Übung 2

1)

$$\mathbb{P}(x = X) = \binom{3}{x}\left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{3-x}$$

$$\mathbb{P}(y = Y) = \left(\frac{1}{2}\right)^Y$$

$$Z = \mathbb{1}_{M=K} Y + \mathbb{1}_{M=Z} X = \frac{1}{3}\left(\frac{1}{2}\right)^z + \left(1 - \frac{1}{3}\right)\binom{3}{x}\left(\frac{1}{3}\right)^z \left(\frac{2}{3}\right)^{3z}$$

$$F_x(x) = P(X \leq x) = \sum_{k=0}^{\lfloor x \rfloor} \binom{3}{x}\left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{3-x}$$

$$F_Y(y) = P(Y \leq y) = \sum_{k=1}^{\lfloor y \rfloor} \left(\frac{1}{2}\right)^k = 1 - \left(\frac{1}{2}\right)^{\lfloor y \rfloor + 1}$$

$$F_Z(z) = \mathbb{P}(Z \leq z) = \mathbb{P}(Z \leq z, M = K) + \mathbb{P}(Z \leq z, M = Z)$$

$$= \underbrace{\frac{\mathbb{P}(Z \leq z, M = K)\,\mathbb{P}(M=K)}{\mathbb{P}(M=K)}}_{\mathbb{P}(Z \leq z \mid M = K)} + \underbrace{\frac{\mathbb{P}(Z \leq z \mid M = Z)\,\mathbb{P}(M=Z)}{\mathbb{P}(M=Z)}}_{\mathbb{P}(Z \leq z \mid M = Z)} =$$

$$= \mathbb{P}(X \leq Z)\,\frac{1}{3} + \mathbb{P}(Y \leq Z)\,\frac{2}{3}$$

2)

$E_1 \ldots k_1$ Bälle in N_1

$E_2 \ldots k_2$ Bälle in N_2

$$P(E_1) = \binom{k}{k_1} \left(\frac{1}{N}\right)^{k_1} \cancel{\left(\frac{N-1}{N}\right)^{k-k_1}} \left(1 - \frac{1}{N}\right)^{k-k_1}$$

$$P(E_2) = \binom{k}{k_2} \left(\frac{1}{N}\right)^{k_2} \left(1 - \frac{1}{N}\right)^{k-k_2}$$

$$P(E_1 \cap E_2) = \binom{k}{k_1\; k_2\; k-(k_1+k_2)} \left(\frac{1}{N}\right)^{k_1} \left(\frac{1}{N}\right)^{k_2} \left(1 - \frac{2}{N}\right)^{k-(k_1+k_2)}$$ Multinominale Verteilung

$$\Downarrow$$

$$P(E_1)\,P(E_2) \neq P(E_1 \cap E_2)$$

3)

Wenn Anzahl der Möglichkeiten $\rightarrow \infty$, dann wird Binomial und Multinomial zu Poisson-Verteilung. $k = \rho N$

$$\Rightarrow P(E_1) = \frac{(\rho)^{k_1}}{k_1!} e^{-\rho}$$

$$\Rightarrow P(E_2) = \frac{(\rho)^{k_2}}{k_2!} e^{-\rho}$$

$$\Rightarrow P(E_1 \cap E_2) = \frac{(\rho)^{k_1}}{k_1!} \cdot \frac{(\rho)^{k_2}}{k_2!} \cdot e^{-2\rho}$$

$$\Downarrow$$

$$P(E_1 \cap E_2) = P(E_1)\,P(E_2)$$

4) P_0 ... Pokemon $\quad$ P_1 ... Person 1 $\quad$ P_2 ... Person 2

A ... Bisasam
B ... Glumanda
C ... Schiggy

$P(A) = P(B) = P(C) = \frac{1}{3}$

$P_0 : \Omega \to \{A, B, C\}$

$$\mathbb{P}(P_0(P_1) = P_0(P_2) = A) = p_A^2$$

$$\Downarrow$$

$$\sum_{i \in \{A,B,C\}} p_i^2 = p_A^2 + p_B^2 + p_C^2$$

$$\Downarrow$$

$$\mathbb{P}(P_0(P_1) = P_0(P_2)) = \mathbb{P}(P_0(P_1) = P_0(P_2) = A) + \mathbb{P}(P_0(P_1) = P_0(P_2) = B) + \mathbb{P}(P_0(P_1) = P_0(P_2) = C) = p_A^2 + p_B^2 + p_C^2$$

=> Maximum wenn zB $p_A = 1, p_B = 0, p_C = 0 \Rightarrow p = 1$

=> Minimal wenn alle gleich $p_A = \frac{1}{3}, p_B = \frac{1}{3}, p_C = \frac{1}{3} \Rightarrow p = \frac{1}{3}$

5)

$A = \{(n, m) : n \leq 50\}$

$B = \{(n, m) : m \geq 10\}$

$$P(A) = P(n \leq 50) = \sum_{n=1}^{50} P_1(n)$$

$$P(B) = P(m \geq 10) = \sum_{m=10}^{100} P_2(m)$$

$$P(A \cap B) = \sum_{n=1}^{50} \sum_{m=10}^{100} P(n) \cdot P(m)$$

$$\Downarrow$$

$$P(A) \cdot P(B) = P(A \cap B)$$

6)

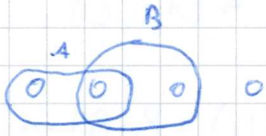
$A = \{(n,m): n+m \leq 20\}$

$B = \{(n,m): m \geq 10\}$

$P_1 = \frac{1}{2}(\delta_k + \delta_L)$

$P_2 = \frac{1}{2}(\delta_{k'} + \delta_{L'})$

Beispiel durch Ausprobieren



$k = 5 \qquad L = 20$

$k' = 5 \qquad L' = 10$

$\mathbb{P}(A) = \{(5,5), (5,10)\} = \frac{1}{2}$

$\mathbb{P}(B) = \{(5,10), (20,10)\} = \frac{1}{2}$

$\mathbb{P}(A \cap B) = \frac{1}{4}$