma^2 = 64 \quad \sigma = 8$$

$$P(X > 60) = 1 - P(X < 60)$$

$$1 - \phi\left(\frac{60-48}{8}\right) \quad 1 - \phi\left(\frac{12}{8}\right) = 1 - \phi(1{,}5)$$

$$= 1 - 0{,}9332 = 0{,}0668$$

Aufgabe 7 – 25-4-17

$H_0 : \mu = \mu_0$

$\mu = 10$

$$\boxed{T_0 = \frac{\bar{X}_n - \mu_0}{S_n \,/\, \sqrt{n}}} = \frac{11{,}79 - 10}{2{,}773 \,/\, \sqrt{8}} = \frac{1{,}79}{0{,}980}$$

$$T_0 = 1{,}825779$$

$$t_{n-1;1-\alpha} = t_{8-1;1-0{,}05} = t_{7;0{,}95} = 1{,}895$$

$$\boxed{T_0 > t_{n-1;1-\alpha} \rightarrow H_0 \text{ verwerfen}}$$

$1{,}825779 > 1{,}895$ ↯ → H_0 nicht verwerfen

Aufgabe 7 – 25-4-17

$$\boxed{S_p^2 = \frac{(m-1)\, s_x^2 + (n-1)\, s_y^2}{m+n-2}}$$

$$S_p^2 = \frac{(8-1)\, 7{,}687 + (10-1) \cdot 10{,}19}{8+10-2} = \frac{145{,}519}{16} = 9{,}0949$$

$$\boxed{\bar{X} - \bar{Y} \pm t_{m+n-2;\,1-\alpha/2} \cdot S_p \sqrt{\frac{1}{m} + \frac{1}{n}}}$$

$$11{,}79 - 15{,}14 \pm t_{16;\,1-0{,}05/2} \cdot \sqrt{9{,}0949} \cdot \sqrt{\frac{1}{10} + \frac{1}{8}}$$

$$-3{,}35 \pm t_{16;\,0{,}975} \cdot 1{,}4305$$

$$-3{,}35 \pm 2{,}120 \cdot 1{,}4305$$

$$[-6{,}38 \, ; \, -0{,}317]$$

Aufgabe 8 – 25-4-17

$\hat{p} = 0{,}45$

$n = 300$

$\alpha = 0{,}05$

$$\hat{p} \pm z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0{,}45 \pm z_{0{,}975} \sqrt{\frac{0{,}45\,(1-0{,}45)}{300}}$$

$$0{,}45 \pm 1{,}96 \cdot 0{,}02872$$

$$[0{,}3937 \, ; \, 0{,}5062]$$

Aufgabe 8 - 25 - 4 - 17

Experiment:

	beob	erwart
Typ 1	42	100 · 0,35 = 35
Typ 2	37	40
Typ 3	21	25
Σ	100	100

$$\frac{100}{7+8+5} = 5$$

$7 \cdot 5 = 35$
$8 \cdot 5 = 40$
$5 \cdot 5 = 25$

$$\sum \frac{(\text{beobachtet} - \text{erwartet})^2}{\text{erwartet}} = \frac{(42-35)^2}{35} + \frac{(37-40)^2}{40} + \frac{(21-25)^2}{25}$$

$$Q_{k-1} = \frac{49}{35} + \frac{9}{40} + \frac{16}{25} = 2{,}265$$

$$\chi^2_{k-1;1-\alpha} \quad \chi^2_{3-1;1-0{,}05} = \chi^2_{2;0{,}95} = 5{,}991$$

$Q_{k-1} > \chi^2_{k-1;1-\alpha} \rightarrow H_0$ verwerfen

$2{,}265 > 5{,}991$ ↯ $\rightarrow H_0$ nicht verwerfen